

Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.

Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$$[\Sigma F_y = 0] \quad N - 981 \cos 20^\circ = 0 \quad N = 922 \text{ N}$$

$$[F_{\max} = \mu_s N] \quad F_{\max} = 0.30(922) = 277 \text{ N}$$

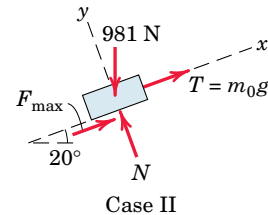
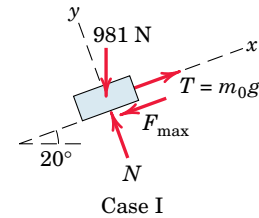
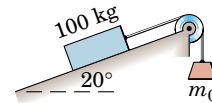
$$[\Sigma F_x = 0] \quad m_0(9.81) - 277 - 981 \sin 20^\circ = 0 \quad m_0 = 62.4 \text{ kg} \quad \text{Ans.}$$

① The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x -direction requires

$$[\Sigma F_x = 0] \quad m_0(9.81) + 277 - 981 \sin 20^\circ = 0 \quad m_0 = 6.01 \text{ kg} \quad \text{Ans.}$$

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{\max} and N be concurrent with the 981-N weight and the tension T .

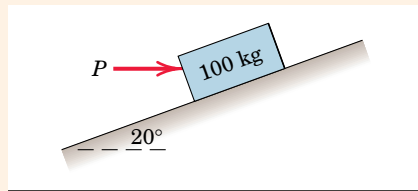


Helpful Hint

- ① We see from the results of Sample Problem 6/1 that the block would slide down the incline without the restraint of attachment to m_0 since $\tan 20^\circ > 0.30$. Thus, a value of m_0 will be required to maintain equilibrium.

Sample Problem 6/3

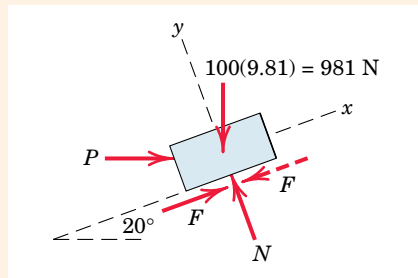
Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution. There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P . It is therefore necessary that we make an assumption, so we will take the friction force to be up the plane, as shown by the solid arrow. From the free-body diagram a balance of forces in both x - and y -directions gives

$$[\Sigma F_x = 0] \quad P \cos 20^\circ + F - 981 \sin 20^\circ = 0$$

$$[\Sigma F_y = 0] \quad N - P \sin 20^\circ - 981 \cos 20^\circ = 0$$



Case I. $P = 500$ N

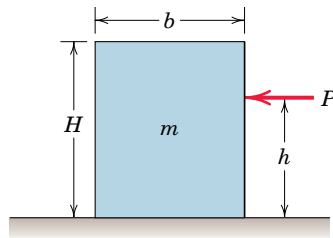
Substitution into the first of the two equations gives

$$F = -134.3 \text{ N}$$

The negative sign tells us that *if* the block is in equilibrium, the friction force

Sample Problem 6/4

The homogeneous rectangular block of mass m , width b , and height H is placed on the horizontal surface and subjected to a horizontal force P which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is μ_k . Determine (a) the greatest value which h may have so that the block will slide without tipping over and (b) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces acts if $h = H/2$.

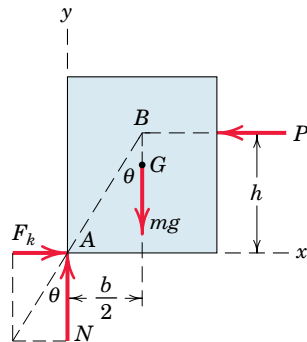


Solution. (a) With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A . The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value $\mu_k N$, and the angle θ becomes $\theta = \tan^{-1} \mu_k$. The resultant of F_k and N passes through a point B through which P must also pass, since three coplanar forces in equilibrium are concurrent. Hence, from the geometry of the block

$$\tan \theta = \mu_k = \frac{b/2}{h} \quad h = \frac{b}{2\mu_k} \quad \text{Ans.}$$

If h were greater than this value, moment equilibrium about A would not be satisfied, and the block would tip over.

Alternatively, we may find h by combining the equilibrium requirements for



Sample Problem 6/5

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.

Solution. The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

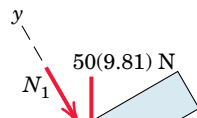
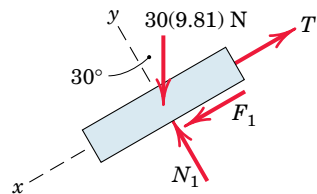
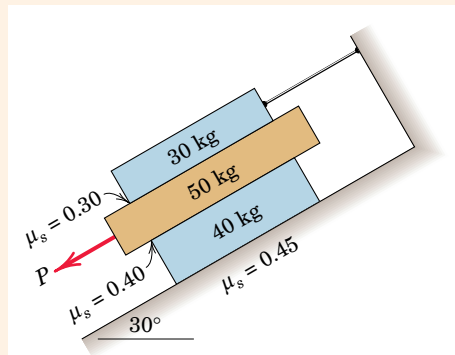
The normal forces, which are in the y -direction, may be determined without reference to the friction forces, which are all in the x -direction. Thus,

$$[\Sigma F_y = 0] \quad (30\text{-kg}) \quad N_1 - 30(9.81) \cos 30^\circ = 0 \quad N_1 = 255 \text{ N}$$

$$(50\text{-kg}) \quad N_2 - 50(9.81) \cos 30^\circ - 255 = 0 \quad N_2 = 680 \text{ N}$$

$$(40\text{-kg}) \quad N_3 - 40(9.81) \cos 30^\circ - 680 = 0 \quad N_3 = 1019 \text{ N}$$

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg



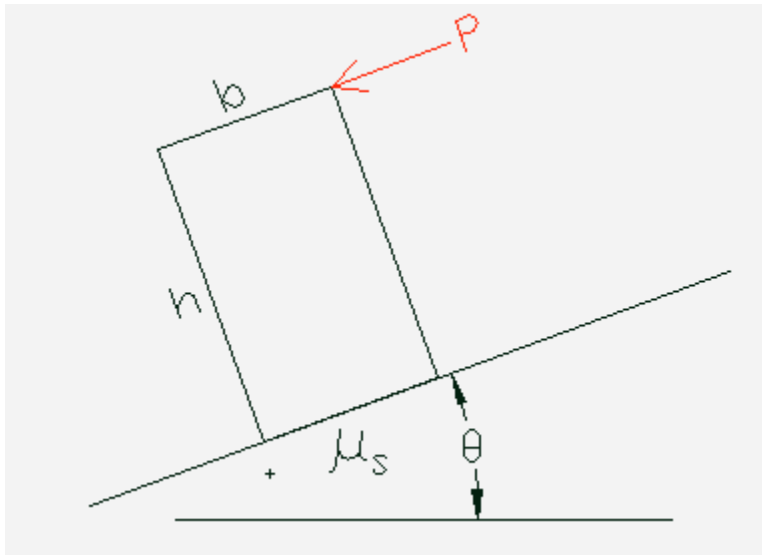
MECH 2110 - Statics & Dynamics

Chapter S6 Problem 5 Solution

Page 357, Engineering Mechanics - Statics, 4th Edition, Meriam and Kraige

Given: A rectangular homogenous block of height h , width b , and mass m , resting on an incline that makes an angle q with the horizontal as shown below. The highest corner of the block is acted upon by a force P that is parallel to and directed down the incline. The coefficient of friction between the block and the incline is μ_s .

Find: The largest ratio of block height to width, h/b , such that steadily increasing the force P will cause the block to begin slipping before it tips.



0. Observations:

1. The block is acted upon by 4 forces, the applied force P , the weight of the block, the normal force exerted by the incline on the block, and the frictional force exerted by the incline on the block.

2. Should the block tip, it will tip about its lowest corner. At this juncture any forces exerted by the incline on the block, must be acting at that corner point. At the critical height to width ratio, where it is just about to both tip and slip, the forces exerted by the incline on the block must also be acting at that corner point.

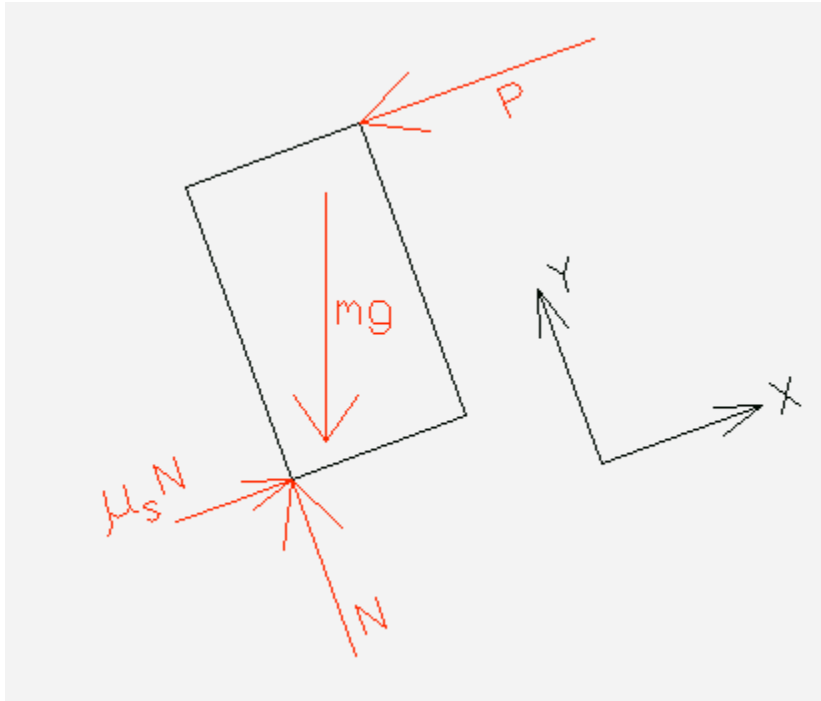
3. As the block is about to begin slipping we can assert that the frictional force exerted by the incline on the block is equal to the static coefficient of friction multiplied by the normal force exerted by the incline on the block.

4. Knowing that two of the four forces act at the corner of the block, makes that corner an excellent candidate as a moment sum point.

1. Mechanical System - Block of height ratio h/b such that it is just about to begin both tipping and slipping as a result of the applied load P . The critical nature of this mechanical system tells us both

the location of the contact forces between the incline and the block and the relationship between the frictional contact force and the normal contact force (see above discussion).

2. Free Body Diagram



The figure provides the free body diagram of the block. The applied load, P , is shown acting parallel to the incline at the upper corner of the block. The weight of the block acts vertically downward through the block center. The contact forces are shown acting at the lower corner of the block, consistent with imminent tipping as discussed above. The normal force, N , is shown acting perpendicular to the incline. The frictional force is shown acting parallel to the incline in a direction opposed to the sliding tendency of the block. As we know that slipping is imminent, the friction force is known to have a magnitude equal to the static coefficient of friction

multiplied by the normal force. As three of the four forces shown on the free body diagram are either parallel or perpendicular to the incline, the coordinate axes have been chosen to be parallel and perpendicular to the incline as shown. The angle between the weight force and the Y -axis (normal to the incline) must be the same as the angle between the horizontal and the X -axis (parallel to the incline), that angle being given as q .

3. Equations

$\sum F_X = P - m g \sin(q) = 0$ { Note that as the incline (X -direction) makes an angle of q with the horizontal,

$\sum F_Y = N - m g \cos(q) = 0$ then the Y -direction must make that same angle with the vertical, gravity direction. }

Summing moments about the lowest corner, C :

$\sum M_C = -P h - m g \sin(q) h/2 + m g \cos(q) b/2 = 0$ { The applied load is in the X -direction. The

distance along the Y -direction (perpendicular) from the moment point to the force is the block height h . The rotational tendency produced by this force about the moment point is counter-clockwise, negative relative to the coordinate axes specified. The weight force of the block acts at the block center. The X -component of the block weight is a Y -distance (perpendicular) of half the block height from the moment sum point. The rotational tendency about the moment point produced by this force component is counter-clockwise or negative. The Y -component of the block weight is a X -distance (perpendicular) of half the block width from the moment sum point. The rotational tendency about

the moment point produced by this force component is clockwise or positive. The contact forces exerted by the incline on the block act through the corner point and do not produce any moment about this point. }

4. Solve

The Y-force equation can be solved for the normal force in terms of the weight and the angle:

$$N = m g \cos(q)$$

The X-force equation can now be solved for the applied load in terms of the weight and the angle:

$$P = m_S N - m g \sin(q)$$

$$= m g \{ m_S \cos(q) - \sin(q) \}$$

The moment equation can now be used to determine a relationship between block height and width:

$$-P h - m g \sin(q) h/2 + m g \cos(q) b/2 = 0$$

$$-h m g \{ m_S \cos(q) - \sin(q) \} - m g \sin(q) h/2 + m g \cos(q) b/2 = 0$$

Dividing all terms by the weight, and grouping the terms involving the block height:

$$-h \{ m_S \cos(q) - \sin(q)/2 \} + \cos(q) b/2 = 0$$

Solving for h/b:

$$h/b = \cos(q) / [2 \{ m_S \cos(q) - \sin(q)/2 \}]$$

$$= \cos(q) / \{ 2 m_S \cos(q) - \sin(q) \}$$

Dividing the top and bottom of this relationship by $\cos(q)$:

$$h/b = 1 / \{ 2 m_S - \tan(q) \}$$

Results

$$\text{Maximum ratio} = h/b = 1 / \{ 2 m_S - \tan(q) \}$$

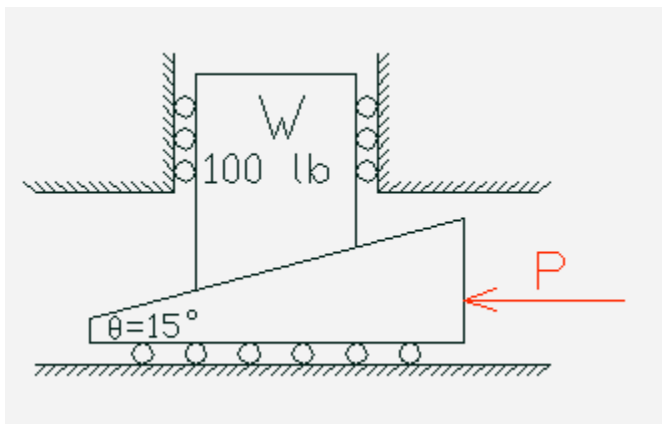
MECH 2110 - Statics & Dynamics

Chapter S6 Problem 53 Solution

Page 376, Engineering Mechanics - Statics, 4th Edition, Meriam and Kraige

Given: The wedge and block system loaded as shown below. The block is of weight, W , equal to 100 lb, while the wedge is of negligible weight. The block is prevented from horizontal movement by frictionless rollers. The surface between the wedge and the block makes an angle of q equal to 15° . The coefficient of static friction, m_s , between the block and the wedge is 0.2. The wedge is supported underneath by a horizontal surface. A horizontal applied force, P , is used to attempt to raise the weight.

Find: The horizontal force, P , required to initiate upward motion of the upper block for two different situations. In the first situation (a), the lower block is supported by rollers of negligible friction. In the second situation (b), the lower block is in direct contact with the supporting floor, and the static coefficient of friction, m_s , between the two surfaces is 0.2. Note that only the first situation is portrayed below.



0. Observations:

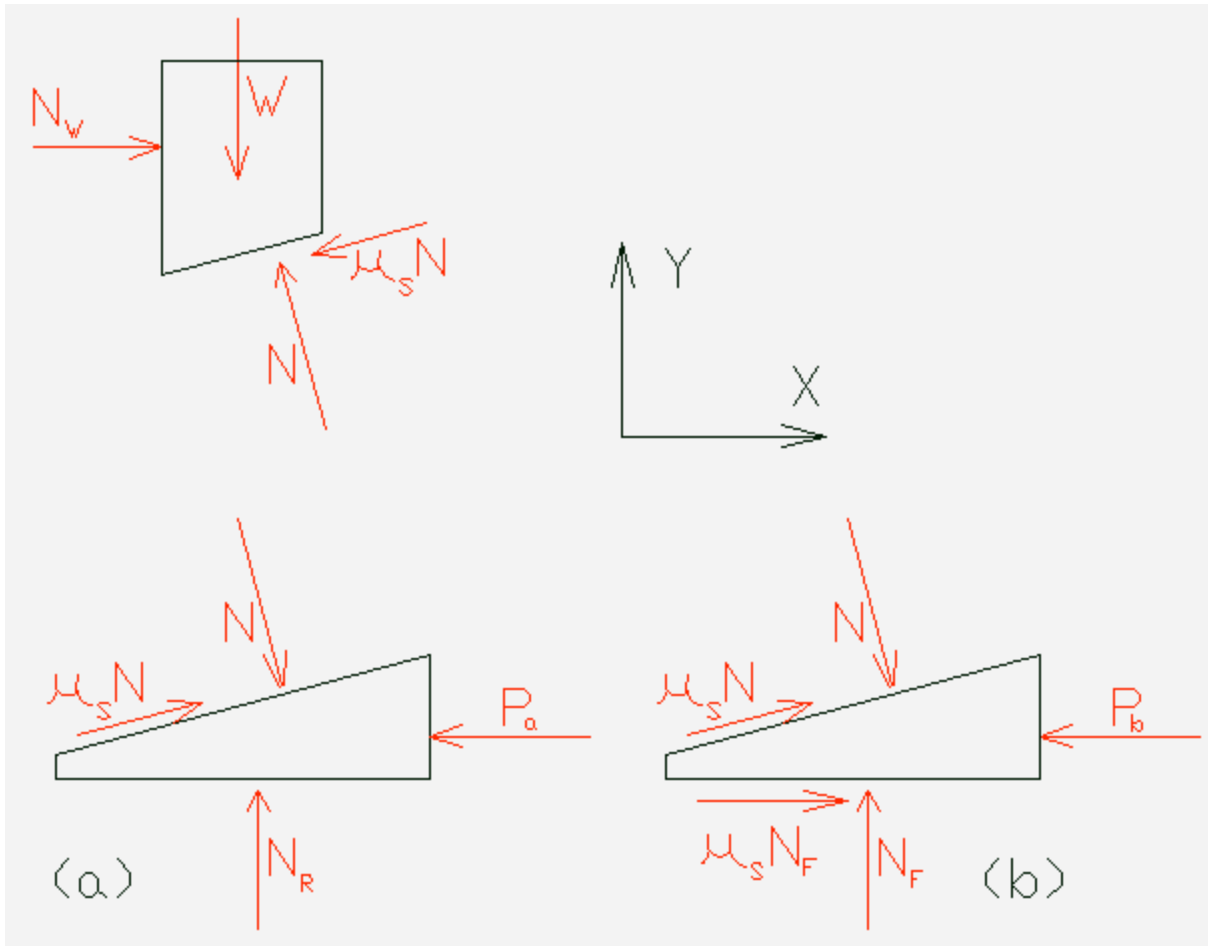
1. In both situations, the force P must be sufficient to just initiate slipping between the wedge and the block. For impending slip at that surface, the frictional force must be equal to the static coefficient of friction multiplied by the normal force.
2. The block is acted on by four forces. Its weight (known) is acting downward through the center of the block. A horizontal support force (unknown) is being exerted by the side rollers on the block. The wedge surface is exerting a frictional and a normal force on the block. The normal force acts diagonally upward in a direction that makes an angle q with the vertical. The frictional force acts tangent to the contacting surface. As the force on the wedge from the block acts up the wedge incline, resisting the motion of the wedge, the force on the block from the wedge must act down the incline. The magnitude of the frictional force is a known percentage of the normal force due to the impending slip condition, thus a single unknown, the magnitude of the normal force, characterizes these two forces. Considering the block as the mechanical system would yield equations involving only two unknowns, the magnitude of the side roller force and the magnitude of the wedge-block normal force.

In fact, the force equation in the vertical direction would involve but a single unknown, the magnitude of the wedge-block normal force. The block appears to be a very appealing choice for a mechanical system.

3. The forces of the wedge on the block can be determined as described in 2. The forces of the block on the wedge, are just equal and opposite to those forces. Having determined these forces, choosing the wedge as our mechanical system becomes attractive. Equilibrium of the wedge should enable us to determine the required applied load, P , in both situations of interest. When the wedge is supported by rollers, it is acted upon by the applied force, the reaction forces from the block, and a vertical upward force transmitted by the rollers. When the wedge is supported directly on the floor, it is acted upon by the applied force, the reaction forces from the block, a vertical upward force transmitted by the floor, and a horizontal frictional force, toward the right, resisting the impending motion, from the floor. As slip is impending, the magnitude of the friction force is known to be equal to the associated static coefficient of friction multiplied by the floor normal force.

1. Mechanical System - Two mechanical systems are of interest, one including only the block, the including only the wedge. We are interested in considering these systems under the conditions where the applied load, P , is just sufficient to begin motion of the block upward. We are interested in two situations for the wedge, when it is supported by rollers and when it is resting directly on a supporting surface.

2. Free Body Diagram



The figure shows the free body diagrams of the block and the wedge. The wedge is shown in both situations, (a) when the rollers are in place, and (b), when the rollers are not in place. When the rollers are present, any frictional force on the bottom of the wedge is negligible. Note the use of the law of action and reaction in relating the forces exerted on the block by the wedge to the forces exerted on the wedge by the block. The coordinate axes used are shown.

3. Equations

Block:

$$\sum F_Y = -W + N \cos(q) - m_S N \sin(q) = 0$$

Wedge:

Situation (a) - Rollers

$$\sum F_X = -P_a + N \sin(q) + m_S N \cos(q) = 0$$

$$\sum F_Y = N_R - N \cos(q) + m_S N \sin(q) = 0$$

Situation (b) - Direct floor contact

$$\sum F_X = -P_b + N \sin(q) + m_S N \cos(q) + m_S N_F = 0$$

$$\sum F_Y = N_F - N \cos(q) + m_S N \sin(q) = 0$$

4. Solve

Block - Y-force equation:

$$N \{ \cos(q) - m_S \sin(q) \} = W$$

$$N = W / \{ \cos(q) - m_S \sin(q) \}$$

$$= 100 \text{ lb} \{ \cos(15^\circ) - 0.2 \sin(15^\circ) \}$$

$$= 109.4 \text{ lb}$$

Situation (a) - Rollers

The X-force equation can be used to determine P_a :

$$P_a = N \sin(q) + m_S N \cos(q)$$

$$= 109.4 \text{ lb} \{ \sin(15^\circ) + 0.2 \cos(15^\circ) \}$$

$$= 49.4 \text{ lb}$$

Situation (b) - Direct floor support

The Y-force equation can be used to determine N_F :

$$N_F = N \cos(q) - m_S N \sin(q)$$

$$= 109.4 \text{ lb} \{ \cos(15^\circ) - 0.2 \sin(15^\circ) \}$$

$$= 100 \text{ lb} \{ \text{As you would expect} \}$$

The X-force equation can now be used to determine P_b :

$$P_b = N \sin(q) + m_S N \cos(q) + m_S N_F$$

$$= N \{ \sin(q) + m_S \cos(q) \} + m_S N_F$$

$$= 109.4 \text{ lb} \{ \sin(15^\circ) + 0.2 \cos(15^\circ) \} + 0.2 \cdot 100 \text{ lb}$$

$$= 69.4 \text{ lb}$$

Results

Force required to initiate motion in situation (a) (rollers) = $P_a = 49.4 \text{ lb}$

Force required to initiate motion in situation (b) (direct floor contact) = $P_b = 69.4 \text{ lb}$

The uniform pole has weight W and length L . If it is placed against the smooth wall and on the rough floor in the position shown, will it remain in this position when it is released? The coefficient of static friction is μ_s

Units Used :

Given:

$$W := 30\text{lb}$$

$$L := 26\text{ft}$$

$$d := 10\text{ft}$$

$$\mu_s := 0.3$$

Solution:

$$\curvearrowleft + \Sigma M_A = 0;$$

$$W \cdot \frac{d}{2} - N_B \cdot \sqrt{L^2 - d^2} = 0$$

$$N_B := \frac{W \cdot d}{2 \cdot \sqrt{L^2 - d^2}}$$

$$N_B = 6.25\text{ lb}$$

$$\rightarrow \Sigma F_x = 0;$$

$$N_B - F_A = 0 \quad F_A := N_B$$

$$F_A = 6.25\text{ lb}$$

$$+\uparrow \Sigma F_y = 0;$$

$$N_A - W = 0 \quad N_A := W$$

$$N_A = 30\text{ lb}$$

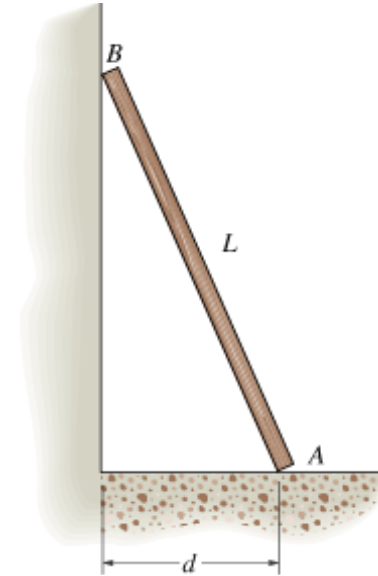
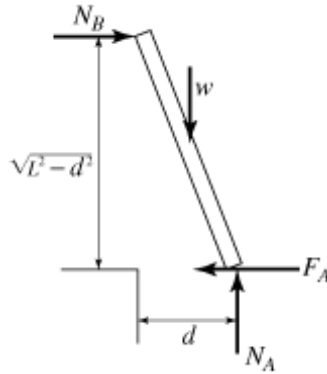
Check

$$F_{A\text{max}} := \mu_s \cdot N_A$$

$$F_{A\text{max}} = 9\text{ lb}$$

$$F_{A\text{max}} > F_A$$

Hence, the pole will remain stationary



The uniform pole has a weight W and length L . Determine the distance d it can be placed from the smooth wall and not slip. The coefficient of the static friction between the floor and the pole is μ_s .

Units Used :

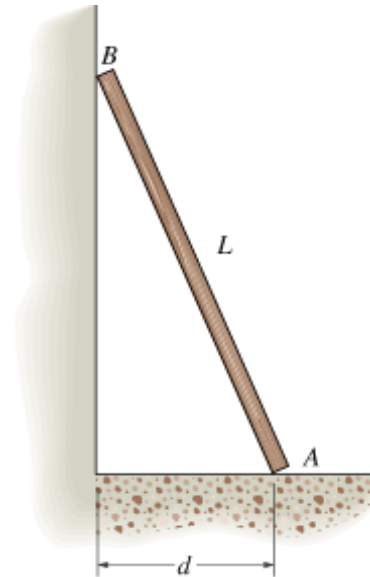
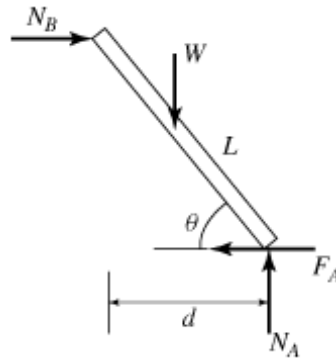
Given:

$$W := 30\text{lb}$$

$$L := 26\text{ft}$$

$$\mu_s := 0.3$$

Solution:



$$+\uparrow \Sigma F_y = 0; \quad N_A - W = 0$$

$$N_A := W \quad N_A = 30\text{lb}$$

$$F_A = F_{A\text{max}} = \mu_s N_A$$

$$F_A := \mu_s N_A \quad F_A = 9.00\text{lb}$$

$$+\rightarrow \Sigma F_x = 0; \quad N_B - F_A = 0$$

$$N_B := F_A \quad N_B = 9\text{lb}$$

$$\curvearrow + \Sigma M_A = 0; \quad W \cdot \frac{L}{2} \cdot \cos(\theta) - N_B \cdot L \cdot \sin(\theta) = 0$$

$$\theta := \text{atan}\left(\frac{1}{2} \cdot \frac{W}{N_B}\right) \quad \theta = 59.04\text{ deg}$$

$$d := L \cdot \cos(\theta) \quad d = 13.4\text{ft}$$