Problem Set 3

Problem 3.1 Center of Gravity for a Bent Rod

Determine the distance x_C and y_C to the center of gravity of the bent rod.

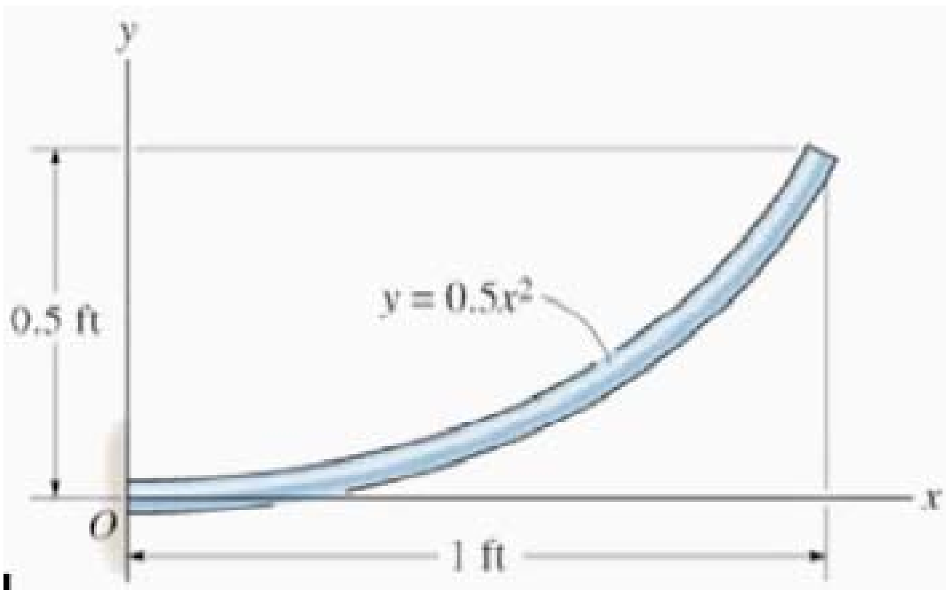


Figure P9.1: Problem 9.1

Problem 3.2 Centroid of a Composite Section

Determine the location y_C of the centroid of the beam's cross sectional area. Neglect the size of the corner welds at A and B for the calculation.

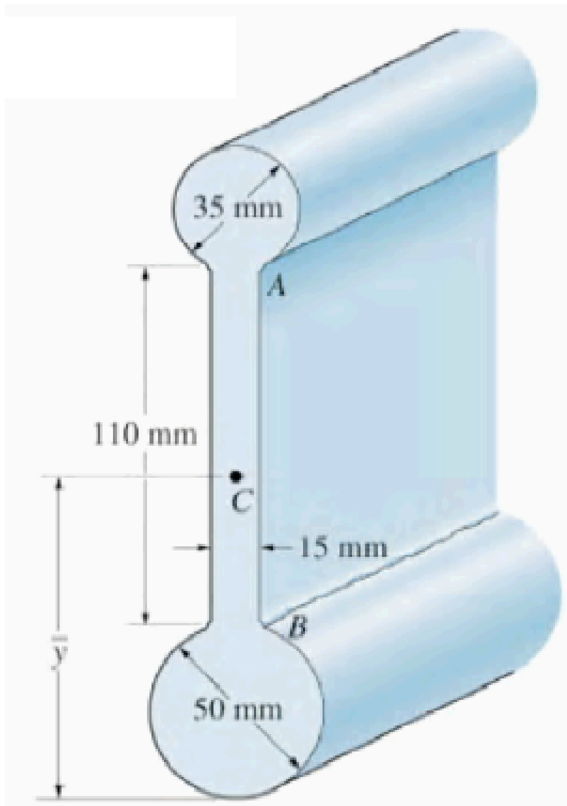


Figure P9.2: Problem 9.2

Problem 3.3 Centroid of a Tapered Cross Section

Locate the centroid y_C of the concrete beam with the tapered cross section shown.

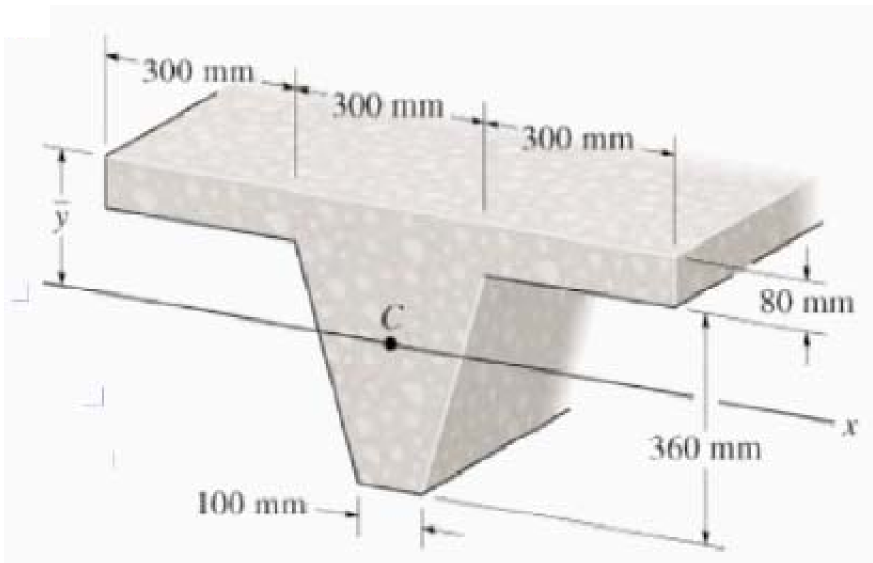


Figure P9.3: Problem 9.3

Problem 3.4

Find the x-coordinate of the centroid of the indicated region where $A = 2$ m and $k = \pi/8$ m^{-1}.

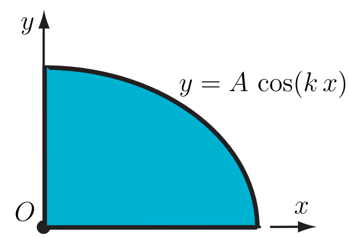


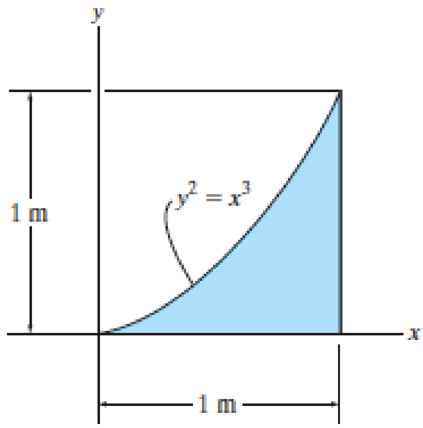
Figure 3.1: Problem 3.1

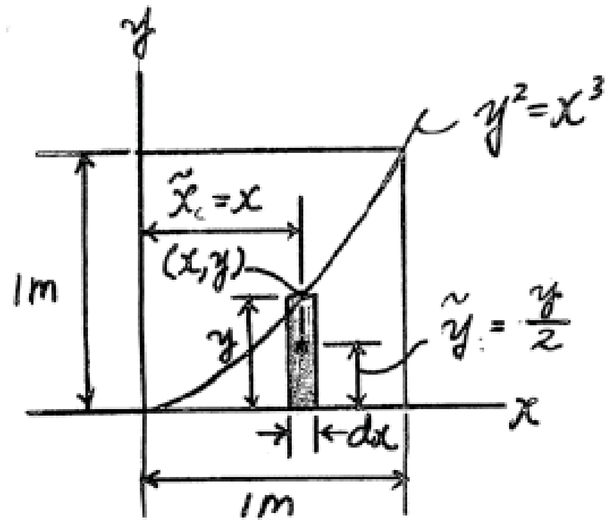
Solution

```
A = 2; % m
k = pi/8; % m^(-1)
% y(x) = A*cos(k*x);
% intersection of y with x-axis
x0 = (pi/2)/k;
% x0 = 4 m

% Area = int(dx dy) where 0<x<x0 and 0<y<A*cos(k*x)
% Ay = int(dy) where 0<y<A*cos(k*x))
Ay = int(1,y,0,A*cos(k*x));
% Ay = 2*cos((pi*x)/8)
% Area = int(Ay dx) where 0<x<x0
Area = int(Ay,x,0,x0);
% Area = 16/pi (m^2)

% first moment of area about y-axis
% My = int(x dx dy) where 0<x<x0 and 0<y<A*cos(k*x)
% Qyy = int(dy) ; 0<y<A*cos(k*x)
Qyy = int(1,y,0,A*cos(k*x));
% My = int(x Qyy dx) where 0<x<x0
My = int(x*Qyy,x,0,x0);
% My = (64*(pi - 2))/pi^2 (m^3)
% centroid xC = My/Area
xC = My/Area;
```

```
% xC = (4*(pi - 2))/pi = 1.454 (m)

% first moment of area about x-axis
% Mx = int(y dx dy) where 0<x<x0 and 0<y<A*cos(k*x)
% Qxy = int(y dy) ; 0<y<A*cos(k*x)
Qxy = int(y,y,0,A*cos(k*x));
% Mx = int(x Qxy dx) where 0<x<x0
Mx = int(x*Qxy,x,0,x0);
% Mx = 8 - 32/pi^2 (m^3)
yC = Mx/Area;
% yC = -(pi*(32/pi^2 - 8))/16 = 0.934 (m)
```

Problem 3.5
Determine the area and the centroid of the area.



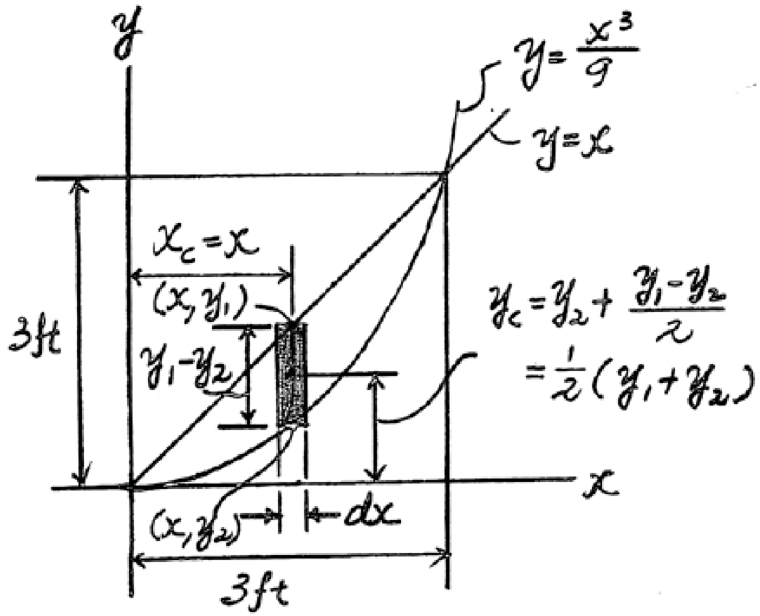


Solution

```
% y(x) = x^(3/2)
% where 0<x<a and 0<y<a
a = 1; % m

% differential element
% dA = y dx = x^(3/2) dx
% centroid of the differential element is at
% x and y/2 = (1/2)  x^(3/2)

% A = int(dA) where 0<x<a
A =  int( x^(3/2),x,0,a);
% A = 0.400 (m^2)

% My = int(x dA) where 0<x<a
My = int(x*x^(3/2),x,0,a);
xC = My/A;
% xC = 0.714 (m)
```

```
% Mx = int(y/2 dA) where 0<x<a
Mx = int((1/2)*x^(3/2)*x^(3/2),x,0,a);
yC = Mx/A;
% yC = 0.312 (m)
```

Problem 3.6
Determine the area and the centroid of the area.



Solution

```
% y1(x) = x
% y2(x) = (x^3)/9
% where 0<x<a and 0<y<a
a = 3; % ft

% differential element
% dA = (y1-y2) dx = (x-(x^3)/9) dx
% centroid of the differential element is at
% x and y = (y1+y2)/2 = (x+(x^3)/9)/2

% A = int(dA) where 0<x<a
A =  int(x-(x^3)/9,x,0,a);
% A = 2.250 (ft^2)

% My = int(x dA) where 0<x<a
```

```
My = int(x*(x-(x^3)/9),x,0,a);
xC = My/A;
% xC = 1.600 (ft)

% Mx = int((y1+y2)/2 dA) where 0<x<a
Mx = int((1/2)*(x+(x^3)/9)*(x-(x^3)/9),x,0,a);
yC = Mx/A;
% yC = 1.143 (ft)
```

Problem 3.7

Determine the location of the centroid C of the area where $a = 6$ in, $b = 6$ in, $c = 3$ in, and $d = 6$ in.

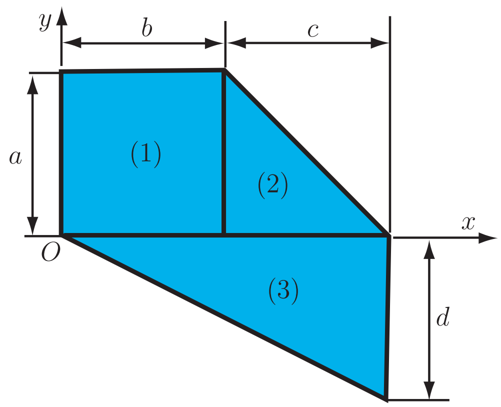


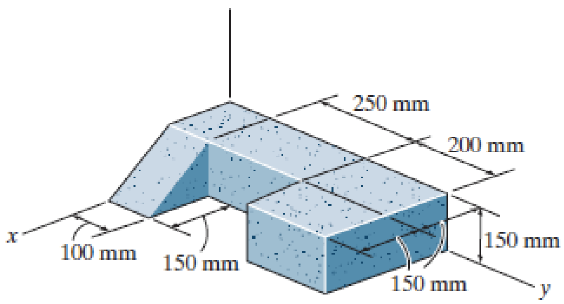
Figure 3.7: Problem 3.7

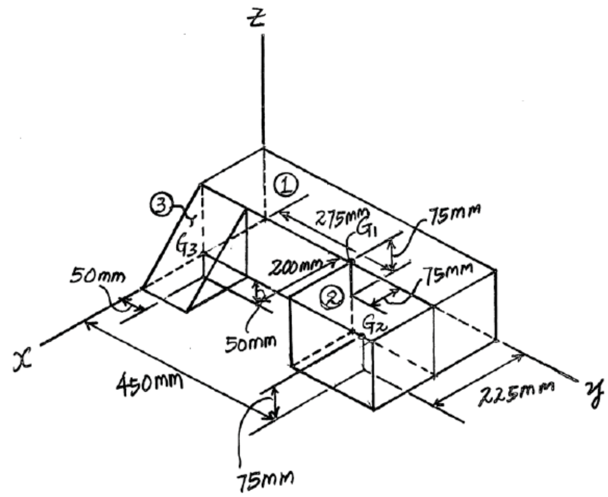
Solution

```
a = 6; % in
b = 6; % in
c = 3; % in
d = 6; % in

% rectangle 1
A1 = a*b;
x1 = b/2;
y1 = a/2;

% triangle 2
A2 = c*a/2;
x2 = b + c/3;
y2 = a/3;

% triangle 3
A3 = (b+c)*d/2;
x3 = (2/3)*(b+c);
y3 = -d/3;
```

```
A = A1 + A2 + A3;

% centroid C(xC, yC)
xC = (x1*A1 + x2*A2 + x3*A3)/A;
yC = (y1*A1 + y2*A2 + y3*A3)/A;

% xC = 4.625 (in)
% yC = 1.000 (in)
```

Problem 3.8
Locate the center of mass of the homogeneous block assembly





Solution

```
% rectangular prism 1
V1 = 150*150*550; % mm^3
x1 = 75;          % mm
y1 = 275;         % mm
z1 = 75;          % mm

% rectangular prism 2
V2 = 150*150*200; % mm^3
x2 = 225;         % mm
y2 = 450;         % mm
z2 = 75;          % mm

% triangular prism 3
V3 = 150*150*100/2; % mm^3
x3 = 200;           % mm
y3 = 50;            % mm
z3 = 50;            % mm

% total area
```

```
V = V1 + V2 + V3;

xC = (x1*V1+x2*V2+x3*V3)/V;
yC = (y1*V1+y2*V2+y3*V3)/V;
zC = (z1*V1+z2*V2+z3*V3)/V;

% V = 1.8e+07 (mm^3)
% xC = 120.312 (mm)
% yC = 304.688 (mm)
% zC = 73.438 (mm)
```

Problem 3.9

Locate the centroid of the paraboloid defined by the equation $x/a = (y/b)^2$ where $a = b = 4$ m. The material is homogeneous.

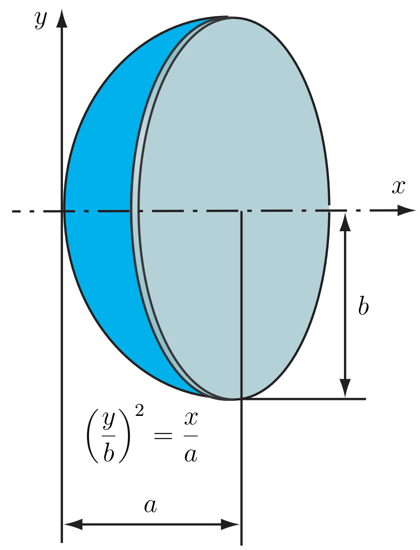


Figure 3.6: Problem 3.6

Solution

```
a = 4; % m
b = 4; % m

y = b*sqrt(x/a);

% dV = pi*y^2 dy
V = int(pi*y^2, x, 0, a);
% V = 32 pi (m^3)

% xC = (1/V) int(x dV)
xC = int(x*pi*y^2, x, 0, a)/V;
% xC = 2.667 (m)
```

Problem 3.10

A circular V-belt has an inner radius $r = 600$ mm and a cross-sectional area with the dimensions $a = 25$ mm, $b = 50$ mm, and $c = 75$ mm. Determine the volume of material required to make the belt.

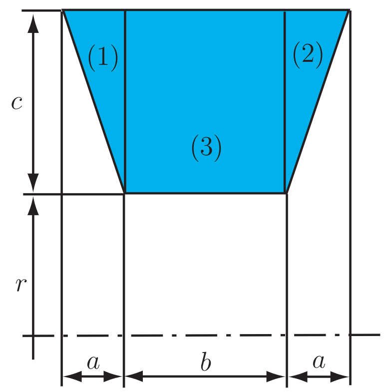


Figure 3.10: Problem 3.10

Solution

```
% Guldinus-Pappus theorem
% V = 2 pi yC A

r = 0.600; % m
a = 0.025; % m
b = 0.050; % m
c = 0.075; % m

% triangle 1
A1 = a*c/2;
y1 = r + (2/3)*c;

% triangle 2
A2 = A1;
y2 = y1;

% rectangle 3
A3 = b*c;
```

```
y3 = r + c/2;

V = 2*pi*(A1*y1+A2*y2+A3*y3);
% V = 0.0226784  (m^3)
```