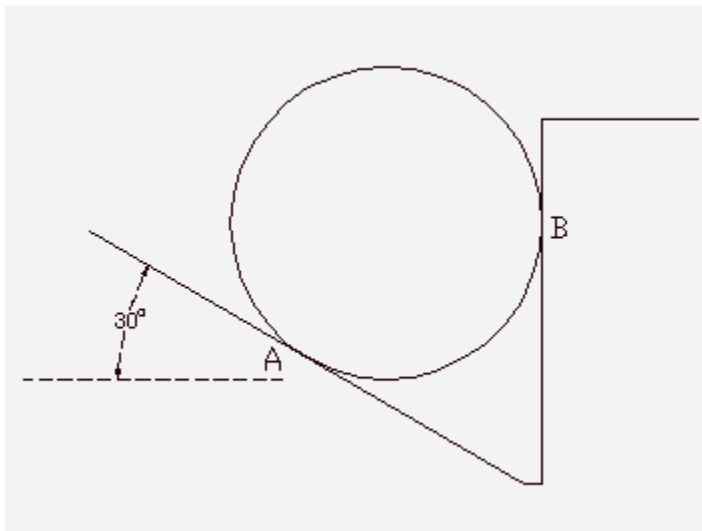
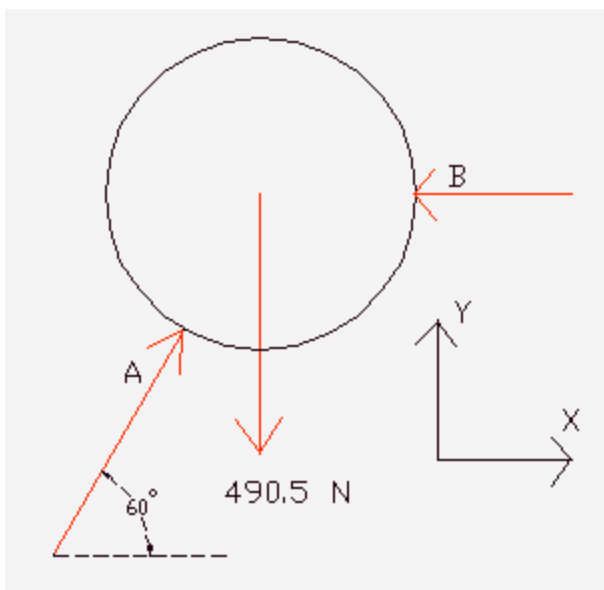


Chapter 3 Problem 1 Page 126



The 50-kg homogenous smooth sphere rests on the 30° incline and bears against the smooth vertical wall B. Calculate the contact forces at A and B.

1. Mechanical System = Sphere. The sphere is the appropriate mechanical system because the only force you know (weight) acts on the sphere and the two forces you want to know (contact forces) also act on the sphere.



2. Free Body Diagram = As smooth walls provide no resistance to movement along the wall, the only force they can exert is normal or perpendicular to the wall. Thus the free body diagram contains three forces, the weight of the ball and two normal contact forces at A and B.

3. Equations = Sum of Forces = Weight (downward) + Contact at B(horizontal) + Contact at A (diagonally upward). Note that as the directions of the forces at A and B are known but their magnitudes are unknown, we express these forces in terms of symbols representing their magnitude multiplied by unit vectors representing their direction.

$$\sum \mathbf{F} = -50 \text{ kg} * 9.81 \text{ m/s}^2 \mathbf{j} - B \mathbf{i} + A (\cos 60 \mathbf{i} + \sin 60 \mathbf{j})$$

j)

$$= (1/2 A - B) \mathbf{i} + (3^{1/2}/2 A - 490.5 \text{ N}) \mathbf{j} = \mathbf{0} = 0 \mathbf{i} + 0 \mathbf{j}$$

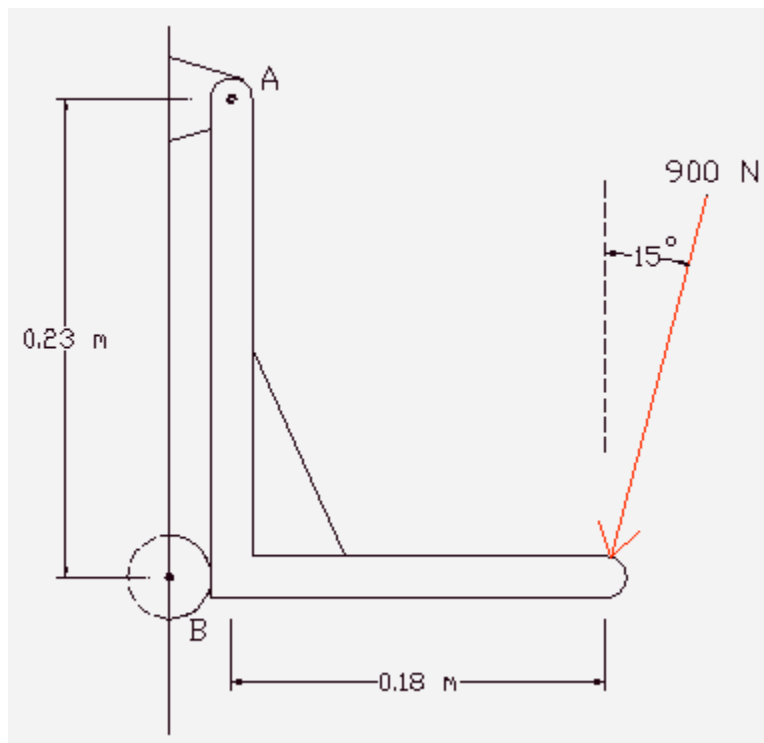
4. Solve the Equations. For two vectors to be equal, each of their components must be equal. Thus each of the two components of the resultant force vector must be zero. Noting that the Y component involves only the unknown A, we can readily solve for A.

$$A = 2 * 490.5 \text{ N} / 3^{1/2} = 566.4 \text{ N}$$

Now that A is known, B is the only unknown in the X component of the sum of the forces.

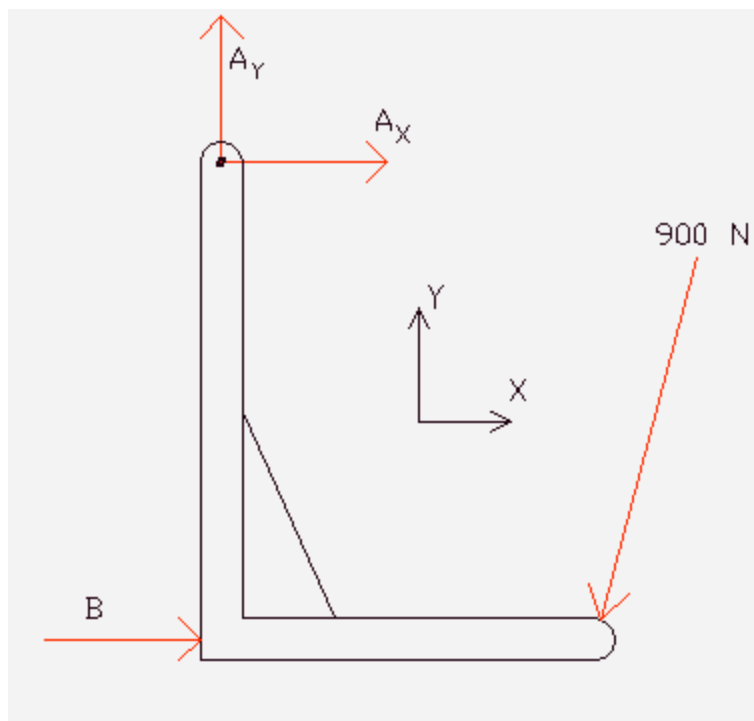
$$B = 1/2 A = 283.2 \text{ N}$$

Chapter 3 Problem 13 Page 129



Calculate the magnitude F of the force supported by the pin at A for the loaded bracket. Neglect the weight of the bracket.

1. Mechanical System = Bracket. The bracket is the appropriate mechanical system as the only force we know (900 N) acts on the bracket and the forces we want to know (at pin A) act on the bracket.



2. Free Body Diagram. 900 N force applied diagonally downward. Weight of the bracket is neglected so no weight force is shown. Contact at B with a smooth roller. The only resistance is to horizontal movement, so we have only a horizontal force. The pin at A resists both horizontal and vertical movement so we have both a horizontal and a vertical component of force at A . Pin A does permit the bracket to rotate or swing freely so there is no couple at that point. Note that we have depicted the forces exerted by the pin A on the bracket. By the law of action and reaction these forces are equal and opposite to the forces exerted by the bracket on the pin. Thus in finding the magnitude of the force exerted by the pin

on the bracket, we are also finding the magnitude of the force exerted on the pin by the bracket.

3. Sum of the Forces = 900 N diagonally downward. Horizontal force of unknown magnitude at B. Force at A with unknown horizontal and vertical components. Note that all unknown values have been represented by appropriate symbols in the free body diagram.

$$\begin{aligned}\sum \mathbf{F} &= 900 \text{ N} (-\sin(15) \mathbf{i} - \cos(15) \mathbf{j}) + B \mathbf{i} + A_X \mathbf{i} + A_Y \mathbf{j} \\ &= (B + A_X - 232.9 \text{ N}) \mathbf{i} + (A_Y - 869.3 \text{ N}) \mathbf{j} = \mathbf{0} = 0 \mathbf{i} + 0 \mathbf{j}\end{aligned}$$

Noting that the force equation contains three unknown quantities but only two components indicates that an additional equation will be required. Thus we must also evaluate the sum of the moments. We can choose to sum moments about any point. We normally choose a point that will make the moment evaluation as simple as possible. If we sum moments about the pin at A, then the position vector to the pin forces at A from that point will be zero, and their contribution to the moment will be zero. This appears to be a good choice of points. The position vector of the force at B from point A can be seen to be 0.23 m straight down. The position vector from point A to the 900 N force can be seen to be 0.23 m straight down and 0.18 m to the right.

$$\sum \mathbf{M}_A = \mathbf{0} \times (A_X \mathbf{i} + A_Y \mathbf{j}) + (-0.23 \text{ m} \mathbf{j}) \times B \mathbf{i} + (-0.23 \text{ m} \mathbf{j} + 0.18 \text{ m} \mathbf{i}) \times (900 \text{ N} (-\sin(15) \mathbf{i} - \cos(15) \mathbf{j}))$$

Recalling that $\mathbf{i} \times \mathbf{i}$ is zero, $\mathbf{i} \times \mathbf{j}$ is \mathbf{k} , $\mathbf{j} \times \mathbf{i}$ is $-\mathbf{k}$ and $\mathbf{j} \times \mathbf{j}$ is zero, and performing the required cross products yields:

$$\begin{aligned}\sum \mathbf{M}_A &= 0.23 \text{ m} B \mathbf{k} - 0.23 \text{ m} 900 \text{ N} \sin(15) \mathbf{k} - 0.18 \text{ m} 900 \text{ N} \cos(15) \mathbf{k} \\ &= (0.23 \text{ m} B - 53.6 \text{ N m} - 156.5 \text{ N m}) \mathbf{k} \\ &= 0.23 \text{ m} B - 210.1 \text{ N m}) \mathbf{k} = \mathbf{0} = 0 \mathbf{k}\end{aligned}$$

4. Solving the Equations. If two vectors are equal, their components must be equal. Thus the X and Y components of the resultant force vector must be zero. The Z component of the resultant moment about point A must be zero. Setting the Y component of the resultant force vector to zero determines the Y component of the pin force, A_Y .

$$A_Y = 869.3 \text{ N}$$

Setting the Z component of the moment equation to zero, determines the force exerted on the bracket at B, B.

$$B = 210.1 \text{ N m} / 0.23 \text{ m} = 913.3 \text{ N}$$

Setting the X component of the resultant force to zero and using the value of the force B just determined, yields:

$$A_X = 232.9 \text{ N} - B = -680.3 \text{ N}$$

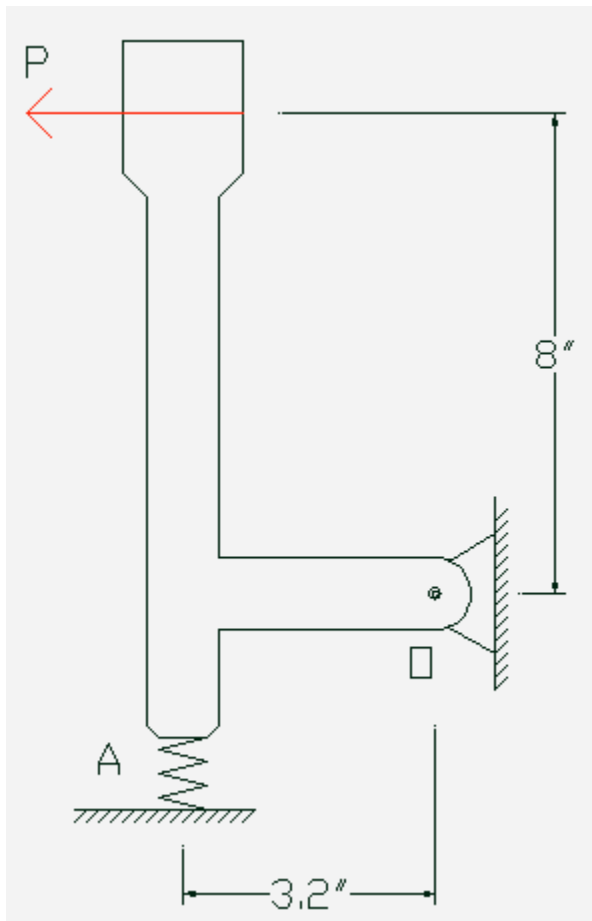
The fact that the result for A_X is negative means that the force exerted on the bracket by the pin in the horizontal direction is in the opposite direction from that shown on the free body diagram. Thus the force exerted on the bracket by the pin is vertically upward (as shown) and horizontally to the left (opposite of shown). The total force exerted by the pin on the bracket is :

$$\mathbf{A} = A_X \mathbf{i} + A_Y \mathbf{j} = -680.3 \text{ N } \mathbf{i} + 869.3 \text{ N } \mathbf{j}$$

The magnitude of this force, which is also the magnitude of the force exerted by the bracket on the pin, is given by the square root of the sum of the squares of the components.

$$\|\mathbf{A}\| = (680.3^2 + 869.3^2)^{1/2} \text{ N} = 1103.9 \text{ N}$$

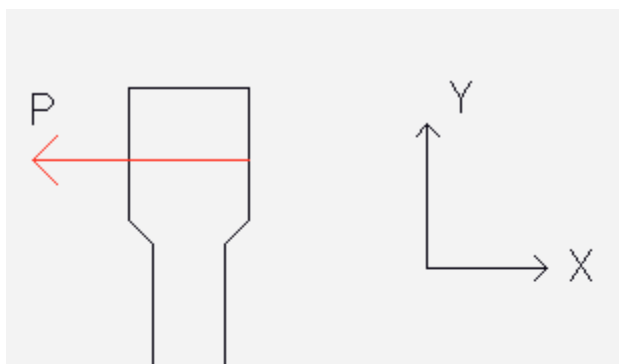
Chapter 3 Problem 15 Page 129



The force P on the handle of the positioning lever produces a vertical compression of 60 lb in the coiled spring at A in the position shown. Determine the corresponding force exerted by the pin at O on the lever.

1. Mechanical System = The handle in the position shown.

2. Free Body Diagram (below).

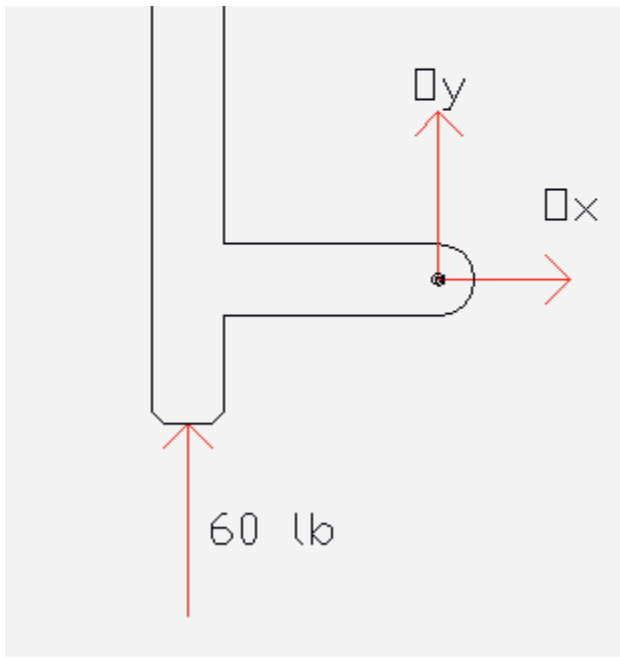


3. Equations

$$\begin{aligned} \mathbf{S F} &= -P \mathbf{i} + 60 \text{ lb } \mathbf{j} + O_x \mathbf{i} + O_y \mathbf{j} \\ &= (O_x - P) \mathbf{i} + (O_y + 60 \text{ lb}) \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \mathbf{S M}_O &= -3.2'' \mathbf{i} \times 60 \text{ lb } \mathbf{j} + 8'' \mathbf{j} \times -P \mathbf{i} \\ &= (8'' P - 192 \text{ lb}) \mathbf{k} = \mathbf{0} \end{aligned}$$

4. Solve



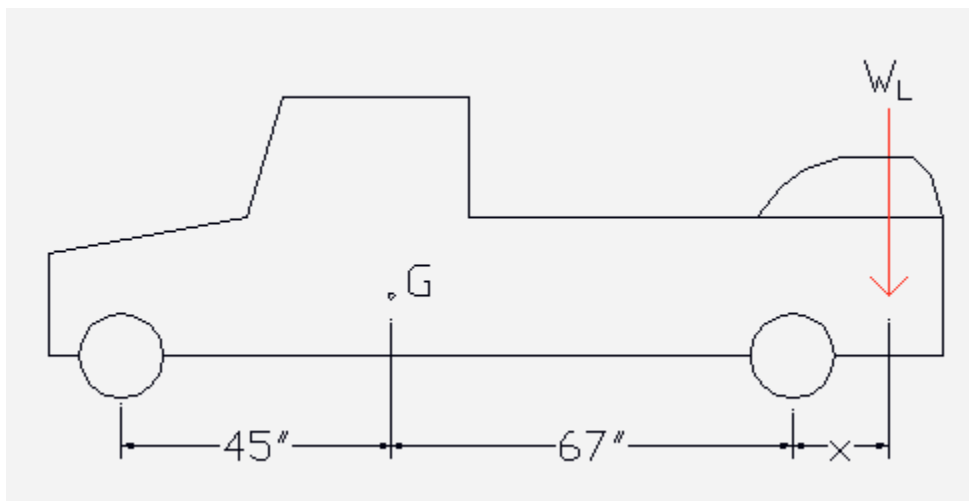
$P = 24 \text{ lb}$ (from moment equation)

$O_x = 24 \text{ lb}$ (from x force equation)

$O_y = -60 \text{ lb}$ (from y force equation)

$$O = (O_x^2 + O_y^2)^{1/2} = 64.62 \text{ lb}$$

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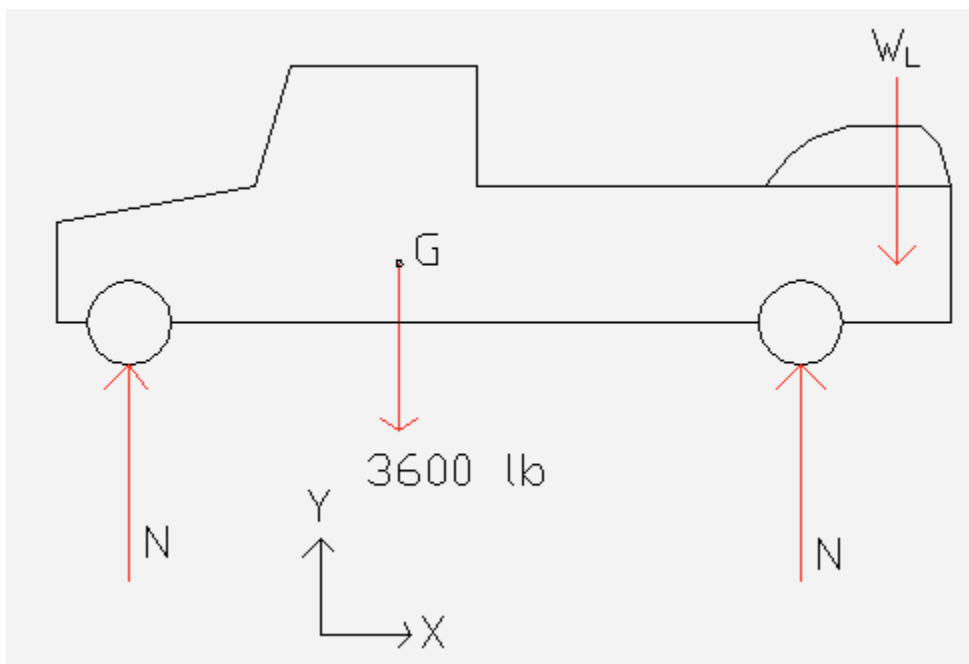


The indicated location (G) of the center of gravity of the 3600-lb pickup truck is for the unladen condition. If a load whose center of gravity is $x = 16$ in. behind the rear axle is added to the truck, determine the load weight W_L for which the normal forces under the front and rear wheels are equal.

equal.

1. Mechanical System = Truck, loaded such that normal forces under front and rear wheels are equal.

2. Free Body Diagram below.



3. Equations:

$$\begin{aligned} \sum \mathbf{F} &= N \mathbf{j} - 3600 \text{ lb } \mathbf{j} + N \mathbf{j} \\ &\quad - W_L \mathbf{j} \\ &= (2N - 3600 \text{ lb} - W_L) \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\mathbf{j} = \mathbf{0}$$

$$\begin{aligned} \sum \mathbf{M}_W &= -128'' \mathbf{i} \times N \mathbf{j} + \\ &\quad -83'' \mathbf{i} \times 3600 \text{ lb } \mathbf{j} + -16'' \mathbf{i} \times N \mathbf{j} \\ &= (298800 \text{ "lb} - \\ &\quad 144'' N) \mathbf{k} = \mathbf{0} \end{aligned}$$

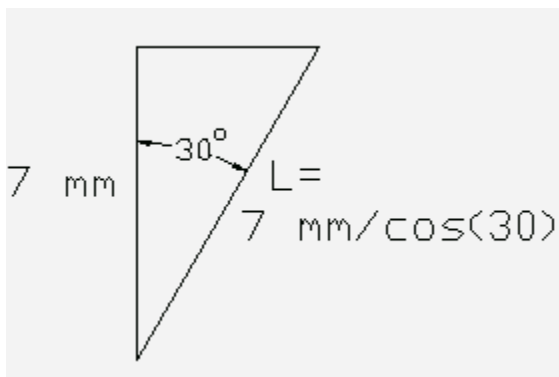
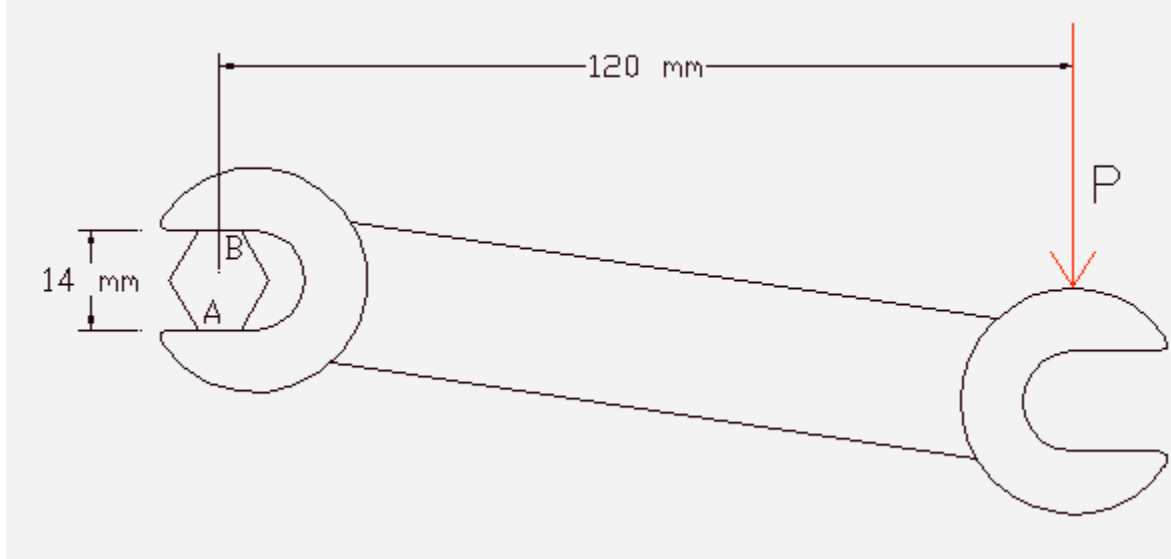
4. Solve:

$$N = 2075 \text{ lb (from moment equation)}$$

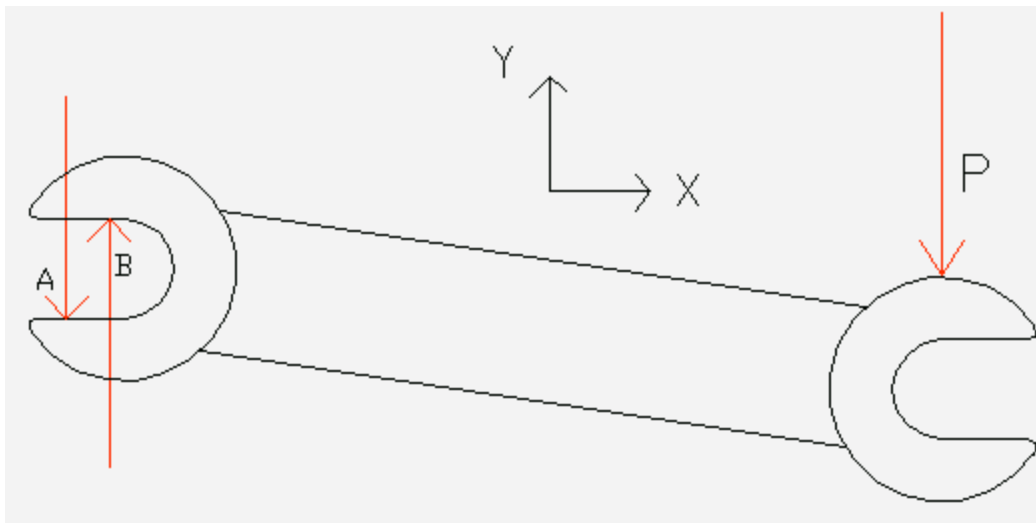
$$W_L = 550 \text{ lb (from Y force equation)}$$

Chapter 3 Problem 37 Page 135

A torque (moment) of 24 N m is required to turn the bolt about its axis. Determine P and the forces between the smooth hardened jaws of the wrench and the corners of A and B of the hexagonal head. Assume that the wrench fits easily on the bolt so that contact is made at corners A and B only.



1. Mechanical System = Wrench. The forces of interest all act on the wrench. The information known is the moment on the bolt produced by the wrench. By the law of action and reaction, this must be the same moment (opposite direction) produced by the bolt on the wrench. Note that as the bolt is hexagonal and spans a vertical distance of 14 mm, each side of the bolt must be of length $(14 \text{ mm}/2)/\cos(30)$ or 8.08 mm (see figure of triangle showing half height of bolt and side length L).



2. Free Body Diagram. The forces acting on the wrench include the applied P and contact forces at A and B. As the contact surfaces are smooth, the contact forces must be normal to the surfaces. Thus at A we have a force vertically downward on the wrench (as the wrench is pushing up on the bolt at A) and at B we have a force vertically upward on the wrench (as the wrench is pushing downward on the bolt).

3. Equations. Sum of the forces includes P and A downward and B upward.

$$\sum \mathbf{F} = -P\mathbf{j} + B\mathbf{j} - A\mathbf{j} = (B - A - P)\mathbf{j} = \mathbf{0} = 0\mathbf{j}$$

In order to use the fact that the moment about the bolt axis is 24 N m, we must also sum moments. As the distance to P is given from the center of the bolt, this becomes a convenient point to sum moments about. Noting that each edge of the hexagonal bolt is 8.08 mm in length, then each contact corner is a horizontal distance of 4.04 mm from the center of the bolt. Letting C represent the bolt center:

$$\begin{aligned} \sum \mathbf{M}_C &= 120 \text{ mm } \mathbf{i} \times -P\mathbf{j} + 4.04 \text{ mm } \mathbf{i} \times B\mathbf{j} - 4.04 \text{ mm } \mathbf{i} \times -A\mathbf{j} \\ &= (-120 \text{ mm } P + 4.04 \text{ mm } (B+A)) \mathbf{k} = \mathbf{0} = 0\mathbf{k} \end{aligned}$$

We also know that net moment from the two forces on the bolt must be 24 N m. Thus the net moment on the wrench from the two bolt forces must also be 24 N m. Thus:

$$4.04 \text{ mm } (B+A) = 24 \text{ N m}$$

4. Solving. Knowing that $4.04 \text{ mm } (B+A) = 24 \text{ N m}$ determines the sum of B+A.

$$A+B = 24 \text{ N m} / 4.04 \text{ mm} = 5938 \text{ N}$$

Now that we know A+B, the fact that the total moment on the wrench must be zero, enables us to determine P.

$$P = 4.04 \text{ mm } (B+A) / 120 \text{ mm} = 24 \text{ N m} / 120 \text{ mm} = 200 \text{ N}$$

The force equation can now be solved for B-A.

$$B - A = P = 200 \text{ N}$$

Adding the expression for B-A and A+B yields:

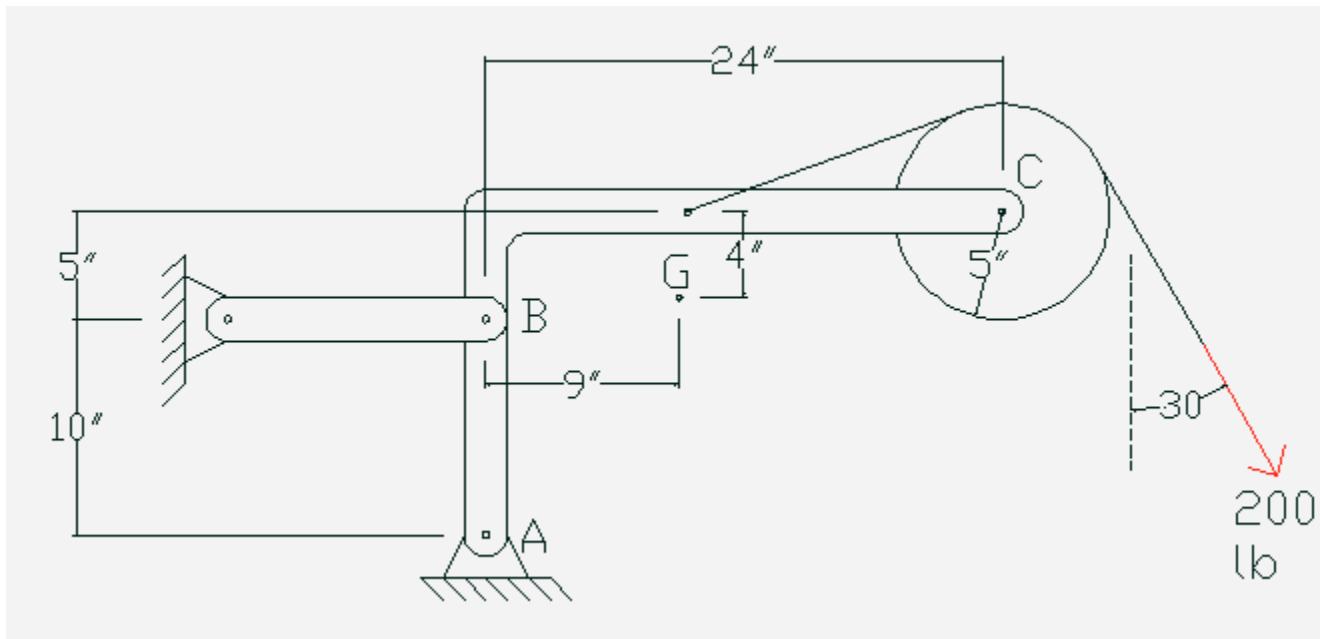
$$2 B = 5938 \text{ N} + 200 \text{ N} = 6138 \text{ N}$$

$$B = 3069 \text{ N}$$

Thus we can now solve for A from the expression for B-A.

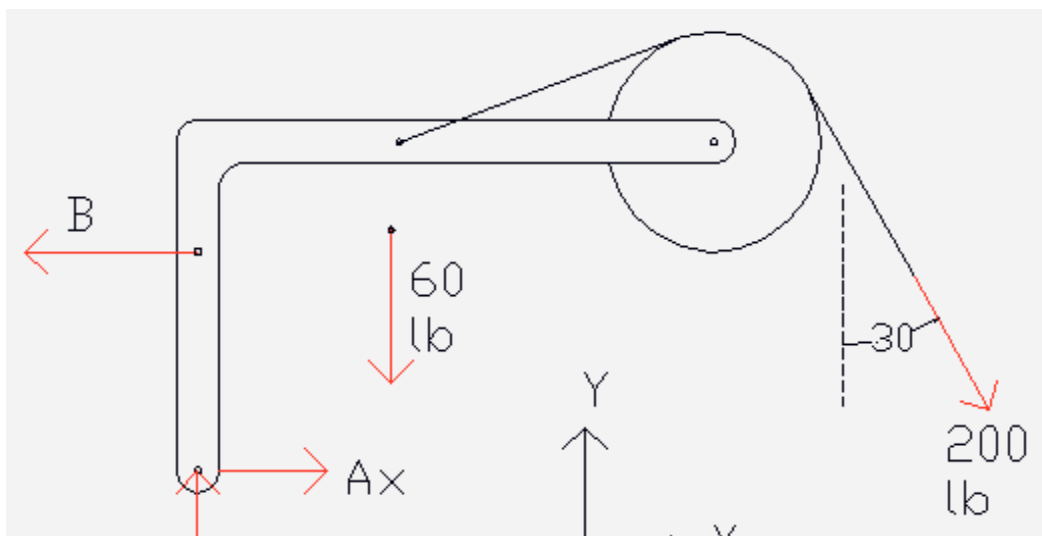
$$A = B - P = 3069 \text{ N} - 200 \text{ N} = 2869 \text{ N}$$

Wow - we get some big forces on that bolt! Now we know why they use hardened jaws!



The member ABC and the pulley together weigh 60 lb with center of gravity at G. Calculate the magnitude of the force supported by the pin at A.

1. Mechanical System = Member ABC, the pulley, the pin connecting the pulley and ABC, and the rope.
2. Free Body Diagram below (note that the horizontal bar attached to B is a two force member and thus exerts a horizontal force).



3. Equations:

$$\begin{aligned} \mathbf{S F} &= -B \mathbf{i} + A_x \mathbf{i} + A_y \mathbf{j} + 200 \text{ lb} (\sin 30^\circ \mathbf{i} - \cos 30^\circ \mathbf{j}) - 60 \text{ lb } \mathbf{j} \\ &= (A_x - B + 100 \text{ lb}) \mathbf{i} + (A_y - 60 \text{ lb} - 100 \cdot 3^{1/2} \text{ lb}) \mathbf{j} = \mathbf{0} \end{aligned}$$

Note that the rope comes off the



Ay

pulley on a tangent making an angle of 30 degrees with the vertical, hence the

contact point on the pulley makes an angle of 30 degrees with the horizontal.

$$\begin{aligned} \sum \mathbf{M}_A &= 10'' \mathbf{j} \times -B \mathbf{i} + (9'' \mathbf{i} \times -60 \text{ lb } \mathbf{j}) + \{(24'' + 5'' \cos 30)\mathbf{i} + (15'' + 5'' \sin 30)\mathbf{j}\} \times 200 \text{ lb} (\sin 30 \mathbf{i} - \\ &\cos 30 \mathbf{j}) \\ &= (10'' B - 540 \text{ in lb} - 6656.9 \text{ in lb}) \mathbf{k} = \mathbf{0} \end{aligned}$$

4. Solve:

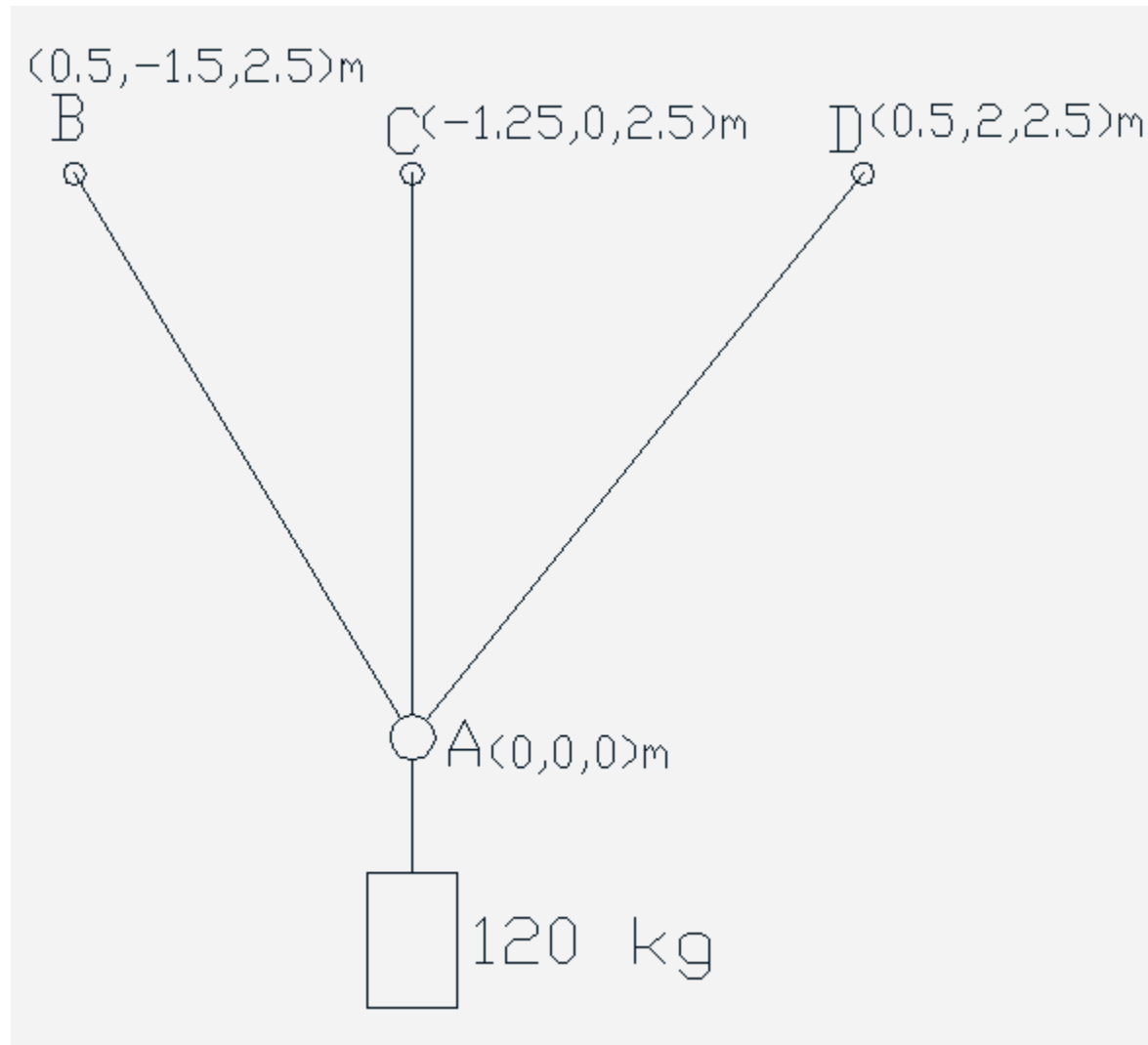
$$B = 719.7 \text{ lb (from moment equation)}$$

$$A_x = 619.7 \text{ lb (from x force equation)}$$

$$A_y = 233.2 \text{ lb (from y force equation)}$$

$$A = (A_x^2 + A_y^2)^{1/2} = 662.1 \text{ lb}$$

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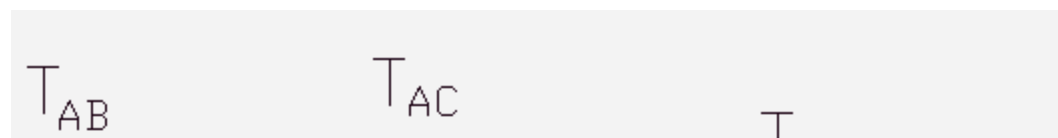


The three cables, AB, AC, and CD support the 120 kg mass. The three

dimensional locations of the points A, B, C, and D are given in the figure. Note that Z is vertically upward, Y is from left to right, while X is from back to front. Determine the tension in cables AB, AC, and AD.

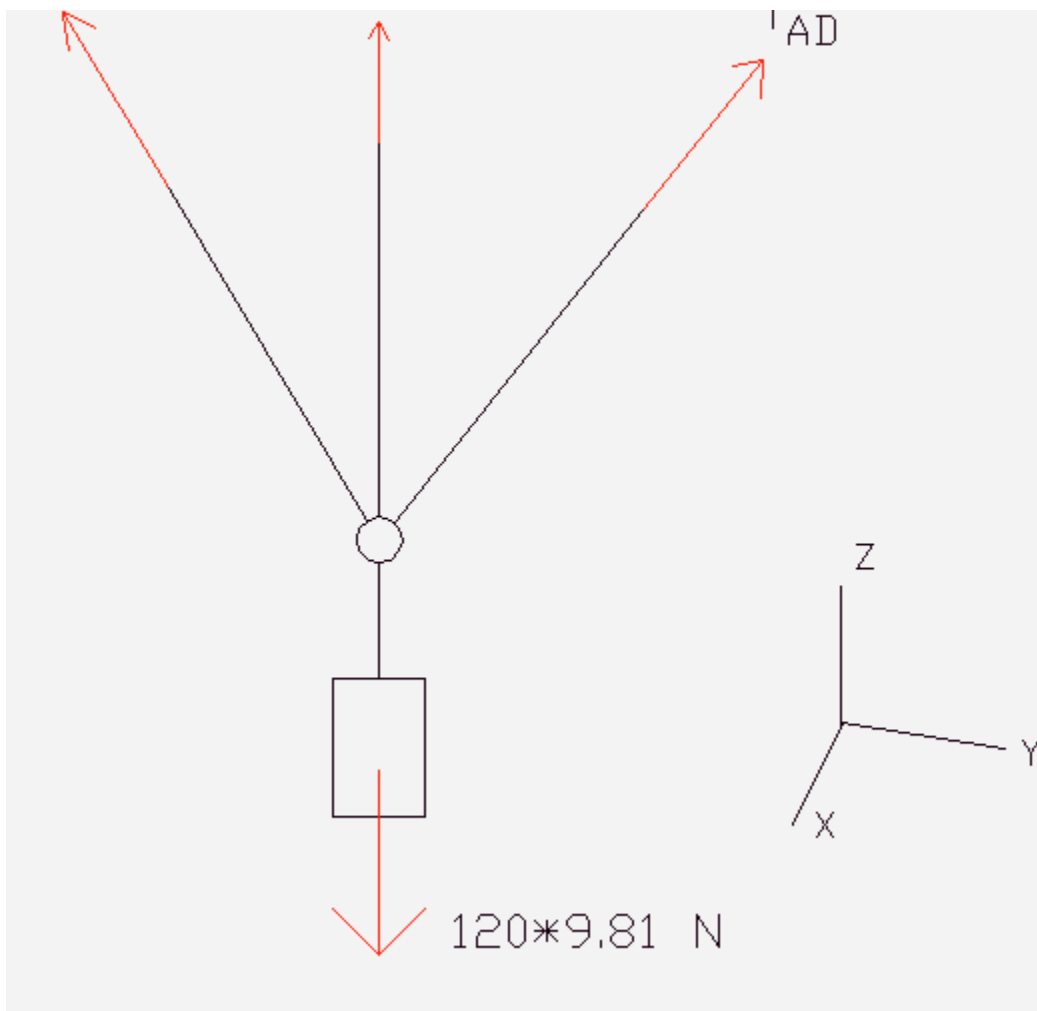
1. Mechanical System = 120 kg mass, ring, and parts of supporting cables.

2. Free Body Diagram below.



3. Equations:

$$\mathbf{S F} = T_{AB} \mathbf{e}_{AB} +$$



$$T_{AC} \mathbf{e}_{AC} + T_{AD} \mathbf{e}_{AD} - 120 \text{ kg } 9.81 \text{ m/s}^2 \mathbf{k} = \mathbf{0}$$

Note that \mathbf{e} represents a unit vector in the direction of the force. As each of the tensions acts along the associated cable, we must evaluate unit vectors parallel to each of the cables.

The position vector from A to B is:

$$\mathbf{r}_{AB} = 2.5 \text{ m } \mathbf{k} - 1.5 \text{ m } \mathbf{j} + 0.5 \text{ m } \mathbf{i}$$

The magnitude of the position vector from A to B is:

$$r_{AB} = (2.5^2 + 1.5^2 + 0.5^2)^{1/2} \text{ m} = 1/2 \ 35^{1/2} \text{ m}$$

The unit vector in the direction from A to B is the position vector from A to B divided by its magnitude:

$$\mathbf{e}_{AB} = \mathbf{r}_{AB} / r_{AB} = (1 \mathbf{i} - 3 \mathbf{j} + 5 \mathbf{k}) / 35^{1/2}$$

The unit vectors in the directions of the other two cables can be developed in a similar fashion:

$$\mathbf{e}_{AC} = (-1 \mathbf{i} + 2 \mathbf{k}) / 5^{1/2}$$

$$\mathbf{e}_{AD} = (1 \mathbf{i} + 4 \mathbf{j} + 5 \mathbf{k}) / 42^{1/2}$$

Using these expressions in the sum of the forces:

$$T_{AB} (1 \mathbf{i} - 3 \mathbf{j} + 5 \mathbf{k}) / 35^{1/2} + T_{AC} (-1 \mathbf{i} + 2 \mathbf{k}) / 5^{1/2}$$

$$+T_{AD} (1 \mathbf{i} + 4 \mathbf{j} + 5 \mathbf{k}) / 42^{1/2}$$

$$- 120 * 9.81 \text{ N } \mathbf{k} = \mathbf{0}$$

For a vector to be zero, all three components must be zero. Thus we obtain the following three equations in matrix form.

$$\begin{array}{l} | 1/35^{1/2} \quad -1/5^{1/2} \quad 1/42^{1/2} | \quad T_{AB} \quad 0 \quad \text{(X components)} \\ | -3/35^{1/2} \quad 0 \quad 4/42^{1/2} | \quad T_{AC} = 0 \quad \text{(Y components)} \\ | 5/35^{1/2} \quad 2/5^{1/2} \quad 5/42^{1/2} | \quad T_{AD} \quad 120 * 9.81 \text{ N} \quad \text{(Z components)} \end{array}$$

4. Solve

Multiplying top equation by 2 and adding to bottom equation:

$$\begin{array}{l} | 1/35^{1/2} \quad -1/5^{1/2} \quad 1/42^{1/2} | \quad T_{AB} \quad 0 \\ | -3/35^{1/2} \quad 0 \quad 4/42^{1/2} | \quad T_{AC} = 0 \\ | 7/35^{1/2} \quad 0 \quad 7/42^{1/2} | \quad T_{AD} \quad 120 * 9.81 \text{ N} \end{array}$$

Multiplying 7 times second equation and adding to three times third equation:

$$\begin{array}{l} | 1/35^{1/2} \quad -1/5^{1/2} \quad 1/42^{1/2} | \quad T_{AB} \quad 0 \\ | -3/35^{1/2} \quad 0 \quad 4/42^{1/2} | \quad T_{AC} = 0 \\ | 0 \quad 0 \quad 49/42^{1/2} | \quad T_{AD} \quad 360 * 9.81 \text{ N} \end{array}$$

Solving the third equation for T_{AD} :

$$T_{AD} = 467.1 \text{ N}$$

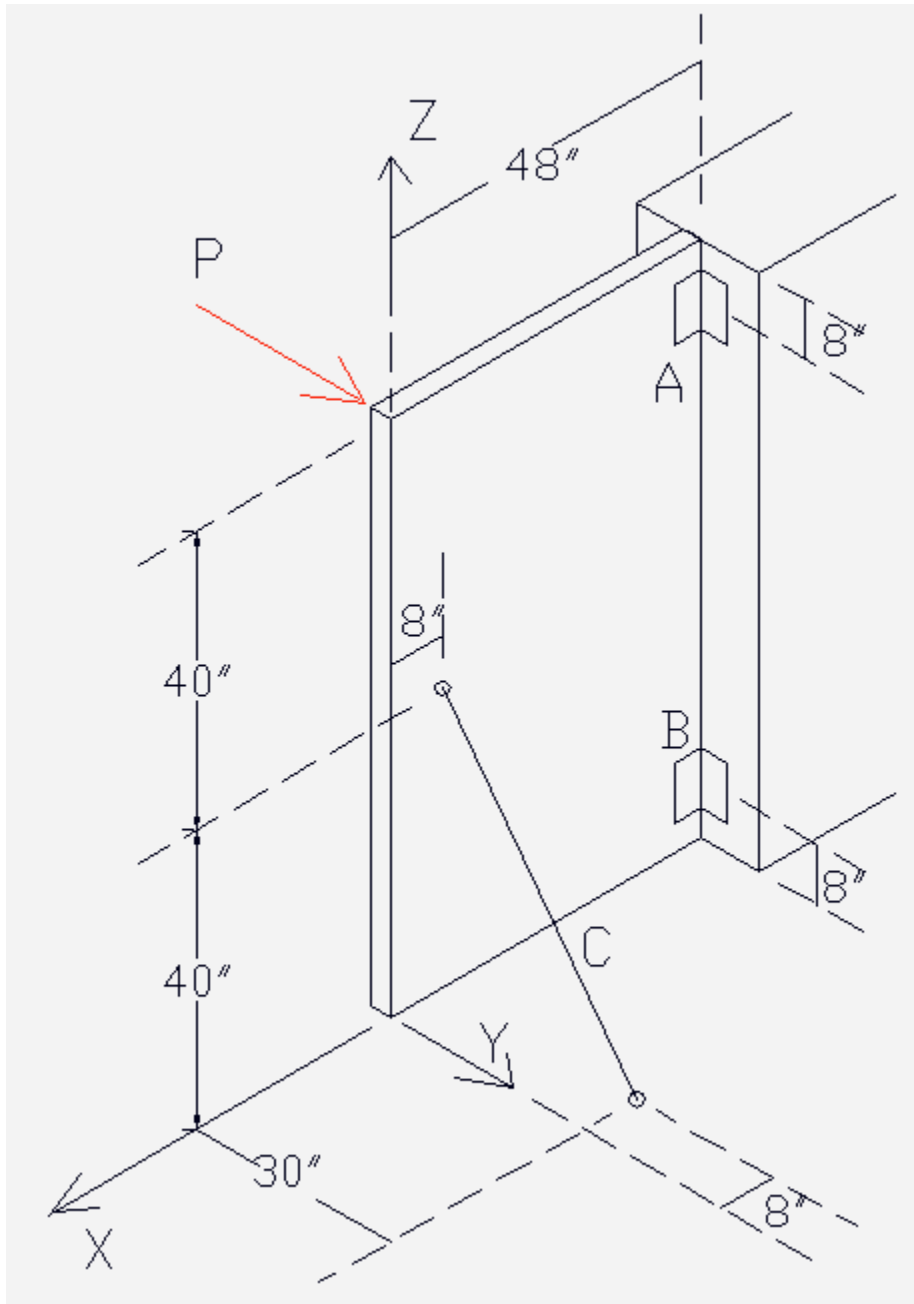
Plugging back into the second equation to solve for T_{AB} :

$$T_{AB} = 568.5 \text{ N}$$

Plugging back into the first equation to solve for T_{AC} :

$$T_{AC} = 376.0 \text{ N}$$

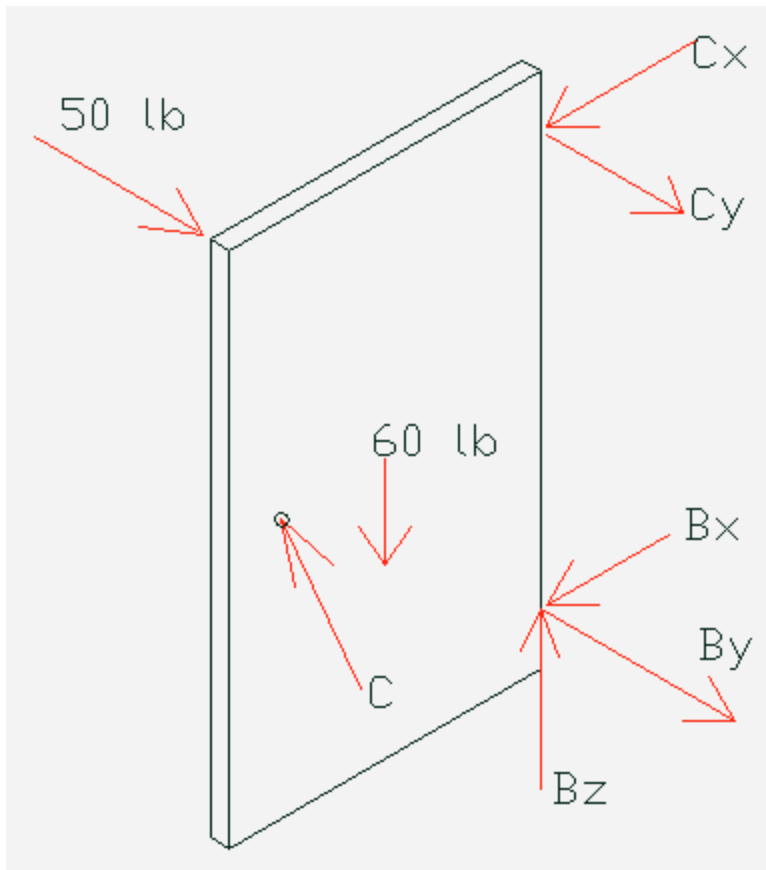
Chapter 3 Problem 77 Page 155



The uniform panel door shown in the isometric drawing weighs 60 lb and is prevented from opening by the strut C, which is a light two-force member whose upper end is secured under the door knob and whose lower end is attached to a rubber cup which does not slip on the floor. Of the door hinges A and B, only B can support force in the vertical z-direction. Calculate the compression in the strut and the horizontal components of the forces supported by the hinges A and B when a horizontal force $P = 50$ lb is applied normal to the plane of the door as shown.

1. Mechanical System = Door

2. Free Body Diagram below.



3. Equations:

Noting that the force in the strut is parallel to the vector $(0, -30'', 40'')$, we obtain the following unit vector for the strut force:

$$\mathbf{e}_C = -3/5 \mathbf{j} + 4/5 \mathbf{k}$$

$$\begin{aligned} \sum \mathbf{F} &= 50 \text{ lb } \mathbf{j} - 60 \text{ lb } \mathbf{k} + C (-3/5 \mathbf{j} + 4/5 \mathbf{k}) \\ &+ B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} + A_x \mathbf{i} + A_y \mathbf{j} \\ &= (A_x + B_x) \mathbf{i} + (50 \text{ lb} - 3/5 C + A_y + B_y) \mathbf{j} + (4/5 C - 60 \text{ lb} + B_z) \mathbf{k} = \mathbf{0} \end{aligned}$$

Note that as the door is uniform, its mass center is located at its geometric center. Thus it is 24'' along the X axis from the hinges and 40'' above the floor (along the Z axis). Summing moments about the hinge at B to eliminate the three components of B yields:

$$\sum \mathbf{M}_B = 64'' \mathbf{k} \times (A_x \mathbf{i} + A_y \mathbf{j}) + 24'' \mathbf{i} \times$$

$$\begin{aligned} &-60 \text{ lb } \mathbf{k} + (48'' \mathbf{i} + 72'' \mathbf{k}) \times 50 \text{ lb } \mathbf{j} + (40'' \mathbf{i} + 32'' \mathbf{k}) \times C (-3/5 \mathbf{j} + 4/5 \mathbf{k}) \\ &= (96/5'' C - 3600 \text{ in } \cdot \text{lb} - 64'' A_y) \mathbf{i} + (64'' A_x + 1440 \text{ in } \cdot \text{lb} - 32'' C) \mathbf{j} + (2400 \text{ in } \cdot \text{lb} - 24'' C) \mathbf{k} = \mathbf{0} \end{aligned}$$

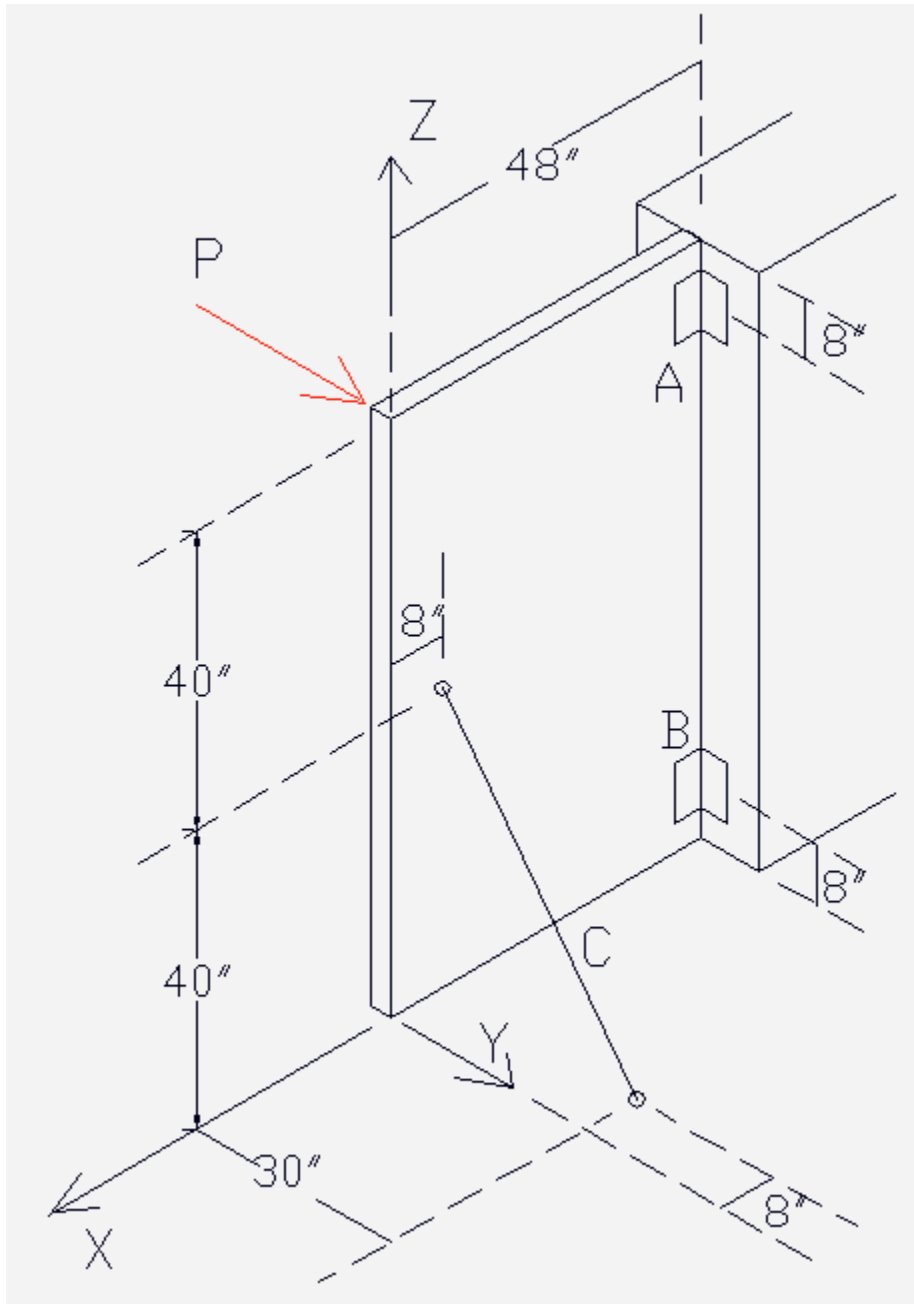
4. Solve:

$$\begin{aligned} C &= 100 \text{ lb} && (\text{z moment equation}) \\ A_x &= 27.5 \text{ lb} && (\text{y moment equation}) \\ A_y &= -26.25 \text{ lb} && (\text{x moment equation}) \\ B_x &= -27.5 \text{ lb} && (\text{x force equation}) \\ B_y &= 36.25 \text{ lb} && (\text{y force equation}) \\ B_z &= -20 \text{ lb} && (\text{z force equation}) \end{aligned}$$

$$A_{xy} = (A_x^2 + A_y^2)^{1/2} = 38.02 \text{ lb}$$

$$B_{xy} = (B_x^2 + B_y^2)^{1/2} = 45.50 \text{ lb}$$

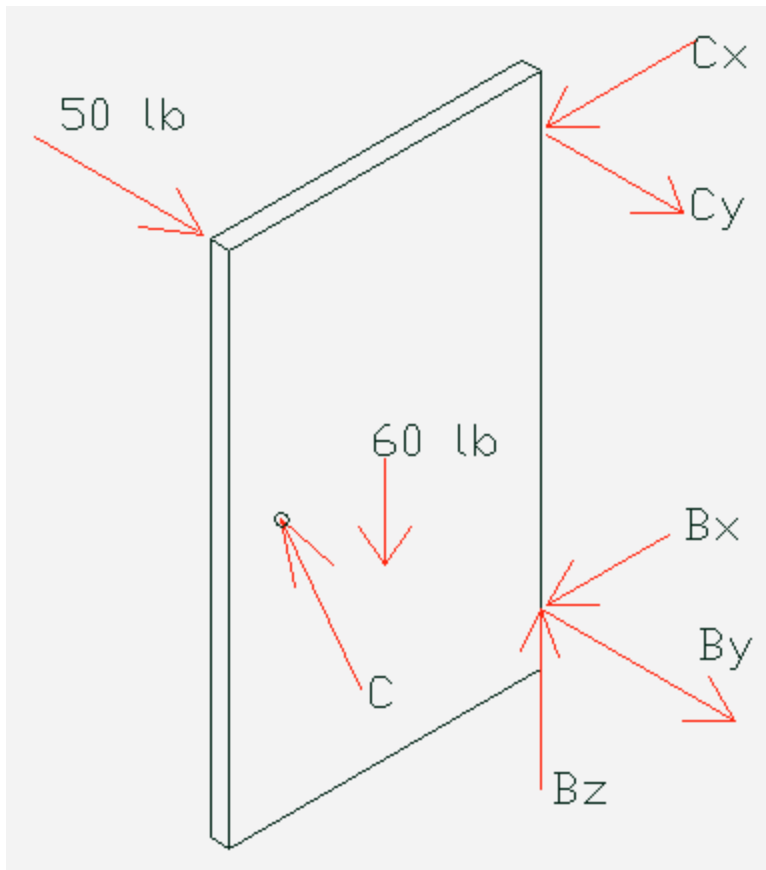
Chapter 3 Problem 77 Page 155



The uniform panel door shown in the isometric drawing weighs 60 lb and is prevented from opening by the strut C, which is a light two-force member whose upper end is secured under the door knob and whose lower end is attached to a rubber cup which does not slip on the floor. Of the door hinges A and B, only B can support force in the vertical z-direction. Calculate the compression in the strut and the horizontal components of the forces supported by the hinges A and B when a horizontal force $P = 50\text{ lb}$ is applied normal to the plane of the door as shown.

1. Mechanical System = Door

2. Free Body Diagram below.



3. Equations:

Noting that the force in the strut is parallel to the vector $(0, -30'', 40'')$, we obtain the following unit vector for the strut force:

$$\mathbf{e}_C = -3/5 \mathbf{j} + 4/5 \mathbf{k}$$

$$\begin{aligned} \sum \mathbf{F} &= 50 \text{ lb } \mathbf{j} - 60 \text{ lb } \mathbf{k} + C (-3/5 \mathbf{j} + 4/5 \mathbf{k}) \\ &+ B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} + A_x \mathbf{i} + A_y \mathbf{j} \\ &= (A_x + B_x) \mathbf{i} + (50 \text{ lb} - 3/5 C + A_y + B_y) \mathbf{j} + (4/5 C - 60 \text{ lb} + B_z) \mathbf{k} = \mathbf{0} \end{aligned}$$

Note that as the door is uniform, its mass center is located at its geometric center. Thus it is 24'' along the X axis from the hinges and 40'' above the floor (along the Z axis). Summing moments about the hinge at B to eliminate the three components of B yields:

$$\sum \mathbf{M}_B = 64'' \mathbf{k} \times (A_x \mathbf{i} + A_y \mathbf{j}) + 24'' \mathbf{i} \times$$

$$\begin{aligned} &-60 \text{ lb } \mathbf{k} + (48'' \mathbf{i} + 72'' \mathbf{k}) \times 50 \text{ lb } \mathbf{j} + (40'' \mathbf{i} + 32'' \mathbf{k}) \times C (-3/5 \mathbf{j} + 4/5 \mathbf{k}) \\ &= (96/5'' C - 3600 \text{ in } \cdot \text{lb} - 64'' A_y) \mathbf{i} + (64'' A_x + 1440 \text{ in } \cdot \text{lb} - 32'' C) \mathbf{j} + (2400 \text{ in } \cdot \text{lb} - 24'' C) \mathbf{k} = \mathbf{0} \end{aligned}$$

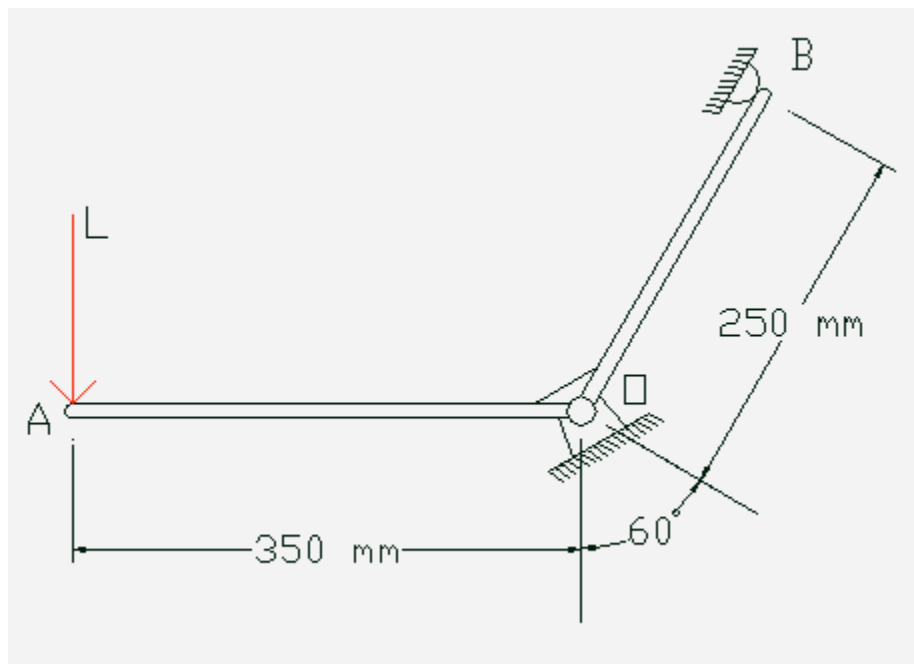
4. Solve:

$$\begin{aligned} C &= 100 \text{ lb} && (\text{z moment equation}) \\ A_x &= 27.5 \text{ lb} && (\text{y moment equation}) \\ A_y &= -26.25 \text{ lb} && (\text{x moment equation}) \\ B_x &= -27.5 \text{ lb} && (\text{x force equation}) \\ B_y &= 36.25 \text{ lb} && (\text{y force equation}) \\ B_z &= -20 \text{ lb} && (\text{z force equation}) \end{aligned}$$

$$A_{xy} = (A_x^2 + A_y^2)^{1/2} = 38.02 \text{ lb}$$

$$B_{xy} = (B_x^2 + B_y^2)^{1/2} = 45.50 \text{ lb}$$

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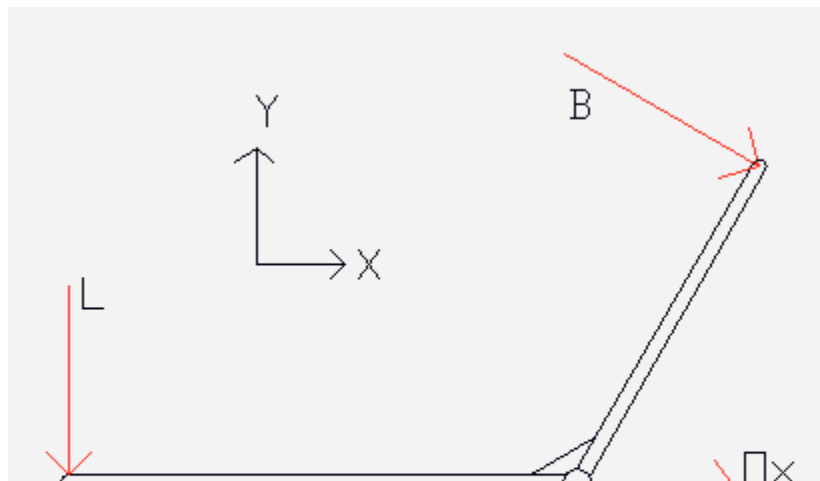


The pin at O can support a maximum force of 3.5 kN. What is the corresponding maximum load L which can be applied to the angled bracket AOB? Note that the bracket consists of a single rigid body composed of two arms. One arm is horizontal and of length 350 mm. The other arm makes an angle of 60 degrees with the horizontal and has a length of 250 mm. The load is applied vertically to the end of the horizontal arm. The bracket is supported by a frictionless pin at O. The pin at O is at the

connection point between the two arms. The bracket is prevented from rotating by a roller support at the end of the arm angled at 60 degrees.

1. Mechanical System = Angle bracket, AOB

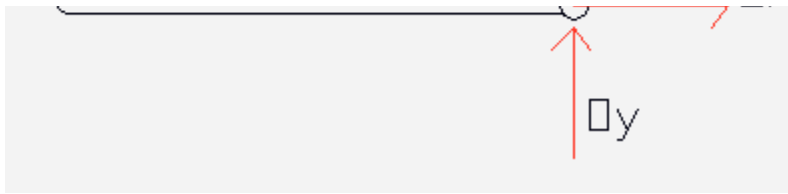
2. Free Body Diagram below. Note that as a roller force is perpendicular to the supporting surface, roller force B is perpendicular to a line that makes an angle of 60 degrees with the horizontal. Thus the roller force B must make an angle of 30 degrees with the horizontal.



3. Equations:

$$\begin{aligned} \mathbf{S F} &= -L \mathbf{j} + O_x \mathbf{i} + O_y \mathbf{j} + B (\cos 30 \mathbf{i} - \sin 30 \mathbf{j}) \\ &= (O_x + B \cos 30) \mathbf{i} + (O_y - L - B \sin 30) \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \mathbf{S M}_O &= -0.35 \text{ m } \mathbf{i} \times -L \mathbf{j} + .25 \text{ m } (\sin 30 \mathbf{i} + \cos 30 \mathbf{j}) \times B (\cos 30 \mathbf{i} - \sin 30 \mathbf{j}) \\ &= (0.35 \text{ m } L - 0.25 \text{ m } B) \mathbf{k} = \mathbf{0} \end{aligned}$$



4. Solve :

$$B = 7/5 L$$

(from moment equation)

$$O_x = - B \cos 30 = -7/10 \sqrt{3} L$$

(from X force equation)

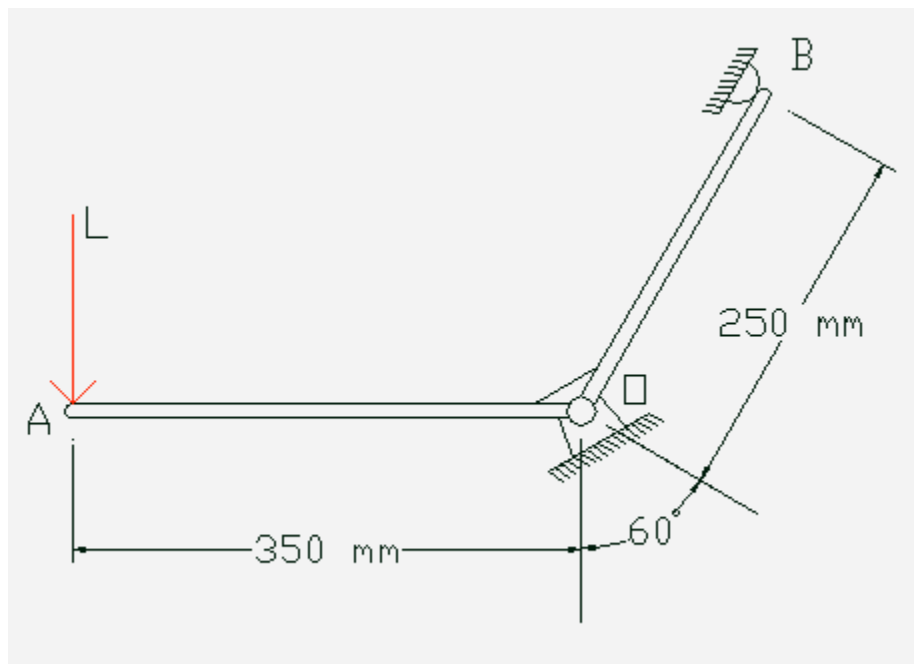
$$O_y = L + B \sin 30 = 17/10 L$$

(from Y force equation)

$$O = (O_x^2 + O_y^2)^{1/2} = L (147/100 + 289/100)^{1/2} = 3.5 \text{ kN}$$

$$L = 3.5 \text{ kN} * 10/436^{1/2} = 1.676 \text{ kN}$$

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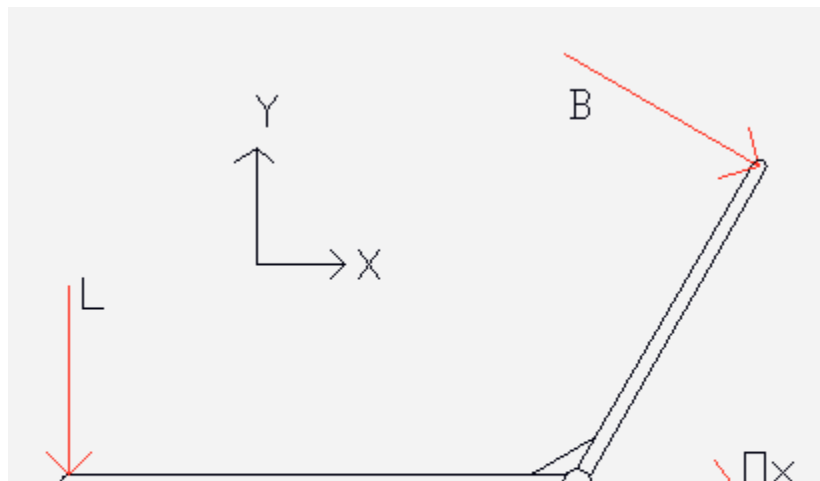


The pin at O can support a maximum force of 3.5 kN. What is the corresponding maximum load L which can be applied to the angled bracket AOB? Note that the bracket consists of a single rigid body composed of two arms. One arm is horizontal and of length 350 mm. The other arm makes an angle of 60 degrees with the horizontal and has a length of 250 mm. The load is applied vertically to the end of the horizontal arm. The bracket is supported by a frictionless pin at O. The pin at O is at the

connection point between the two arms. The bracket is prevented from rotating by a roller support at the end of the arm angled at 60 degrees.

1. Mechanical System = Angle bracket, AOB

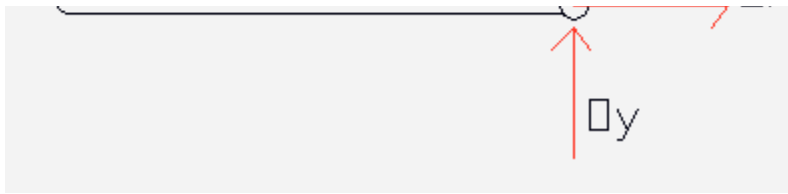
2. Free Body Diagram below. Note that as a roller force is perpendicular to the supporting surface, roller force B is perpendicular to a line that makes an angle of 60 degrees with the horizontal. Thus the roller force B must make an angle of 30 degrees with the horizontal.



3. Equations:

$$\begin{aligned} \mathbf{S F} &= -L \mathbf{j} + O_x \mathbf{i} + O_y \mathbf{j} + B (\cos 30 \mathbf{i} - \sin 30 \mathbf{j}) \\ &= (O_x + B \cos 30) \mathbf{i} + (O_y - L - B \sin 30) \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \mathbf{S M}_O &= -0.35 \text{ m } \mathbf{i} \times -L \mathbf{j} + .25 \text{ m } (\sin 30 \mathbf{i} + \cos 30 \mathbf{j}) \times B (\cos 30 \mathbf{i} - \sin 30 \mathbf{j}) \\ &= (0.35 \text{ m } L - 0.25 \text{ m } B) \mathbf{k} = \mathbf{0} \end{aligned}$$



4. Solve :

$$B = 7/5 L$$

(from moment equation)

$$O_x = - B \cos 30 = -7/10 \sqrt{3} L$$

(from X force equation)

$$O_y = L + B \sin 30 = 17/10 L$$

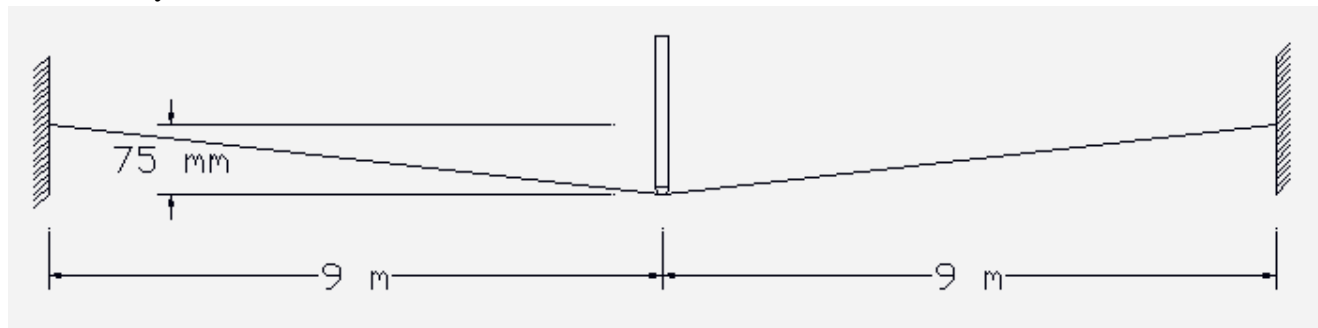
(from Y force equation)

$$O = (O_x^2 + O_y^2)^{1/2} = L (147/100 + 289/100)^{1/2} = 3.5 \text{ kN}$$

$$L = 3.5 \text{ kN} * 10/436^{1/2} = 1.676 \text{ kN}$$

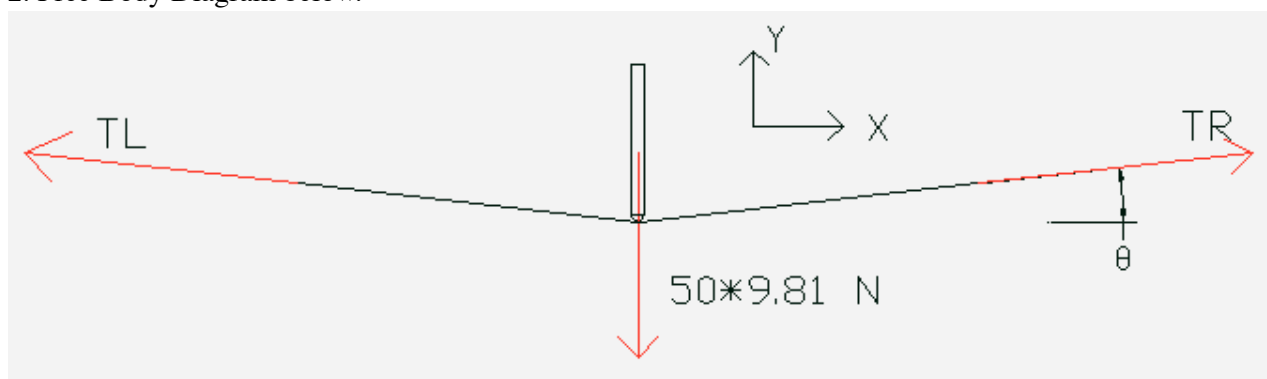
Chapter 3 Problem 98 Page 165

A 50-kg acrobat pedals her unicycle across the taut but slightly elastic cable. If the deflection at the center of the 18-m span is 75 mm, determine the tension in the cable. Neglect the effects of the weights of the cable and the unicycle. Note that the deflection of the cable and the acrobat have not been drawn to scale.



1. Mechanical System = acrobat, unicycle, and part of cable supporting the acrobat.

2. Free Body Diagram below.



3. Equations:

$$\begin{aligned} \sum \mathbf{F} &= T_L (-\cos q \mathbf{i} + \sin q \mathbf{j}) - 50 \cdot 9.81 \text{ N } \mathbf{j} + T_R (\cos q \mathbf{i} + \sin q \mathbf{j}) \\ &= \cos q (T_R - T_L) \mathbf{i} + \{ \sin q (T_R + T_L) - 490.5 \text{ N} \} \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\tan q = 0.075/9 \quad \sin q = 0.075/(9^2 + 0.075^2)^{1/2} \quad \cos q = 9/(9^2 + 0.075^2)^{1/2}$$

4. Solve

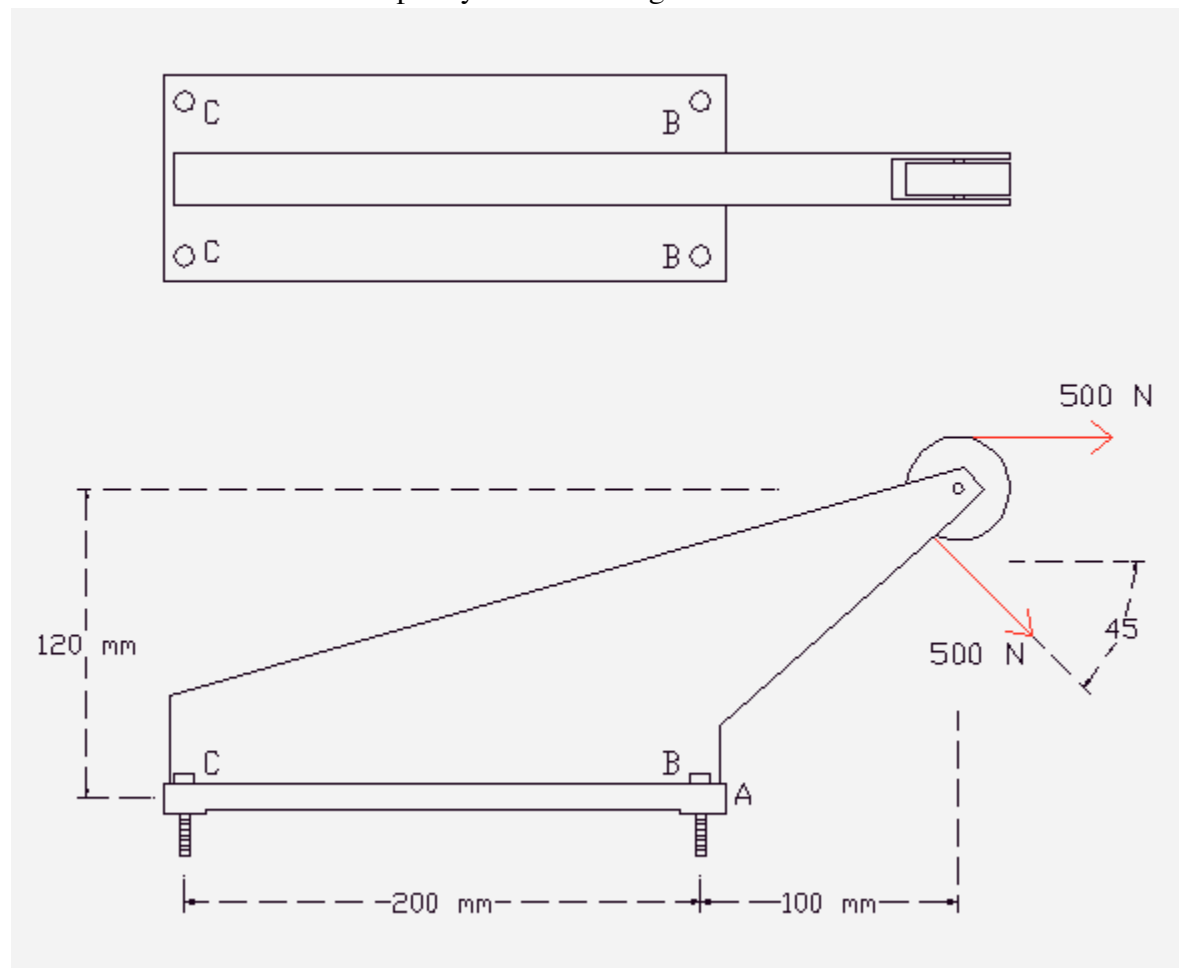
$$T_R = T_L \quad (\text{x force equation})$$

$$T_R = T_L = 490.5 \text{ N} / 2 / \sin q = 29430 \text{ N} \quad (\text{y force equation})$$

Wow!

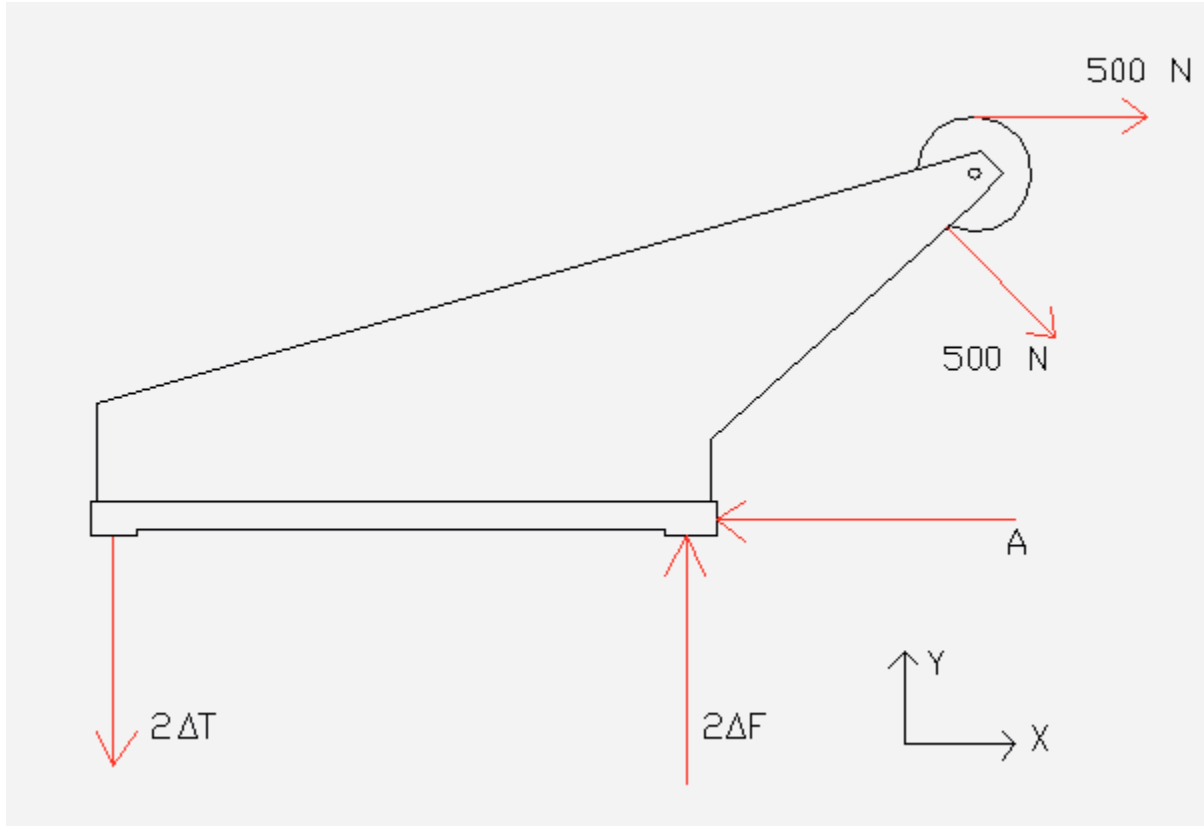
Chapter 3 Problem 100 Page 166

The pulley bracket is secured to its foundation by two bolts B and two bolts C with an initial tension in each of the four bolts prior to application of the 500-N tension in the cable. Determine the increase DT in tension in each of the bolts C and the increase DF in force under each side of the bracket at B resulting from application of the cable tension. Assume that the bolt holes are oversized with all horizontal force supported by the small ledge at A. Also assume that the vertical reactions on the base are concentrated at the center lines of the bolts. These assumptions would be typical design considerations. The following figure shows both a top view and a side view of the bracket. Neither view shows the supporting ledge at A or the floor to which the bracket is bolted. From the top view you can see the tops of the four bolts, however the pulley cable is not shown in the top view. The side view shows the pulley bracket, side views of the bolts, the pulley, and the forces from the pulley cable. The side view does not show the floor or the supporting ledge at A. The side view shows all of the relevant dimensions. The pulley radius is not given but will be shown to be irrelevant.



1. Mechanical System = Bracket and pulley.

2. Free Body Diagram below. Side view. Note that only the changes in forces are shown as these changes must also satisfy the equilibrium conditions. Note the factors of 2 to account for the two bolts at each position.



3. Equations:

$$\begin{aligned} \sum \mathbf{F} &= -2 DT \mathbf{j} + 2 DF \mathbf{j} - A \mathbf{i} + 500 N \mathbf{i} + 500 N (\cos 45 \mathbf{i} - \sin 45 \mathbf{j}) \\ &= (500 N + 250 \sqrt{2} N - A) \mathbf{i} + (2 DF - 2 DT - 250 \sqrt{2} N) \mathbf{j} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \sum \mathbf{M}_B &= -0.2 \text{ m } \mathbf{i} \times -2 DT \mathbf{j} + \{ 0.1 \text{ m } \mathbf{i} + (0.12 \text{ m} + R) \mathbf{j} \} \times 500 N \mathbf{i} + \{ (0.1 \text{ m} - R/\sqrt{2}) \mathbf{i} + (0.12 \text{ m} - \\ &R/\sqrt{2}) \mathbf{j} \} \times \{ 250 \sqrt{2} N \mathbf{i} - 250 \sqrt{2} N \mathbf{j} \} \\ &= \{ 0.4 \text{ m } DT - 60 \text{ N m} - 55 \sqrt{2} \text{ Nm} \} \mathbf{k} = \mathbf{0} \end{aligned}$$

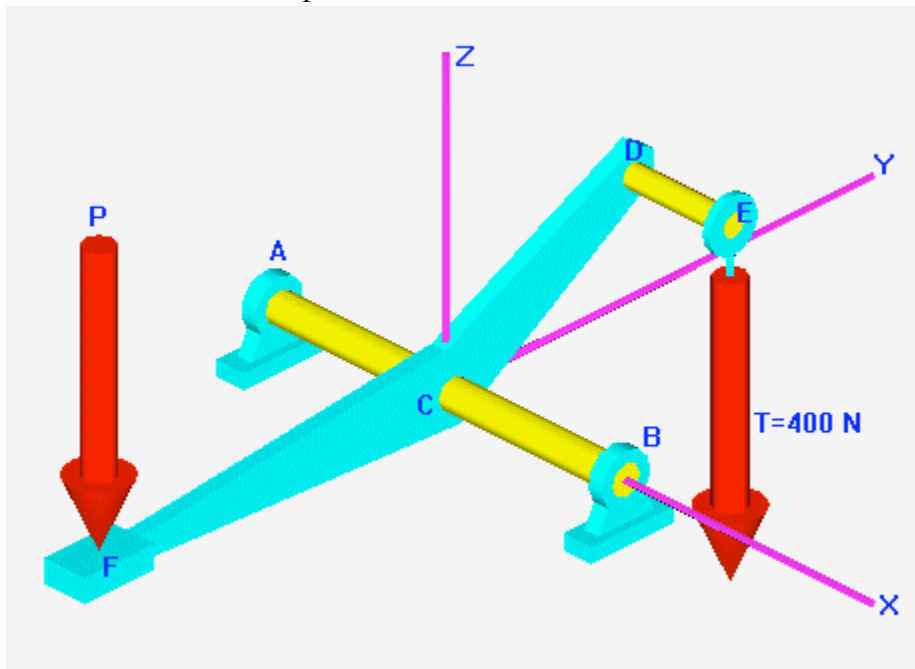
R is the radius of the pulley and drops out of the final expression for the moment sum.

4. Solve:

$$\begin{aligned} DT &= 344.5 \text{ N} && (\text{z moment equation}) \\ DF &= 521.2 \text{ N} && (\text{y force equation}) \end{aligned}$$

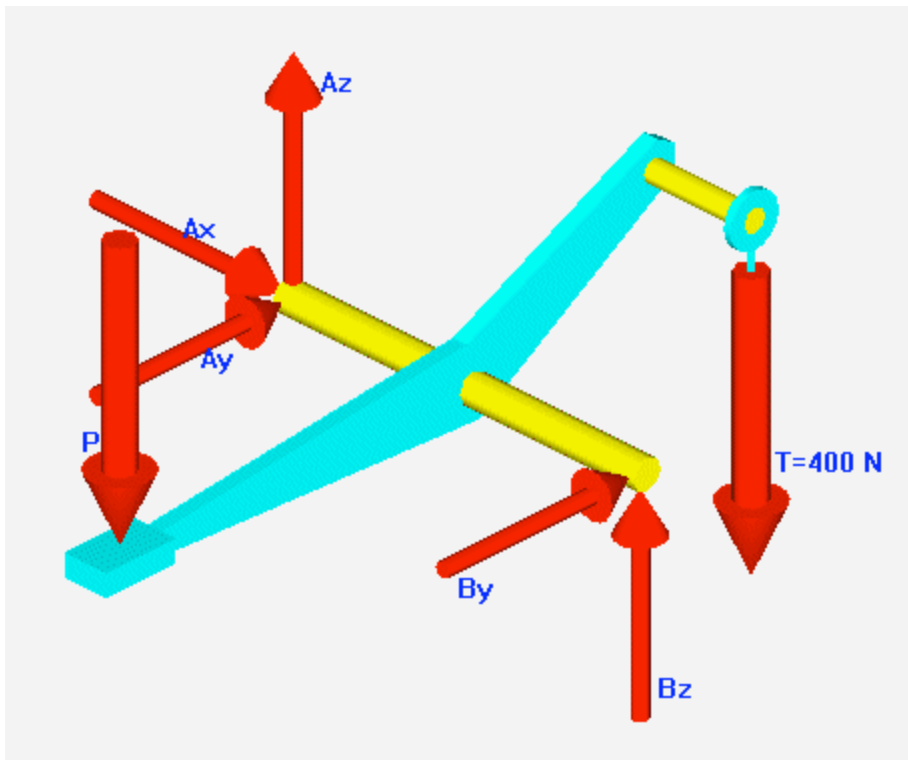
Chapter 3 Problem 103 Page 167

A vertical force P on the foot pedal of the bell crank is required to produce a tension T of 400 N in the vertical control rod. Determine the corresponding bearing reactions at the support points A and B. The bell crank is shown below in light blue and yellow. Some of the relevant points are labeled in blue. A suggested set of axes are shown in purple. The origin of those axes is at the point C. The forces P and T are shown in red. These forces act vertically downward ($-Z$). The force P is applied at a point F which is 200 mm down the $-Y$ axis from C. The point D is distance of 120 mm from C and lies along a line in the YZ plane. This line makes an angle of 30 degrees with the positive Y axis. Thus the Y coordinate of point D is $120 \text{ mm} \cdot \cos(30)$ while the Z coordinate of D is $120 \text{ mm} \cdot \sin(30)$. The control rod force passes through point E which is a distance of 60 mm along the positive X axis from the point D. The two supports at A and B are also shown in light blue. The support points at A and B are each 100 mm from C. The vector from C to A points in the $-X$ direction, while the vector from C to B points in the $+X$ direction.



1. Mechanical System = Bell Crank.

2. Free Body Diagram below. Note that from the figure in the text it would appear that the bearing at B cannot support any force along the x axis.



3. Equations:

$$\begin{aligned} \sum \mathbf{F} &= -P \mathbf{k} - 400 \text{ N } \mathbf{k} + A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} + B_y \mathbf{j} + B_z \mathbf{k} \\ &= A_x \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z - P - 400 \text{ N}) \mathbf{k} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \sum \mathbf{M}_A &= (0.1 \text{ m } \mathbf{i} - 0.2 \text{ m } \mathbf{j}) \times -P \mathbf{k} + 0.2 \text{ m } \mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) + \{ 0.16 \text{ m } \mathbf{i} + 0.12 \text{ m}(\cos 30^\circ \mathbf{j} + \sin 30^\circ \mathbf{k}) \} \times -400 \text{ N } \mathbf{k} \\ &= (0.2 \text{ m } P - 24 \sqrt{3} \text{ Nm}) \mathbf{i} + (0.1 \text{ m } P - 0.2 \text{ m } B_z + 64 \text{ Nm}) \mathbf{j} + (0.2 \text{ m } B_y) \mathbf{k} = \mathbf{0} \end{aligned}$$

4. Solve:

$$\begin{aligned} B_y &= 0 && \text{(z moment equation)} \\ P &= 207.8 \text{ N} && \text{(x moment equation)} \\ B_z &= 423.9 \text{ N} && \text{(y moment equation)} \\ A_x &= 0 && \text{(x force equation)} \\ A_y &= 0 && \text{(y force equation)} \\ A_z &= 183.9 \text{ N} && \text{(z force equation)} \end{aligned}$$

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.

- ① **Solution.** The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

Solution I (scalar algebra). For the x - y axes as shown we have

$$[\Sigma F_x = 0] \quad 8 + T \cos 40^\circ + C \sin 20^\circ - 16 = 0$$

$$0.766T + 0.342C = 8 \quad (a)$$

$$[\Sigma F_y = 0] \quad T \sin 40^\circ - C \cos 20^\circ - 3 = 0$$

$$0.643T - 0.940C = 3 \quad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \quad C = 3.03 \text{ kN} \quad \text{Ans.}$$

- ② **Solution II (scalar algebra).** To avoid a simultaneous solution, we may use axes x' - y' with the first summation in the y' -direction to eliminate reference to T . Thus,

$$[\Sigma F_{y'} = 0] \quad -C \cos 20^\circ - 3 \cos 40^\circ - 8 \sin 40^\circ + 16 \sin 40^\circ = 0$$

$$C = 3.03 \text{ kN} \quad \text{Ans.}$$

$$[\Sigma F_{x'} = 0] \quad T + 8 \cos 40^\circ - 16 \cos 40^\circ - 3 \sin 40^\circ - 3.03 \sin 20^\circ = 0$$

$$T = 9.09 \text{ kN} \quad \text{Ans.}$$

Solution III (vector algebra). With unit vectors **i** and **j** in the x - and y -directions, the zero summation of forces for equilibrium yields the vector equation

$$[\Sigma \mathbf{F} = \mathbf{0}] \quad 8\mathbf{i} + (T \cos 40^\circ)\mathbf{i} + (T \sin 40^\circ)\mathbf{j} - 3\mathbf{j} + (C \sin 20^\circ)\mathbf{i} \\ - (C \cos 20^\circ)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

Equating the coefficients of the **i**- and **j**-terms to zero gives

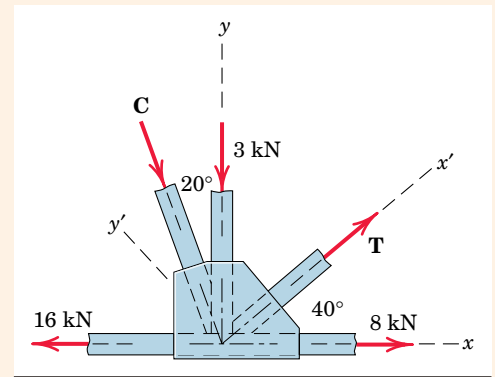
$$8 + T \cos 40^\circ + C \sin 20^\circ - 16 = 0$$

$$T \sin 40^\circ - 3 - C \cos 20^\circ = 0$$

which are the same, of course, as Eqs. (a) and (b), which we solved above.

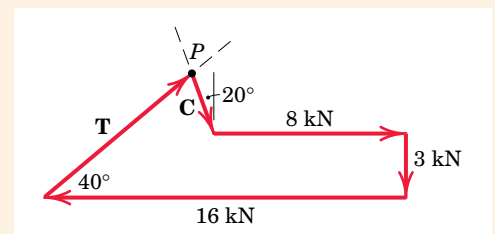
Solution IV (geometric). The polygon representing the zero vector sum of the five forces is shown. Equations (a) and (b) are seen immediately to give the projections of the vectors onto the x - and y -directions. Similarly, projections onto the x' - and y' -directions give the alternative equations in Solution II.

- ③ A graphical solution is easily obtained. The known vectors are laid off head-to-tail to some convenient scale, and the directions of **T** and **C** are then drawn to close the polygon. The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of **T** and **C** directly from the drawing to whatever degree of accuracy we incorporate in the construction.



Helpful Hints

- ① Since this is a problem of concurrent forces, no moment equation is necessary.
- ② The selection of reference axes to facilitate computation is always an important consideration. Alternatively in this example we could take a set of axes along and normal to the direction of **C** and employ a force summation normal to **C** to eliminate it.



- ③ The known vectors may be added in any order desired, but they must be added before the unknown vectors.

Sample Problem 3/2

Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C .

Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A , which includes the only known force. With the unspecified pulley radius designated by r , the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

$$\begin{aligned} \textcircled{1} [\Sigma M_O = 0] \quad T_1 r - T_2 r = 0 \quad T_1 = T_2 \\ [\Sigma F_y = 0] \quad T_1 + T_2 - 1000 = 0 \quad 2T_1 = 1000 \quad T_1 = T_2 = 500 \text{ lb} \end{aligned}$$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 250 \text{ lb}$$

For pulley C the angle $\theta = 30^\circ$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

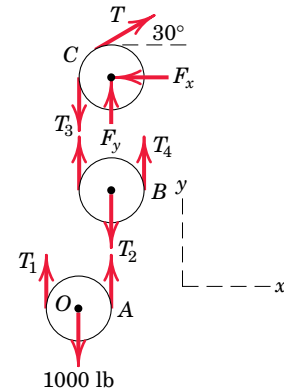
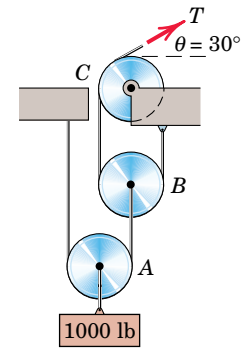
$$T = T_3 \quad \text{or} \quad T = 250 \text{ lb} \quad \text{Ans.}$$

Equilibrium of the pulley in the x - and y -directions requires

$$[\Sigma F_x = 0] \quad 250 \cos 30^\circ - F_x = 0 \quad F_x = 217 \text{ lb}$$

$$[\Sigma F_y = 0] \quad F_y + 250 \sin 30^\circ - 250 = 0 \quad F_y = 125 \text{ lb}$$

$$[F = \sqrt{F_x^2 + F_y^2}] \quad F = \sqrt{(217)^2 + (125)^2} = 250 \text{ lb} \quad \text{Ans.}$$



Helpful Hint

- ① Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Sample Problem 3/3

The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B . By means of the cable at C it is desired to elevate end B to a position 3 m above end A . Determine the required tension P , the reaction at A , and the angle θ made by the beam with the horizontal in the elevated position.

Solution. In constructing the free-body diagram, we note that the reaction on the roller at A and the weight are vertical forces. Consequently, in the absence of other horizontal forces, P must also be vertical. From Sample Problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C .

Moment equilibrium about A eliminates force R and gives

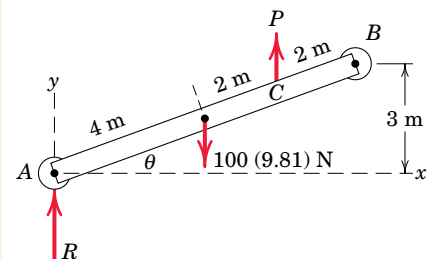
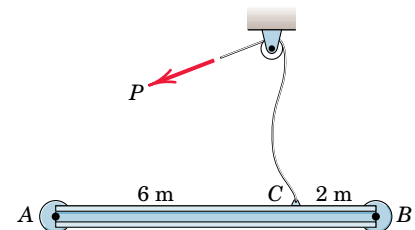
$$\textcircled{1} [\Sigma M_A = 0] \quad P(6 \cos \theta) - 981(4 \cos \theta) = 0 \quad P = 654 \text{ N} \quad \text{Ans.}$$

Equilibrium of vertical forces requires

$$[\Sigma F_y = 0] \quad 654 + R - 981 = 0 \quad R = 327 \text{ N} \quad \text{Ans.}$$

The angle θ depends only on the specified geometry and is

$$\sin \theta = 3/8 \quad \theta = 22.0^\circ \quad \text{Ans.}$$



Helpful Hint

- ① Clearly the equilibrium of this parallel force system is independent of θ .

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.

Algebraic solution. The system is symmetrical about the vertical x - y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x , A_y , and T , which may be found from the three equations of equilibrium. We begin with a moment equation about A , which eliminates two of the three unknowns from the equation.

① In applying the moment equation about A , it is simpler to consider the moments of the x - and y -components of \mathbf{T} than it is to compute the perpendicular distance from \mathbf{T} to A . Hence, with the counterclockwise sense as positive we write

$$\textcircled{2} \quad [\Sigma M_A = 0] \quad (T \cos 25^\circ)0.25 + (T \sin 25^\circ)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $T = 19.61$ kN *Ans.*

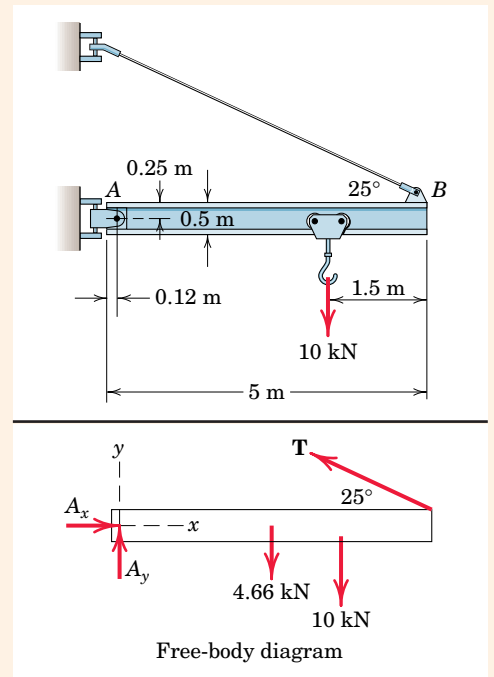
Equating the sums of forces in the x - and y -directions to zero gives

$$[\Sigma F_x = 0] \quad A_x - 19.61 \cos 25^\circ = 0 \quad A_x = 17.77 \text{ kN}$$

$$[\Sigma F_y = 0] \quad A_y + 19.61 \sin 25^\circ - 4.66 - 10 = 0 \quad A_y = 6.37 \text{ kN}$$

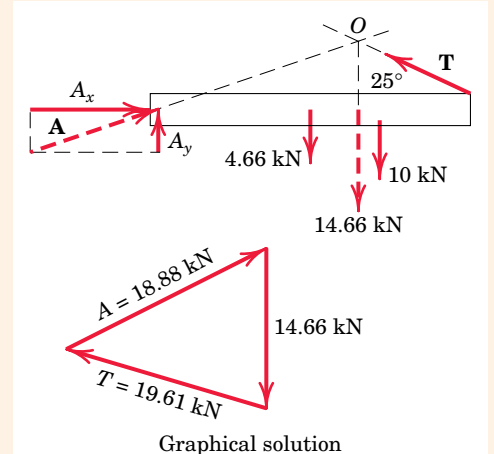
$$\textcircled{3} \quad [A = \sqrt{A_x^2 + A_y^2}] \quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN} \quad \textit{Ans.}$$

Graphical solution. The principle that three forces in equilibrium must be concurrent is utilized for a graphical solution by combining the two known vertical forces of 4.66 and 10 kN into a single 14.66-kN force, located as shown on the modified free-body diagram of the beam in the lower figure. The position of this resultant load may easily be determined graphically or algebraically. The intersection of the 14.66-kN force with the line of action of the unknown tension \mathbf{T} defines the point of concurrency O through which the pin reaction \mathbf{A} must pass. The unknown magnitudes of \mathbf{T} and \mathbf{A} may now be found by adding the forces head-to-tail to form the closed equilibrium polygon of forces, thus satisfying their zero vector sum. After the known vertical load is laid off to a convenient scale, as shown in the lower part of the figure, a line representing the given direction of the tension \mathbf{T} is drawn through the tip of the 14.66-kN vector. Likewise a line representing the direction of the pin reaction \mathbf{A} , determined from the concurrency established with the free-body diagram, is drawn through the tail of the 14.66-kN vector. The intersection of the lines representing vectors \mathbf{T} and \mathbf{A} establishes the magnitudes T and A necessary to make the vector sum of the forces equal to zero. These magnitudes are scaled from the diagram. The x - and y -components of \mathbf{A} may be constructed on the force polygon if desired.



Helpful Hints

- ① The justification for this step is Varignon's theorem, explained in Art. 2/4. Be prepared to take full advantage of this principle frequently.
- ② The calculation of moments in two-dimensional problems is generally handled more simply by scalar algebra than by the vector cross product $\mathbf{r} \times \mathbf{F}$. In three dimensions, as we will see later, the reverse is often the case.
- ③ The direction of the force at A could be easily calculated if desired. However, in designing the pin A or in checking its strength, it is only the magnitude of the force that matters.



Sample Problem 3/5

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A in the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.

Solution. The free-body diagram of the shaft is first drawn where the contact forces acting on the shaft at B are shown normal to the wall surfaces. In addition to the weight $W = mg = 200(9.81) = 1962$ N, the force exerted by the floor on the ball joint at A is represented by its x -, y -, and z -components. These components are shown in their correct physical sense, as should be evident from the requirement that A be held in place. The vertical position of B is found from $7 = \sqrt{2^2 + 6^2 + h^2}$, $h = 3$ m. Right-handed coordinate axes are assigned as shown.

Vector solution. We will use A as a moment center to eliminate reference to the forces at A . The position vectors needed to compute the moments about A are

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k} \quad \text{and} \quad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} \text{ m}$$

where the mass center G is located halfway between A and B .

The vector moment equation gives

$$[\Sigma \mathbf{M}_A = \mathbf{0}] \quad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$$

$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = \mathbf{0}$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of \mathbf{i} , \mathbf{j} , and \mathbf{k} to zero and solving give

$$\textcircled{2} \quad B_x = 654 \text{ N} \quad \text{and} \quad B_y = 1962 \text{ N} \quad \text{Ans.}$$

The forces at A are easily determined by

$$[\Sigma \mathbf{F} = \mathbf{0}] \quad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$$

$$\text{and} \quad A_x = 654 \text{ N} \quad A_y = 1962 \text{ N} \quad A_z = 1962 \text{ N}$$

$$\text{Finally} \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850 \text{ N} \quad \text{Ans.}$$

Scalar solution. Evaluating the scalar moment equations about axes through A parallel, respectively, to the x - and y -axes, gives

$$[\Sigma M_{A_x} = 0] \quad 1962(3) - 3B_y = 0 \quad B_y = 1962 \text{ N}$$

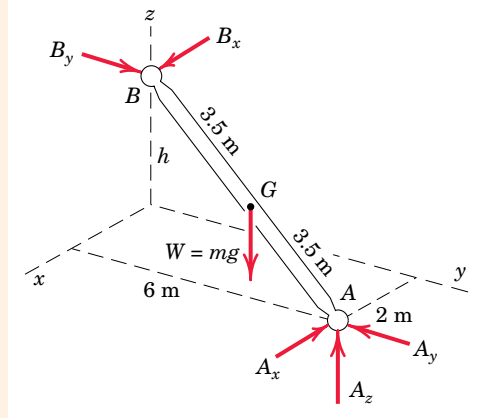
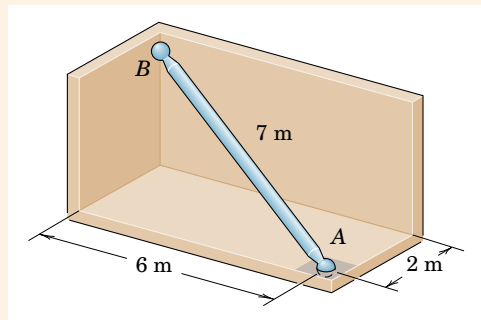
$$\textcircled{3} \quad [\Sigma M_{A_y} = 0] \quad -1962(1) + 3B_x = 0 \quad B_x = 654 \text{ N}$$

The force equations give, simply,

$$[\Sigma F_x = 0] \quad -A_x + 654 = 0 \quad A_x = 654 \text{ N}$$

$$[\Sigma F_y = 0] \quad -A_y + 1962 = 0 \quad A_y = 1962 \text{ N}$$

$$[\Sigma F_z = 0] \quad A_z - 1962 = 0 \quad A_z = 1962 \text{ N}$$

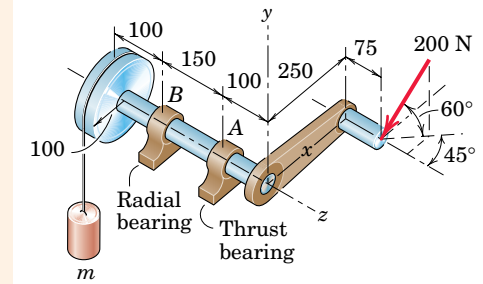


Helpful Hints

- ① We could, of course, assign all of the unknown components of force in the positive mathematical sense, in which case A_x and A_y would turn out to be negative upon computation. The free-body diagram describes the physical situation, so it is generally preferable to show the forces in their correct physical senses wherever possible.
- ② Note that the third equation $-2B_y + 6B_x = 0$ merely checks the results of the first two equations. This result could be anticipated from the fact that an equilibrium system of forces concurrent with a line requires only two moment equations (Category 2 under *Categories of Equilibrium*).
- ③ We observe that a moment sum about an axis through A parallel to the z -axis merely gives us $6B_x - 2B_y = 0$, which serves only as a check as noted previously. Alternatively we could have first obtained A_z from $\Sigma F_z = 0$ and then taken our moment equations about axes through B to obtain A_x and A_y .

Sample Problem 3/6

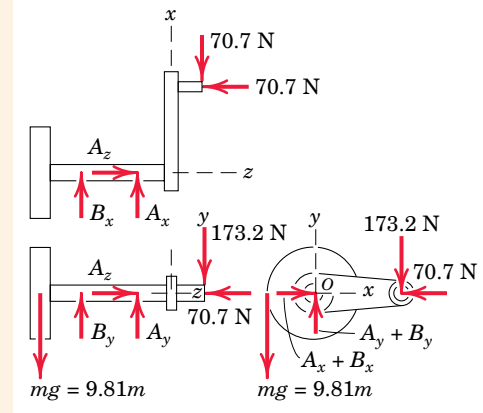
A 200-N force is applied to the handle of the hoist in the direction shown. The bearing A supports the thrust (force in the direction of the shaft axis), while bearing B supports only radial load (load normal to the shaft axis). Determine the mass m which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.



Dimensions in millimeters

- Solution.** The system is clearly three-dimensional with no lines or planes of symmetry, and therefore the problem must be analyzed as a general space system of forces. A scalar solution is used here to illustrate this approach, although a solution using vector notation would also be satisfactory. The free-body diagram of the shaft, lever, and drum considered a single body could be shown by a space view if desired, but is represented here by its three orthogonal projections.

The 200-N force is resolved into its three components, and each of the three views shows two of these components. The correct directions of A_x and B_x may be seen by inspection by observing that the line of action of the resultant of the two 70.7-N forces passes between A and B . The correct sense of the forces A_y and B_y cannot be determined until the magnitudes of the moments are obtained, so they are arbitrarily assigned. The x - y projection of the bearing forces is shown in terms of the sums of the unknown x - and y -components. The addition of A_z and the weight $W = mg$ completes the free-body diagrams. It should be noted that the three views represent three two-dimensional problems related by the corresponding components of the forces.



- ② From the x - y projection

$$[\Sigma M_O = 0] \quad 100(9.81m) - 250(173.2) = 0 \quad m = 44.1 \text{ kg} \quad \text{Ans.}$$

From the x - z projection

$$[\Sigma M_A = 0] \quad 150B_x + 175(70.7) - 250(70.7) = 0 \quad B_x = 35.4 \text{ N}$$

$$[\Sigma F_x = 0] \quad A_x + 35.4 - 70.7 = 0 \quad A_x = 35.4 \text{ N}$$

- ③ The y - z view gives

$$[\Sigma M_A = 0] \quad 150B_y + 175(173.2) - 250(44.1)(9.81) = 0 \quad B_y = 520 \text{ N}$$

$$[\Sigma F_y = 0] \quad A_y + 520 - 173.2 - (44.1)(9.81) = 0 \quad A_y = 86.8 \text{ N}$$

$$[\Sigma F_z = 0] \quad A_z = 70.7 \text{ N}$$

The total radial forces on the bearings become

$$[A_r = \sqrt{A_x^2 + A_y^2}] \quad A_r = \sqrt{(35.4)^2 + (86.8)^2} = 93.5 \text{ N} \quad \text{Ans.}$$

- ④ $[B = \sqrt{B_x^2 + B_y^2}] \quad B = \sqrt{(35.4)^2 + (520)^2} = 521 \text{ N} \quad \text{Ans.}$

Helpful Hints

- ① If the standard three views of orthographic projection are not entirely familiar, then review and practice them. Visualize the three views as the images of the body projected onto the front, top, and end surfaces of a clear plastic box placed over and aligned with the body.
- ② We could have started with the x - z projection rather than with the x - y projection.
- ③ The y - z view could have followed immediately after the x - y view since the determination of A_y and B_y may be made after m is found.
- ④ Without the assumption of zero moment supported by each bearing about a line normal to the shaft axis, the problem would be statically indeterminate.

Sample Problem 3/7

The welded tubular frame is secured to the horizontal x - y plane by a ball-and-socket joint at A and receives support from the loose-fitting ring at B . Under the action of the 2-kN load, rotation about a line from A to B is prevented by the cable CD , and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension T in the cable, the reaction at the ring, and the reaction components at A .

Solution. The system is clearly three-dimensional with no lines or planes of symmetry, and therefore the problem must be analyzed as a general space system of forces. The free-body diagram is drawn, where the ring reaction is shown in terms of its two components. All unknowns except \mathbf{T} may be eliminated by a moment sum about the line AB . The direction of AB is specified by the unit

$$\textcircled{1} \text{ vector } \mathbf{n} = \frac{1}{\sqrt{6^2 + 4.5^2}} (4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}). \text{ The moment of } \mathbf{T} \text{ about } AB$$

is the component in the direction of AB of the vector moment about the point A and equals $\mathbf{r}_1 \times \mathbf{T} \cdot \mathbf{n}$. Similarly the moment of the applied load F about AB is $\mathbf{r}_2 \times \mathbf{F} \cdot \mathbf{n}$. With $CD = \sqrt{46.2}$ m, the vector expressions for \mathbf{T} , \mathbf{F} , \mathbf{r}_1 , and \mathbf{r}_2 are

$$\mathbf{T} = \frac{T}{\sqrt{46.2}} (2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \quad \mathbf{F} = 2\mathbf{j} \text{ kN}$$

$$\textcircled{2} \quad \mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j} \text{ m} \quad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k} \text{ m}$$

The moment equation now becomes

$$\begin{aligned} [\Sigma M_{AB} = 0] \quad & (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}} (2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) \\ & + (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0 \end{aligned}$$

Completion of the vector operations gives

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \quad T = 2.83 \text{ kN} \quad \text{Ans.}$$

and the components of T become

$$T_x = 0.833 \text{ kN} \quad T_y = 1.042 \text{ kN} \quad T_z = -2.50 \text{ kN}$$

We may find the remaining unknowns by moment and force summations as follows:

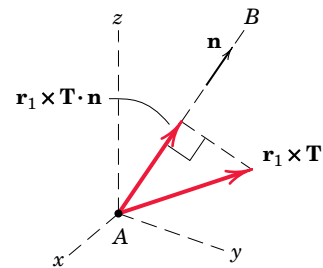
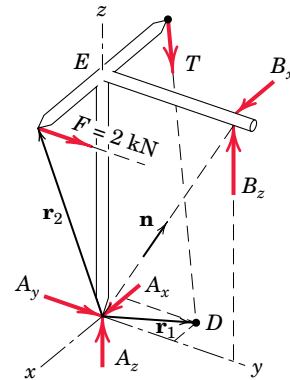
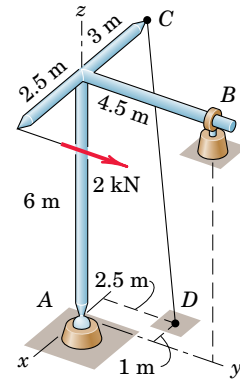
$$[\Sigma M_z = 0] \quad 2(2.5) - 4.5B_x - 1.042(3) = 0 \quad B_x = 0.417 \text{ kN} \quad \text{Ans.}$$

$$[\Sigma M_x = 0] \quad 4.5B_z - 2(6) - 1.042(6) = 0 \quad B_z = 4.06 \text{ kN} \quad \text{Ans.}$$

$$[\Sigma F_x = 0] \quad A_x + 0.417 + 0.833 = 0 \quad A_x = -1.250 \text{ kN} \quad \text{Ans.}$$

$$\textcircled{3} [\Sigma F_y = 0] \quad A_y + 2 + 1.042 = 0 \quad A_y = -3.04 \text{ kN} \quad \text{Ans.}$$

$$[\Sigma F_z = 0] \quad A_z + 4.06 - 2.50 = 0 \quad A_z = -1.556 \text{ kN} \quad \text{Ans.}$$



Helpful Hints

- ① The advantage of using vector notation in this problem is the freedom to take moments directly about any axis. In this problem this freedom permits the choice of an axis that eliminates five of the unknowns.
- ② Recall that the vector \mathbf{r} in the expression $\mathbf{r} \times \mathbf{F}$ for the moment of a force is a vector from the moment center to *any* point on the line of action of the force. Instead of \mathbf{r}_1 , an equally simple choice would be the vector \overrightarrow{AC} .
- ③ The negative signs associated with the A -components indicate that they are in the opposite direction to those shown on the free-body diagram.