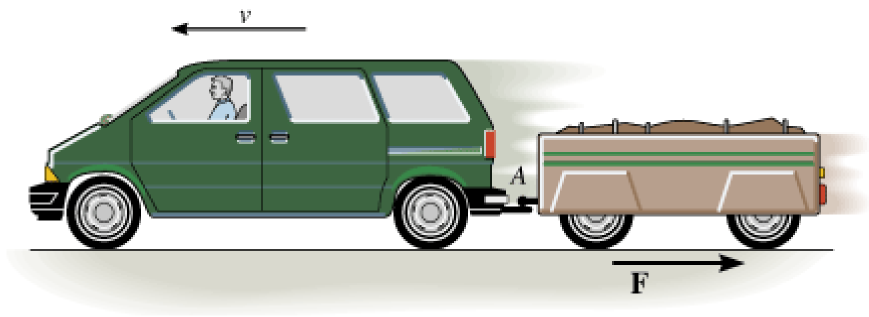
The van is travelling at velocity v when the coupling of the trailer at A fails. If the trailer has mass M and coasts a distance d before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



Given:

$\mathbf{v} := 20\,\mathbf{kph}$

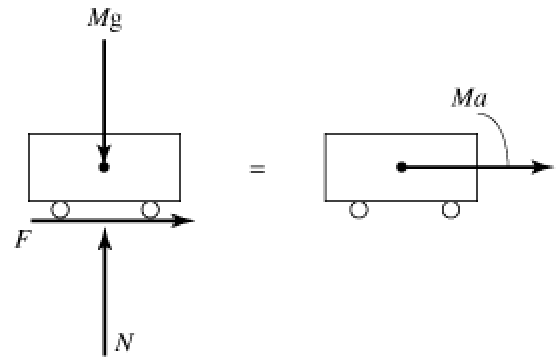
$\mathbf{M} := 250\,\mathbf{kg}$

$\mathbf{d} := 45\,\mathbf{m}$

Solution:

$$\mathbf{v}^2 = \mathbf{v_o}^2 + 2\cdot \mathbf{a_c}\cdot\left(\mathbf{d} - \mathbf{d_o}\right)$$

$$\mathbf{a_c} := \frac{1}{2}\cdot\frac{\mathbf{v}^2 - 0}{\mathbf{d} - 0} \qquad \mathbf{a_c} = 0.3429\,\frac{\mathbf{m}}{\mathbf{s}^2}$$



$$\xrightarrow{+}\ \Sigma \mathbf{F_x} = \mathbf{M}\cdot\mathbf{a_c} \qquad \mathbf{F} := \mathbf{M}\cdot\mathbf{a_c} \qquad \mathbf{F} = 85.7\ \mathbf{N}$$

The position of a particle is defined by $\mathbf{r} = \{a\cos(bt)\,\mathbf{i} + c\sin(bt)\,\mathbf{j}\}$. Determine the magnitudes of the velocity and acceleration of the particle when $t = t_1$. Also, prove that the path of the particle is elliptical.

Given : $a := 5\,m$ $\quad b := 2\cdot\frac{rad}{s}$ $\quad c := 4\cdot m$ $\quad t_1 := 1\,s$

Velocities

$v_{x1} := -a\cdot b\cdot \sin(b\cdot t_1)$ $\quad v_{y1} := c\cdot b\cdot \cos(b\cdot t_1)$ $\quad v_1 := \sqrt{v_{x1}^2 + v_{y1}^2}$

$v_{x1} = -9.09\,\frac{m}{s}$ $\quad v_{y1} = -3.33\,\frac{m}{s}$ $\quad v_1 = 9.68\,\frac{m}{s}$

Accelerations

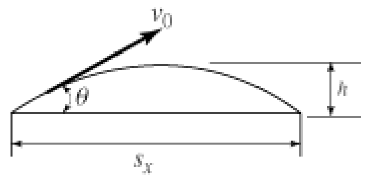
$a_{x1} := -a\cdot b^2\cdot \cos(b\cdot t_1)$ $\quad a_{y1} := -c\cdot b^2\cdot \sin(b\cdot t_1)$ $\quad a_1 := \sqrt{a_{x1}^2 + a_{y1}^2}$

$a_{x1} = 8.32\,\frac{m}{s^2}$ $\quad a_{y1} = -14.55\,\frac{m}{s^2}$ $\quad a_1 = 16.76\,\frac{m}{s^2}$

Path

$\frac{x}{a} = \cos(b\cdot t)$ $\quad \frac{y}{c} = \sin(b\cdot t)$ $\quad$ **Thus** $\quad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{c}\right)^2 = 1$ $\quad$ **QED**

The nozzle of a garden hose discharges water at the rate v_0. If the nozzle is held at ground level and directed at angle θ from the ground, determine the maximum height reached by the water and the horizontal distance from the nozzle to where the water strikes the ground.



Given: $v_0 := 15\frac{m}{s}$ $\theta := 30deg$ $g := 9.81\frac{m}{s^2}$

Solution:

$a_x = 0$ $a_y = -g$

$v_x = v_0 \cdot \cos(\theta)$ $v_y = -g \cdot t + v_0 \cdot \sin(\theta)$

$s_x = v_0 \cdot \cos(\theta) \cdot t$ $s_y = \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$

Maximum height $0 = -g \cdot t + v_0 \cdot \sin(\theta)$ $t := \frac{v_0 \cdot \sin(\theta)}{g}$ $t = 0.76\ s$

$h := \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$ $h = 2.87\ m$

Horizontal distance $0 = \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$ $t := \frac{2 \cdot v_0 \cdot \sin(\theta)}{g}$ $t = 1.53\ s$

$d := v_0 \cdot \cos(\theta) \cdot t$ $d = 19.86\ m$

The position of a particle along a straight line is given by $s_p = at^3+bt^2+ct$. Determine its maximum acceleration and maximum velocity during the time interval $0 <= t <= t_f$.

Given: $\mathbf{a} := 1\frac{\mathbf{ft}}{\mathbf{s}^3}$ $\mathbf{b} := -9\cdot\frac{\mathbf{ft}}{\mathbf{s}^2}$ $\mathbf{c} := 15\cdot\frac{\mathbf{ft}}{\mathbf{s}}$ $\mathbf{t_f} := 10\mathbf{s}$ $\mathbf{t_0} := 0\mathbf{s}$

Solution:

$$s_p = a\cdot t^3 + b\cdot t^2 + c\cdot t$$

$$v_p = \frac{ds_p}{dt} = 3\cdot a\cdot t^2 + 2\cdot b\cdot t + c$$

$$a_p = \frac{dv_p}{dt} = \frac{d^2\cdot s_p}{dt^2} = 6\cdot a\cdot t + 2\cdot b$$

Since the acceleration is linear in time then the maximum will occur at the start or at the end.
We check both possibilities.

$$a_{max} := \max\left(6\cdot a\cdot t_0 + b, 6\cdot a\cdot t_f + 2\cdot b\right)$$

$$a_{max} = 42\frac{ft}{s^2}$$

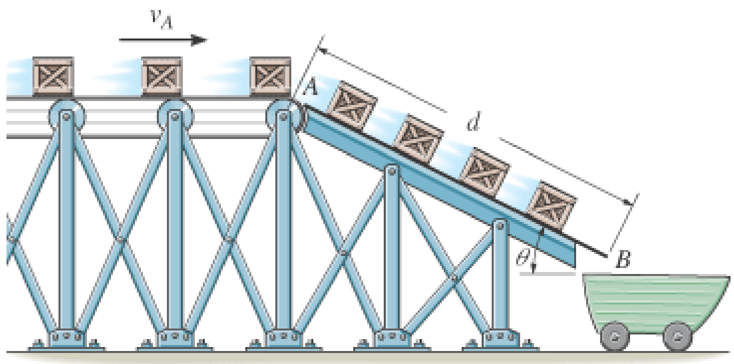
The maximum velocity can occur at the beginnig, at the end, or where the acceleration is zero.
We will check all three locations.

$$t_{cr} := \frac{-b}{3\cdot a} \qquad t_{cr} = 3\ s$$

$$v_{max} := \max\left(3\cdot a\cdot t_0^{\ 2} + 2\cdot b\cdot t_0 + c, 3\cdot a\cdot t_f^{\ 2} + 2\cdot b\cdot t_f + c, 3\cdot a\cdot t_{cr}^{\ 2} + 2\cdot b\cdot t_{cr} + c\right)$$

$$v_{max} = 135\frac{ft}{s}$$

The conveyor belt delivers each crate of mass M to the ramp at A such that the crate's speed is v_A directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is μ_k, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.



Given:

$$\mathbf{M} := 12\mathbf{kg}$$

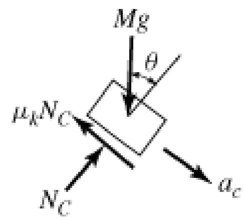
$$\mathbf{v_A} := 2.5\frac{\mathbf{m}}{\mathbf{s}}$$

$$\mathbf{d} := 3\mathbf{m}$$

$$\mu_\mathbf{k} := 0.3$$

$$\theta := 30\mathbf{deg}$$

$$\mathbf{g} := 9.81\frac{\mathbf{m}}{\mathbf{s}^2}$$



Solution:

$$\mathbf{N_C} - \mathbf{M}\cdot\mathbf{g}\cdot\cos(\theta) = 0 \qquad \mathbf{N_C} := \mathbf{M}\cdot\mathbf{g}\cdot\cos(\theta)$$

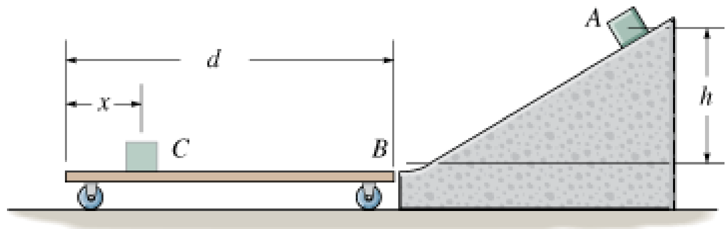
$$\mathbf{M}\cdot\mathbf{g}\cdot\sin(\theta) - \mu_\mathbf{k}\cdot\mathbf{N_C} = \mathbf{M}\cdot\mathbf{a} \qquad \mathbf{a} := \mathbf{g}\cdot\sin(\theta) - \mu_\mathbf{k}\cdot\frac{\mathbf{N_C}}{\mathbf{M}} \qquad \mathbf{a} = 2.36\frac{\mathbf{m}}{\mathbf{s}^2}$$

$$\mathbf{v_B} := \sqrt{\mathbf{v_A}^2 + 2\cdot\mathbf{a}\cdot\mathbf{d}} \qquad \mathbf{v_B} = 4.52\frac{\mathbf{m}}{\mathbf{s}}$$

Box A of weight W is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *fixed from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is μ_κ

Given:

$\mathbf{W} := 30\,\mathbf{lb}$



$\mu_k := 0.6$

$\mathbf{h} := 4\,\mathbf{ft}$

$\mathbf{d} := 10\,\mathbf{ft}$

Solution:

$0 = \mathbf{W} \cdot \mathbf{h} - \mu_k \cdot \mathbf{W} \cdot (\mathbf{d} - \mathbf{x}) = 0$ $\quad \mathbf{x} := \mathbf{d} - \frac{\mathbf{h}}{\mu_k}$ $\quad \mathbf{x} = 3.33\,\mathbf{ft}$

The suitcase of weight W slides down the curved ramp for which the coefficient of kinetic friction is μ_k. If at the instant it reaches point A it has speed v_A determine the normal force on the suitcase and the rate of increase of its speed.

Given:

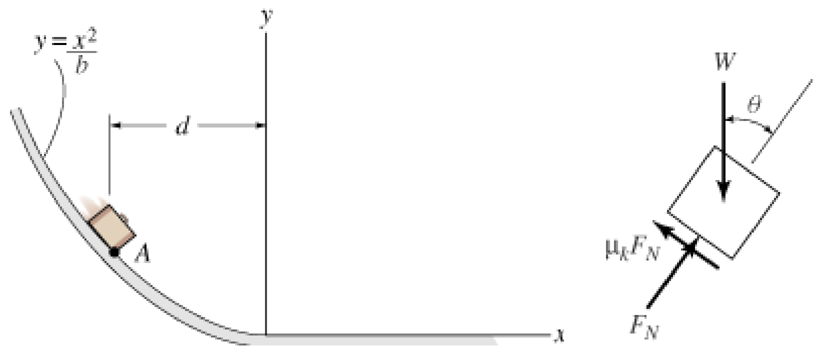
$\mathbf{W} := 10\,\mathbf{lb}$

$\mu_k := 0.2$

$v_A := 5\,\dfrac{\mathbf{ft}}{\mathbf{s}}$

$\mathbf{d} := 6\,\mathbf{ft}$

$\mathbf{b} := 8\,\mathbf{ft}$



Solution : First find the radius of curvature ρ, and angle θ.

$$\rho = \frac{\left[1 + \left(\frac{d}{dx}y\right)^2\right]^{\frac{3}{2}}}{\frac{d^2}{dx^2}y} \qquad \rho := \frac{\left[1 + \left(2\cdot\frac{d}{b}\right)^2\right]^{\frac{3}{2}}}{\frac{2}{b}} \qquad \rho = 23.44\,\mathbf{ft}$$

$$\tan(\theta) = \frac{d}{dx}y = 2\cdot\frac{d}{b} \qquad \theta := \operatorname{atan}\left(2\cdot\frac{d}{b}\right) \qquad \theta = 56.31\,\mathbf{deg}$$

Now do the dynamics **Guesses** $F_N := 1\,\mathbf{lb}$ $\quad v' := 1\,\dfrac{\mathbf{ft}}{\mathbf{s}^2}$

Given $$F_N - W\cdot\cos(\theta) = \frac{W}{g}\cdot\frac{v_A^2}{\rho} \qquad \mu_k\cdot F_N - W\cdot\sin(\theta) = -\frac{W}{g}\cdot v'$$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} := \operatorname{Find}(F_N, v') \qquad F_N = 5.88\,\mathbf{lb} \qquad v' = 22.99\,\frac{\mathbf{ft}}{\mathbf{s}^2}$$