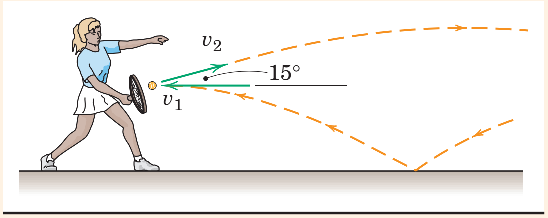
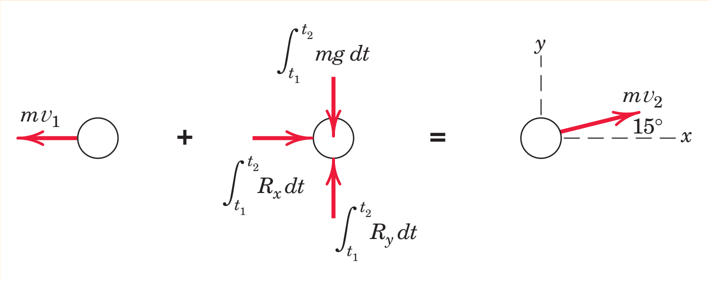
Sample Problem 3/19



A tennis player strikes the tennis ball with her racket when the ball is at the uppermost point of its trajectory as shown. The horizontal velocity of the ball just before impact with the racket is $v_1 = 50$ ft/sec and just after impact its velocity is $v_2 = 70$ ft/sec directed at the 15° angle as shown. If the 4-oz ball is in contact with the racket for 0.02 sec, determine the magnitude of the average force **R** exerted by the racket on the ball. Also determine the angle β made by **R** with the horizontal.

Solution. We construct the impulse-momentum diagrams for the ball as follows:

① 

② $\left[m(v_x)_1 + \int_{t_1}^{t_2} \Sigma F_x\,dt = m(v_x)_2\right]$ $\quad -\dfrac{4/16}{32.2}(50) + R_x(0.02) = \dfrac{4/16}{32.2}(70 \cos 15°)$

$$\left[m(v_y)_1 + \int_{t_1}^{t_2} \Sigma F_y\,dt = m(v_y)_2\right]$$

$$\frac{4/16}{32.2}(0) + R_y(0.02) - (4/16)(0.02) = \frac{4/16}{32.2}(70 \sin 15°)$$

We can now solve for the impact forces as

$$R_x = 45.7 \text{ lb}$$

$$R_y = 7.28 \text{ lb}$$

We note that the impact force $R_y = 7.28$ lb is considerably larger than the 0.25-lb weight of the ball. Thus, the weight mg, a nonimpulsive force, could have been neglected as small in comparison with R_y. Had we neglected the weight, the computed value of R_y would have been 7.03 lb.

We now determine the magnitude and direction of **R** as

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{45.7^2 + 7.28^2} = 46.2 \text{ lb} \qquad \textit{Ans.}$$

$$\beta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{7.28}{45.7} = 9.06° \qquad \textit{Ans.}$$

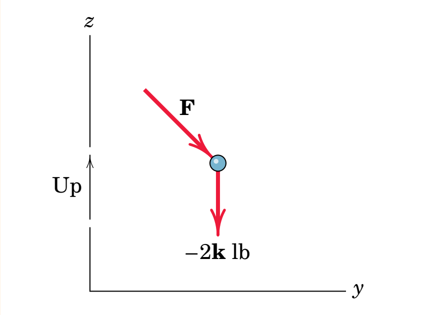
Helpful Hints

① Recall that for the impulse-momentum diagrams, initial linear momentum goes in the first diagram, all external linear impulses go in the second diagram, and final linear momentum goes in the third diagram.

② For the linear impulse $\int_{t_1}^{t_2} R_x\,dt$, the average impact force R_x is a constant, so that it can be brought outside the integral sign, resulting in $R_x \int_{t_1}^{t_2} dt = R_x(t_2 - t_1) = R_x \Delta t$. The linear impulse in the y-direction has been similarly treated.

Sample Problem 3/20

A 2-lb particle moves in the vertical y-z plane (z up, y horizontal) under the action of its weight and a force $\mathbf{F}$ which varies with time. The linear momentum of the particle in pound-seconds is given by the expression $\mathbf{G} = \frac{3}{2}(t^2 + 3)\mathbf{j} - \frac{2}{3}(t^3 - 4)\mathbf{k}$, where t is the time in seconds. Determine $\mathbf{F}$ and its magnitude for the instant when $t = 2$ sec.



Solution. The weight expressed as a vector is $-2\mathbf{k}$ lb. Thus, the force-momentum equation becomes

① $[\Sigma\mathbf{F} = \dot{\mathbf{G}}]$ $$\mathbf{F} - 2\mathbf{k} = \frac{d}{dt}\left[\frac{3}{2}(t^2 + 3)\mathbf{j} - \frac{2}{3}(t^3 - 4)\mathbf{k}\right]$$
$$= 3t\mathbf{j} - 2t^2\mathbf{k}$$

For $t = 2$ sec, $$\mathbf{F} = 2\mathbf{k} + 3(2)\mathbf{j} - 2(2^2)\mathbf{k} = 6\mathbf{j} - 6\mathbf{k} \text{ lb}$$ *Ans.*

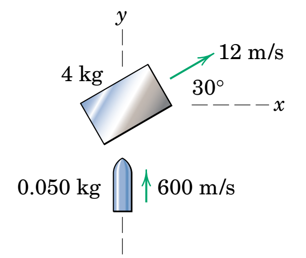
Thus, $$F = \sqrt{6^2 + 6^2} = 6\sqrt{2} \text{ lb}$$ *Ans.*

Helpful Hint

① Don't forget that $\Sigma\mathbf{F}$ includes *all* external forces acting on the particle, including the weight.

Sample Problem 3/23

The 50-g bullet traveling at 600 m/s strikes the 4-kg block centrally and is embedded within it. If the block slides on a smooth horizontal plane with a velocity of 12 m/s in the direction shown prior to impact, determine the velocity $\mathbf{v}_2$ of the block and embedded bullet immediately after impact.



Solution. Since the force of impact is internal to the system composed of the block and bullet and since there are no other external forces acting on the system in the plane of motion, it follows that the linear momentum of the system is conserved. Thus,

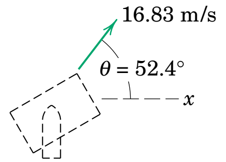
① $[\mathbf{G}_1 = \mathbf{G}_2]$ $\quad 0.050(600\mathbf{j}) + 4(12)(\cos 30^\circ\mathbf{i} + \sin 30^\circ\mathbf{j}) = (4 + 0.050)\mathbf{v}_2$

$$\mathbf{v}_2 = 10.26\mathbf{i} + 13.33\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

The final velocity and its direction are given by

$[v = \sqrt{v_x{}^2 + v_y{}^2}]$ $\qquad v_2 = \sqrt{(10.26)^2 + (13.33)^2} = 16.83 \text{ m/s}$ $\qquad$ *Ans.*

$[\tan\theta = v_y/v_x]$ $\qquad \tan\theta = \dfrac{13.33}{10.26} = 1.299 \qquad \theta = 52.4^\circ$ $\qquad$ *Ans.*

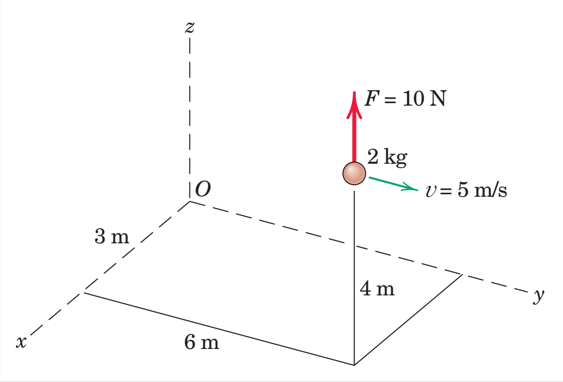


Helpful Hint

① Working with the vector form of the principle of conservation of linear momentum is clearly equivalent to working with the component form.

Sample Problem 3/24

A small sphere has the position and velocity indicated in the figure and is acted upon by the force F. Determine the angular momentum $\mathbf{H}_O$ about point O and the time derivative $\dot{\mathbf{H}}_O$.



Solution. We begin with the definition of angular momentum and write

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 2(5\mathbf{j})$$

$$= -40\mathbf{i} + 30\mathbf{k} \text{ N}\cdot\text{m/s} \qquad \textit{Ans.}$$

From Eq. 3/31,

$$\dot{\mathbf{H}}_O = \mathbf{M}_O$$

$$= \mathbf{r} \times \mathbf{F}$$

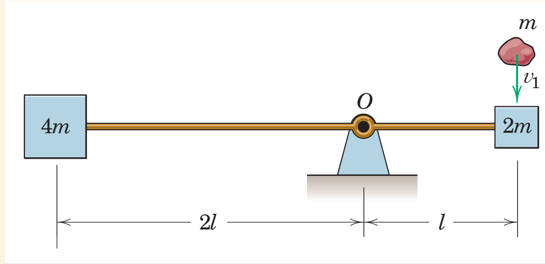
$$= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 10\mathbf{k}$$

$$= 60\mathbf{i} - 30\mathbf{j} \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

As with moments of forces, the position vector must run *from* the reference point (O in this case) *to* the line of action of the linear momentum $m\mathbf{v}$. Here $\mathbf{r}$ runs directly to the particle.

Sample Problem 3/26

The assembly of the light rod and two end masses is at rest when it is struck by the falling wad of putty traveling with speed v_1 as shown. The putty adheres to and travels with the right-hand end mass. Determine the angular velocity $\dot{\theta}_2$ of the assembly just after impact. The pivot at O is frictionless, and all three masses may be assumed to be particles.



Solution. If we ignore the angular impulses associated with the weights during the collision process, then system angular momentum about O is conserved during the impact.

$$(H_O)_1 = (H_O)_2$$

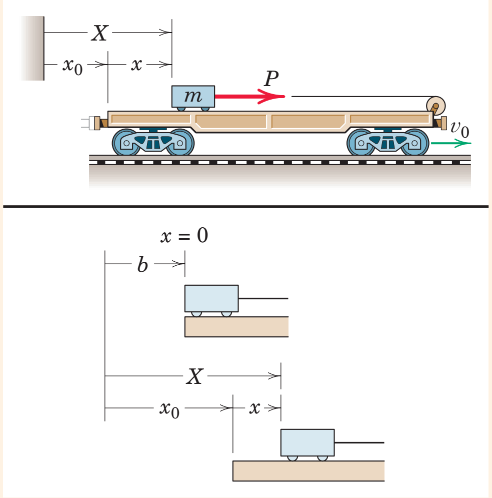
$$mv_1 l = (m + 2m)(l\dot{\theta}_2)l + 4m(2l\dot{\theta}_2)2l$$

$$\dot{\theta}_2 = \frac{v_1}{19l} \text{ CW} \qquad \textit{Ans.}$$

Note that each angular-momentum term is written in the form mvd, and the final transverse velocities are expressed as radial distances times the common final angular velocity $\dot{\theta}_2$.

Sample Problem 3/33

The flatcar moves with a constant speed v_0 and carries a winch which produces a constant tension P in the cable attached to the small carriage. The carriage has a mass m and rolls freely on the horizontal surface starting from rest relative to the flatcar at $x = 0$, at which instant $X = x_0 = b$. Apply the work-energy equation to the carriage, first, as an observer moving with the frame of reference of the car and, second, as an observer on the ground. Show the compatibility of the two expressions.



Solution. To the observer on the flatcar, the work done by P is

① $$U_{\text{rel}} = \int_0^x P\,dx = Px \qquad \text{for constant } P$$

The change in kinetic energy relative to the car is

$$\Delta T_{\text{rel}} = \frac{1}{2}m(\dot{x}^2 - 0)$$

The work-energy equation for the moving observer becomes

$[U_{\text{rel}} = \Delta T_{\text{rel}}]$ $$Px = \frac{1}{2}m\dot{x}^2$$

To the observer on the ground, the work done by P is

$$U = \int_b^X P\,dX = P(X - b)$$

The change in kinetic energy for the ground measurement is

② $$\Delta T = \frac{1}{2}m(\dot{X}^2 - v_0^2)$$

The work-energy equation for the fixed observer gives

$[U = \Delta T]$ $$P(X - b) = \frac{1}{2}m(\dot{X}^2 - v_0^2)$$

To reconcile this equation with that for the moving observer, we can make the following substitutions:

$$X = x_0 + x, \qquad \dot{X} = v_0 + \dot{x}, \qquad \ddot{X} = \ddot{x}$$

Thus,

$$P(X - b) = Px + P(x_0 - b) = Px + m\ddot{x}(x_0 - b)$$

③ $$= Px + m\ddot{x}v_0 t = Px + mv_0\dot{x}$$

and

$$\dot{X}^2 - v_0^2 = (v_0^2 + \dot{x}^2 + 2v_0\dot{x} - v_0^2) = \dot{x}^2 + 2v_0\dot{x}$$

The work-energy equation for the fixed observer now becomes

$$Px + mv_0\dot{x} = \frac{1}{2}m\dot{x}^2 + mv_0\dot{x}$$

which is merely $Px = \frac{1}{2}m\dot{x}^2$, as concluded by the moving observer. We see, therefore, that the difference between the two work-energy expressions is

$$U - U_{\text{rel}} = T - T_{\text{rel}} = mv_0\dot{x}$$

Helpful Hints

① The only coordinate which the moving observer can measure is x.

② To the ground observer, the initial velocity of the carriage is v_0 so its initial kinetic energy is $\frac{1}{2}mv_0^2$.

③ The symbol t stands for the time of motion from $x = 0$ to $x = x$. The displacement $x_0 - b$ of the carriage is its velocity v_0 times the time t or $x_0 - b = v_0 t$. Also, since the constant acceleration times the time equals the velocity change, $\ddot{x}t = \dot{x}$.