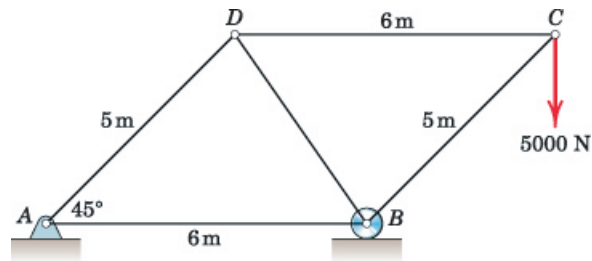
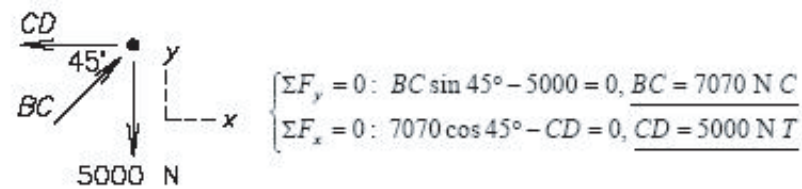


Determine the force in members: (a) BC, (b) CD, (c) AD, (d) BD, (e) AB of the loaded truss. (Note: the compression force must be negative.)

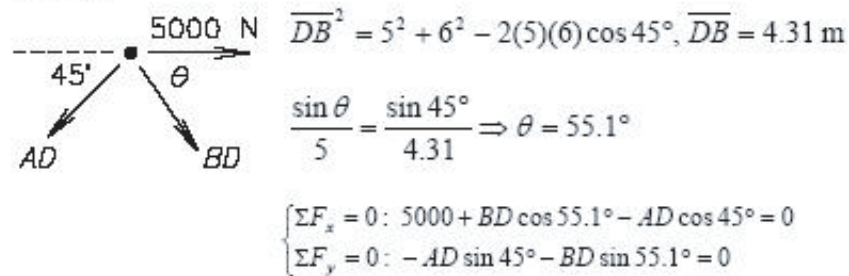


We can begin at joint C without finding the external reactions.

Joint C:

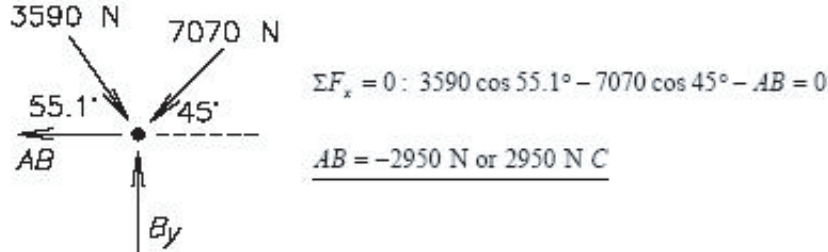


Joint D:



Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N C}}$
 $\underline{AD = 4170 \text{ N T}}$

Joint B:



Problem 6-7

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

kN := 1000N

Given:

$F_1 := 3\text{kN}$

$F_2 := 8\text{kN}$

$F_3 := 4\text{kN}$

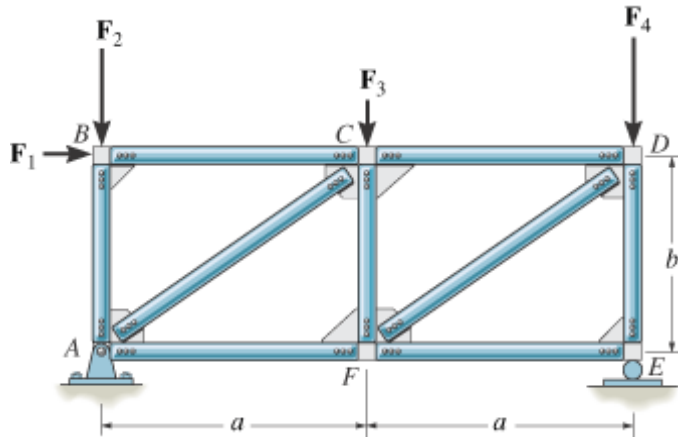
$F_4 := 10\text{kN}$

$a := 2\text{m}$

$b := 1.5\text{m}$

Solution:

$$\curvearrowright + \Sigma M_A = 0 ; -F_1(b) - F_3(a) - F_4(2a) + E_y(2a) = 0$$



$$E_y := \frac{F_1 \cdot (b) + F_3 \cdot (a) + F_4 \cdot (2a)}{2a}$$

$$E_y = 13.125 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - F_2 - F_3 - F_4 + E_y = 0$$

$$A_y := F_2 + F_3 + F_4 - E_y$$

$$A_y = 8.875 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - F_1 = 0 \quad A_x := F_1$$

$$A_x = 3 \text{ kN}$$

Joint B:

$$\rightarrow \Sigma F_x = 0; \quad F_1 - F_{BC} = 0 \quad F_{BC} := F_1$$

$$F_{BC} = 3 \text{ kN} \quad (C)$$

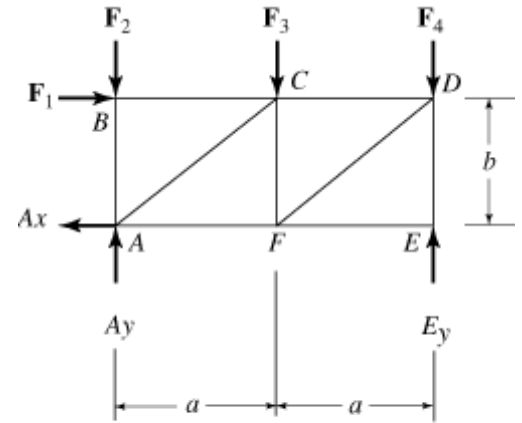
$$+\uparrow \Sigma F_y = 0; \quad F_{BA} - F_2 = 0 \quad F_{BA} := F_2$$

$$F_{BA} = 8 \text{ kN} \quad (C)$$

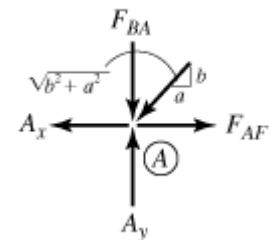
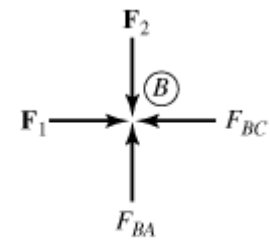
Joint A:

$$+\uparrow \Sigma F_y = 0; \quad A_y - F_{BA} - \frac{b}{\sqrt{b^2 + a^2}} F_{AC} = 0$$

$$F_{AC} := \frac{A_y - F_{BA}}{\frac{b}{\sqrt{b^2 + a^2}}}$$



At joint B



$$F_{AC} = 1.46 \text{ kN} \quad (\text{C})$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x - \frac{a}{\sqrt{b^2 + a^2}} F_{AC} + F_{AF} = 0$$

$$F_{AF} := A_x + \frac{F_{AC} \cdot a}{\sqrt{b^2 + a^2}}$$

$$F_{AF} = 4.17 \text{ kN} \quad (\text{T})$$

Joint C:

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} + \frac{a}{\sqrt{b^2 + a^2}} \cdot F_{AC} - F_{CD} = 0$$

$$F_{CD} := F_{BC} + \frac{F_{AC} \cdot a}{\sqrt{b^2 + a^2}}$$

$$F_{CD} = 4.17 \text{ kN} \quad (\text{C})$$

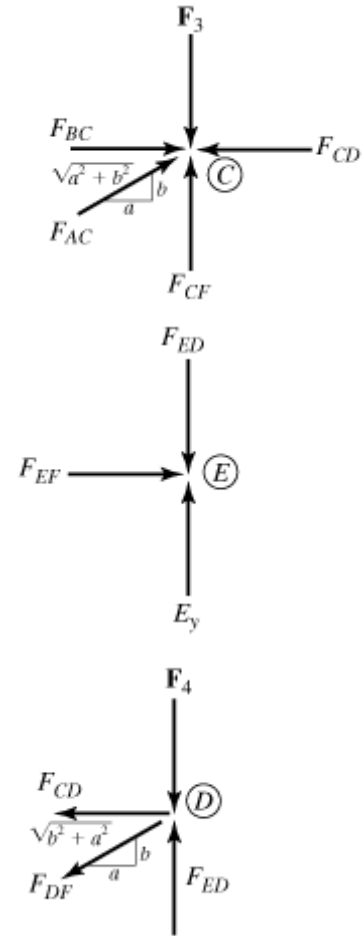
$$+\uparrow \Sigma F_y = 0; \quad F_{CF} - F_3 + \frac{F_{AC} \cdot b}{\sqrt{b^2 + a^2}} = 0$$

$$F_{CF} := F_3 - \frac{F_{AC} \cdot b}{\sqrt{b^2 + a^2}}$$

$$F_{CF} = 3.13 \text{ kN} \quad (\text{C})$$

Joint E:

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{EF} := 0 \text{ kN} \quad F_{EF} = 0.00 \text{ kN}$$



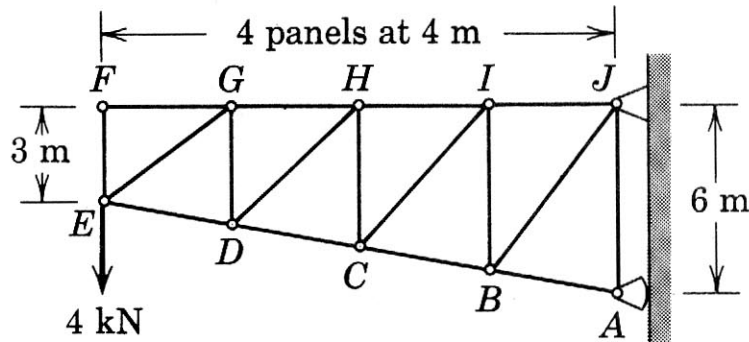
$$+\uparrow \Sigma F_y = 0; \quad E_y - F_{ED} = 0$$
$$F_{ED} := E_y \quad F_{ED} = 13.1 \text{ kN} \quad (\text{C})$$

Joint D:

$$+\uparrow \Sigma F_y = 0; \quad F_{ED} - F_4 - \frac{F_{DF} \cdot b}{\sqrt{b^2 + a^2}} = 0$$
$$F_{DF} := \frac{(F_{ED} - F_4) \cdot \sqrt{b^2 + a^2}}{b}$$
$$F_{DF} = 5.21 \text{ kN} \quad (\text{T})$$

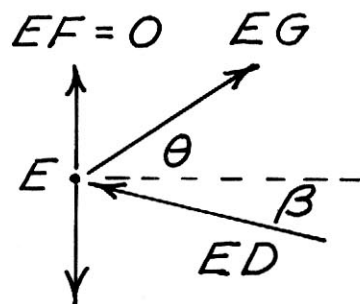
ART. 4/3 METHOD OF JOINTS

Determine the forces in members FG , EG , and GD for the simple truss.



By inspection of joint F , $FG = EF = 0$

Joint E



$$\theta = \tan^{-1} \frac{3}{4}, \quad \sin \theta = \frac{3}{5}, \quad \cos \theta = \frac{4}{5}$$

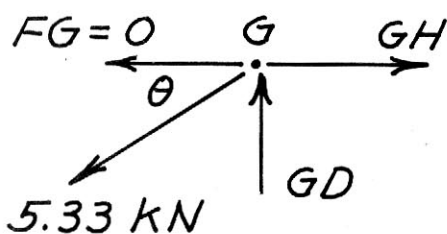
$$\beta = \tan^{-1} \frac{3}{16} = 10.62^\circ$$

$$\Sigma F_x = 0: \quad EG \left(\frac{4}{5}\right) - ED \cos 10.62^\circ = 0$$

$$\Sigma F_y = 0: \quad EG \left(\frac{3}{5}\right) + ED \sin 10.62^\circ - 4 = 0$$

Solve to obtain $EG = 5.33 \text{ kN T}$

Joint G



$$\Sigma F_y = 0: \quad GD - 5.33 \left(\frac{3}{5}\right) = 0$$

$$\boxed{GD = 3.20 \text{ kN C}}$$

Sample Problem 4/1

Compute the force in each member of the loaded cantilever truss by the method of joints.

Solution. If it were not desired to calculate the external reactions at D and E , the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$$[\Sigma M_E = 0] \quad 5T - 20(5) - 30(10) = 0 \quad T = 80 \text{ kN}$$

$$[\Sigma F_x = 0] \quad 80 \cos 30^\circ - E_x = 0 \quad E_x = 69.3 \text{ kN}$$

$$[\Sigma F_y = 0] \quad 80 \sin 30^\circ + E_y - 20 - 30 = 0 \quad E_y = 10 \text{ kN}$$

Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A . Equilibrium requires

$$[\Sigma F_y = 0] \quad 0.866AB - 30 = 0 \quad AB = 34.6 \text{ kN } T \quad \text{Ans.}$$

$$[\Sigma F_x = 0] \quad AC - 0.5(34.6) = 0 \quad AC = 17.32 \text{ kN } C \quad \text{Ans.}$$

① where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C . The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$$[\Sigma F_y = 0] \quad 0.866BC - 0.866(34.6) = 0 \quad BC = 34.6 \text{ kN } C \quad \text{Ans.}$$

$$[\Sigma F_x = 0] \quad BD - 2(0.5)(34.6) = 0 \quad BD = 34.6 \text{ kN } T \quad \text{Ans.}$$

Joint C now contains only two unknowns, and these are found in the same way as before:

$$[\Sigma F_y = 0] \quad 0.866CD - 0.866(34.6) - 20 = 0$$

$$CD = 57.7 \text{ kN } T \quad \text{Ans.}$$

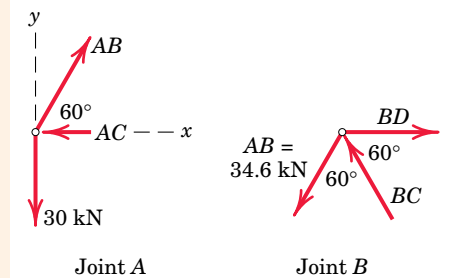
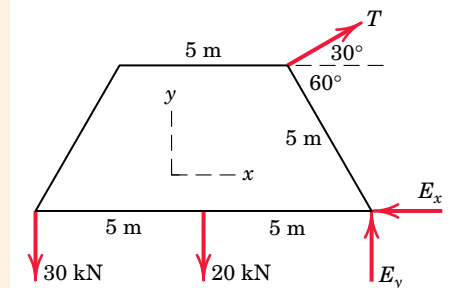
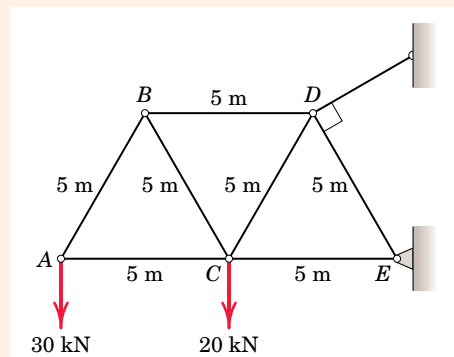
$$[\Sigma F_x = 0] \quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$$

$$CE = 63.5 \text{ kN } C \quad \text{Ans.}$$

Finally, from joint E there results

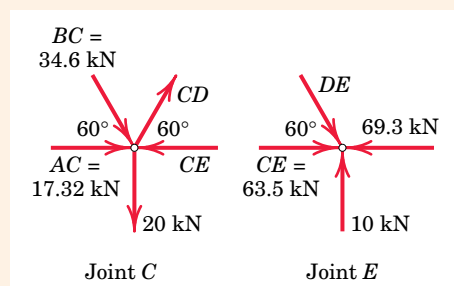
$$[\Sigma F_y = 0] \quad 0.866DE = 10 \quad DE = 11.55 \text{ kN } C \quad \text{Ans.}$$

and the equation $\Sigma F_x = 0$ checks.



Helpful Hint

① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).



Sample Problem 4/5

The space truss consists of the rigid tetrahedron $ABCD$ anchored by a ball-and-socket connection at A and prevented from any rotation about the x -, y -, or z -axes by the respective links 1, 2, and 3. The load L is applied to joint E , which is rigidly fixed to the tetrahedron by the three additional links. Solve for the forces in the members at joint E and indicate the procedure for the determination of the forces in the remaining members of the truss.

Solution. We note first that the truss is supported with six properly placed constraints, which are the three at A and the links 1, 2, and 3. Also, with $m = 9$ members and $j = 5$ joints, the condition $m + 6 = 3j$ for a sufficiency of members to provide a noncollapsible structure is satisfied.

The external reactions at A , B , and D can be calculated easily as a first step, although their values will be determined from the solution of all forces on each of the joints in succession.

①

We start with a joint on which at least one known force and not more than three unknown forces act, which in this case is joint E . The free-body diagram of joint E is shown with all force vectors arbitrarily assumed in their positive tension directions (away from the joint). The vector expressions for the three unknown forces are

②

$$\mathbf{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}), \quad \mathbf{F}_{EC} = \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}), \quad \mathbf{F}_{ED} = \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k})$$

Equilibrium of joint E requires

$$[\Sigma \mathbf{F} = \mathbf{0}] \quad \mathbf{L} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED} = \mathbf{0} \quad \text{or}$$

$$-L\mathbf{i} + \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}) + \frac{F_{EC}}{5}(-3\mathbf{i} + 4\mathbf{k}) + \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k}) = \mathbf{0}$$

Rearranging terms gives

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\mathbf{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\mathbf{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of the \mathbf{i} -, \mathbf{j} -, and \mathbf{k} -unit vectors to zero gives the three equations

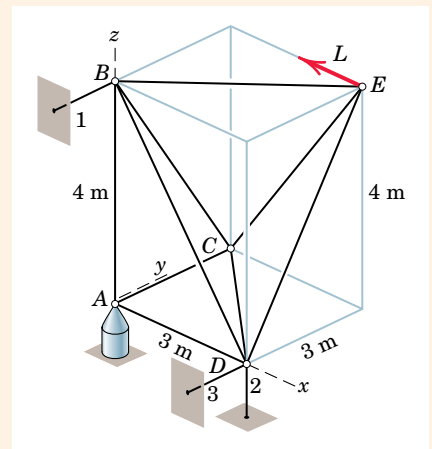
$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \quad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \quad F_{EC} + F_{ED} = 0$$

Solving the equations gives us

$$F_{EB} = -L/\sqrt{2} \quad F_{EC} = -5L/6 \quad F_{ED} = 5L/6 \quad \text{Ans.}$$

Thus, we conclude that F_{EB} and F_{EC} are compressive forces and F_{ED} is tension.

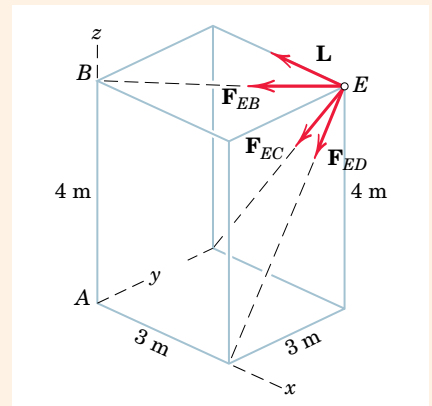
Unless we have computed the external reactions first, we must next analyze joint C with the known value of F_{EC} and the three unknowns F_{CB} , F_{CA} , and F_{CD} . The procedure is identical with that used for joint E . Joints B , D , and A are then analyzed in the same way and in that order, which limits the scalar unknowns to three for each joint. The external reactions computed from these analyses must, of course, agree with the values which can be determined initially from an analysis of the truss as a whole.



Helpful Hints

① *Suggestion:* Draw a free-body diagram of the truss as a whole and verify that the external forces acting on the truss are $\mathbf{A}_x = L\mathbf{i}$, $\mathbf{A}_y = L\mathbf{j}$, $\mathbf{A}_z = (4L/3)\mathbf{k}$, $\mathbf{B}_y = \mathbf{0}$, $\mathbf{D}_y = -L\mathbf{j}$, $\mathbf{D}_z = -(4L/3)\mathbf{k}$.

② With this assumption, a negative numerical value for a force indicates compression.



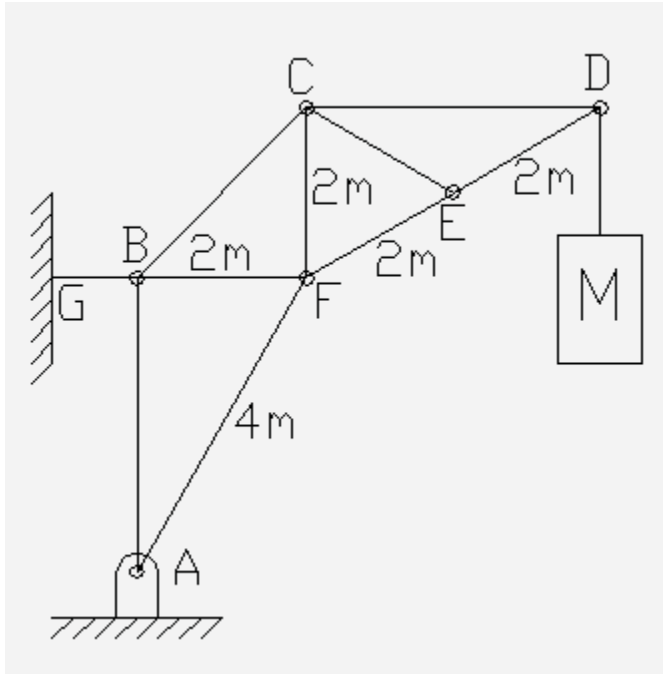
MECH 2110 - Statics & Dynamics

Chapter S4 Problem 23 Solution

Page 188, Engineering Mechanics - Statics, 4th Edition, Meriam and Kraige

Given: The truss shown below supporting a mass M equal to 500 kg.

Find: The force in each member of the truss.



0. Observations:

A. All connections between the members are pins. Each member is connected at two points. All external loads are applied at connection points. The weights of the members are small compared to the external loads. The system may be classified as a truss. All members are two force members, transmitting force along the line between the member connection points.

B. In order to determine the directions of all of the members (and hence all of the forces), we need to determine the coordinates of each of the pins. Setting our origin at point A, X positive to the right, Y positive upward, all coordinates in meters:

A (0,0)

B $(0, (4^2 - 2^2)^{1/2}) = (0, 12^{1/2})$

C $(2, 12^{1/2} + 2)$

D $(2 + 12^{1/2}, 12^{1/2} + 2)$

F $(2, 12^{1/2})$

E $(2 + 3^{1/2}, 12^{1/2} + 1)$ { midway between D and F, average of those two points }

G $(X_G, 12^{1/2})$ { where X_G is some unspecified negative number. }

C. From the above coordinates we can determine the direction of each member (and hence each force). This can be done by subtracting the coordinates of two points along the line of action of the force and then evaluating the unit vector parallel to that direction.

X-Direction: BF, BG, and CD

Y-Direction: AB and CF

$$\mathbf{e}_{EF} \text{ and } \mathbf{e}_{DE} = \pm \left\{ 3^{1/2}/2 \mathbf{i} + 0.5 \mathbf{j} \right\} \quad \{ 30 \text{ degree angle} \}$$

$$\mathbf{e}_{BC} = \pm \left\{ 1/2^{1/2} \mathbf{i} + 1/2^{1/2} \mathbf{j} \right\} \quad \{ 45 \text{ degree angle} \}$$

$$\mathbf{e}_{AF} = \pm \left\{ 0.5 \mathbf{i} + 3^{1/2}/2 \mathbf{j} \right\} \quad \{ 60 \text{ degree angle} \}$$

$$\mathbf{e}_{CE} = \pm \left\{ 3^{1/2}/2 \mathbf{i} - 0.5 \mathbf{j} \right\} \quad \{ -30 \text{ degree angle} \}$$

D. We are asked to determine the force in each and every member of the truss. This will require the consideration of several mechanical systems. One approach would be to successively consider different connecting pins. If we can sequentially identify pins that are connected to no more than two members transmitting loads that have not yet been determined, we can readily determine the forces in those members (method of joints). In so doing, we must exploit the fact that the force transmitted by a two force member is along the line between the two connection points. We must further exploit the fact that the forces acting at the two ends of the member are oppositely directed. We can see that considering the pins in the following sequence will enable us to determine the member forces as follows:

Pin D (Members CD and DE)

Pin E (Members CE and EF)

Pin C (members CF and BC)

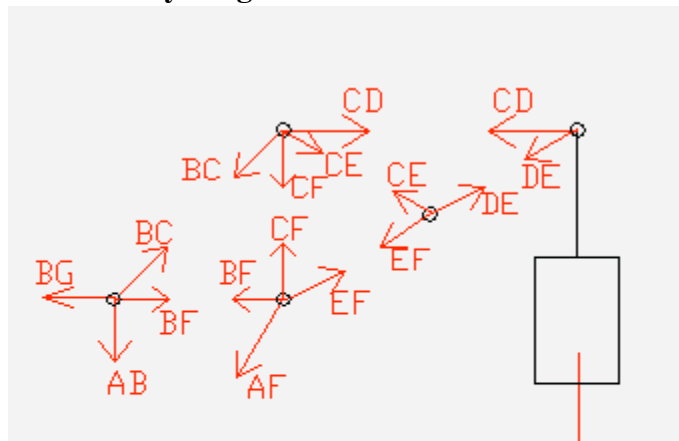
Pin F (Members BF and AF)

Pin B (Members BG and AF)

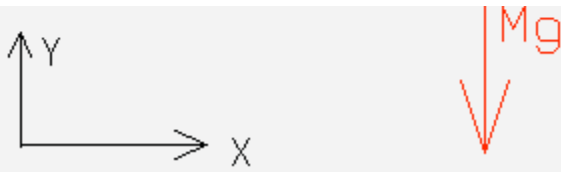
For convenience we will assume all members to be in tension. In this way any negative results will indicate a member transmitting a compressive force.

1. Mechanical System - Successive consideration of 5 mechanical systems, beginning with pin D and following sequentially with pins E, C, F and B (each considered individually). Note that the mechanical system including pin D also includes the supported mass and the cable that connects pin D to the mass. The other mechanical system include only the pins.

2. Free Body Diagram



The figure provides the free body diagrams of the required five mechanical systems. The coordinate axes used are shown. Note that as every member of a truss is a two force member, each of the forces exerted by a truss member on a pin, is parallel to the member. Further note that the forces exerted by either end of a two force member must be equal in magnitude and opposite in direction. Both of these facts are reflected in the free body diagrams. Additionally, all forces are shown pulling the



pins closer together. This reflects an assumption that each and every member of the truss is in tension (being stretched) and is exerting a resisting force pulling the attached pins toward one another. This is convenient as a positive result for any member force indicates tension,

while a negative result indicates compression in that member. This approach is reflected in the signs of the various terms appearing in the equilibrium equations for each of the pins. As noted above, by sequentially considering pins D, E, C, F, and B, all member forces can be evaluated.

3. Equations

Note the use of the various unit vectors in expressing the force components. The force components are obtained by multiplying the magnitude of the force by the corresponding component of the associated unit vector. The choice of the positive or negative sign is made based upon the free body diagrams of the pins.

Pin D:

$$\sum F_X = -CD - 3^{1/2}/2 DE = 0$$

$$\sum F_Y = -1/2 DE - Mg = 0$$

Pin E

$$\sum F_X = 3^{1/2}/2 DE - 3^{1/2}/2 EF - 3^{1/2}/2 CE = 0$$

$$\sum F_Y = 1/2 DE - 1/2 EF + 1/2 CE = 0$$

Pin C

$$\sum F_X = CD + 3^{1/2}/2 CE - 1/2^{1/2} BC = 0$$

$$\sum F_Y = -1/2 CE - CF - 1/2^{1/2} BC = 0$$

Pin F

$$\sum F_X = -BF + 3^{1/2}/2 EF - 1/2 AF = 0$$

$$\sum F_Y = CF + 1/2 EF - 3^{1/2}/2 AF = 0$$

Pin B

$$\sum F_X = BF - BG + 1/2^{1/2} BC = 0$$

$$\sum F_Y = -AB + 1/2^{1/2} BC = 0$$

4. Solve

From the Y equation for pin D:

$$DE = -2 Mg = -2 * 500 \text{ kg } 9.81 \text{ m/s}^2 = -9810 \text{ N}$$

$$DE = 9.81 \text{ kN compression}$$

From the X equation for pin D:

$$CD = -3^{1/2}/2 DE = 8500 \text{ N}$$

$$CD = 8.50 \text{ kN tension.}$$

Multiplying the Y equation for pin E by the square root of 3, and then subtracting the X equation for pin E from that result, we observe that all of the terms except the one involving CE vanish:

$$3^{1/2} CE = 0$$

$$CE = 0$$

Using this result in the y equation for pin E:

$$EF = DE = -9810 \text{ N}$$

$$EF = 9.81 \text{ kN compression}$$

From the X equation for pin C:

$$BC = 2^{1/2} CD + (3/2)^{1/2} CE = 12010 \text{ N}$$

$$BC = 12.01 \text{ kN tension}$$

From the Y equation for pin C:

$$CF = -1/2 CE - 1/2^{1/2} BC = -8500 \text{ N}$$

$$CF = 8.50 \text{ kN compression}$$

From the Y equation for pin F:

$$AF = 1/3^{1/2} EF + 2/3^{1/2} CF = -15470 \text{ N}$$

$$AF = 15.47 \text{ kN compression}$$

From the X equation for pin F:

$$BF = 3^{1/2}/2 EF - 1/2 AF = -759 \text{ N}$$

$$BF = 0.759 \text{ kN compression}$$

From the X equation for pin B

$$BG = BF + 1/2^{1/2} BC = 7740 \text{ N}$$

$$BG = 7.74 \text{ kN tension}$$

From the Y equation for pin B:

$$AB = 1/2^{1/2} BC = 8500 \text{ N}$$

$$AB = 8.50 \text{ kN tension}$$

Results

$$DE = 9810 \text{ N compression}$$

$$CD = 8.50 \text{ kN tension}$$

$$CE = 0$$

$$EF = 9.81 \text{ kN compression}$$

$$BC = 12.01 \text{ kN tension}$$

$$CF = 8.50 \text{ kN compression}$$

$$AF = 15.47 \text{ kN compression}$$

BF = 0.759 kN compression

BG = 7.74 kN tension

AB = 8.50 kN tension