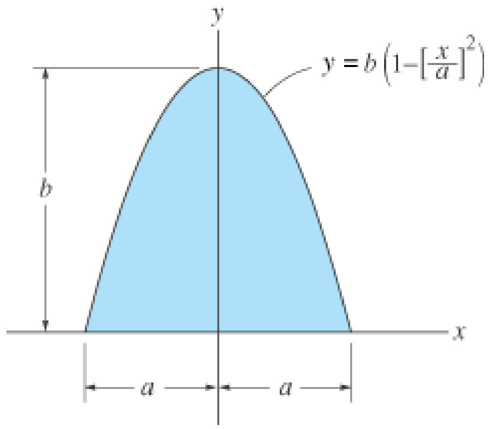
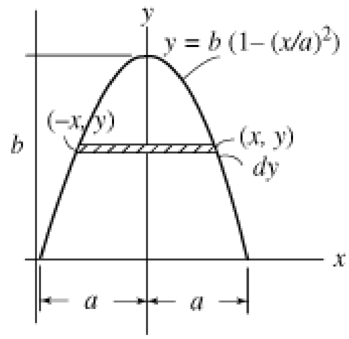
Problem 10-1

Determine the moment of inertia of the shaded area about the *x* axis.

Given:





$\mathbf{a} := 2\mathbf{m}$

$\mathbf{b} := 4\mathbf{m}$

Solution:

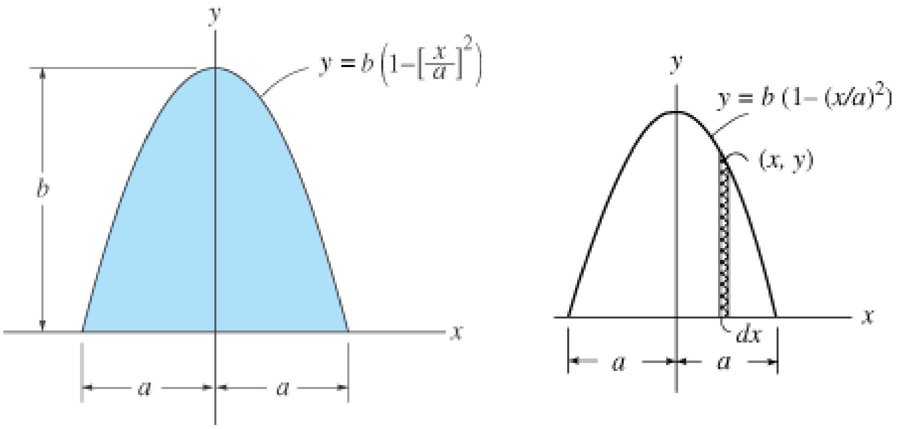
$$\mathbf{I_x} := 2\int_0^{\mathbf{b}} \mathbf{y}^2 \cdot \mathbf{a}\sqrt{1-\frac{\mathbf{y}}{\mathbf{b}}}\,\mathbf{dy}$$

$$\mathbf{I_x} = 39.0\,\mathbf{m}^4$$

Problem 10-2

Determine the moment of inertia of the shaded area about the y axis.

Given:



a := 2**m**

b := 4**m**

Solution:

$$I_y := 2\int_0^a x^2 b\left[1-\left(\frac{x}{a}\right)^2\right] dx$$

$$I_y = 8.53\ m^4$$

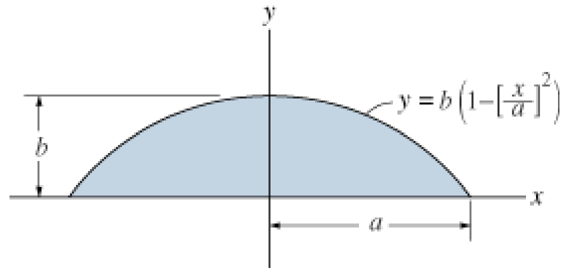
Problem 10-3

Determine the moment of inertia of the area about the *x* axis. Solve the problem in two ways, using rectangular differential elements:
(a) having a thickness of *dx*, and (b) having a thickness of *dy*.

Given :

$\mathbf{a} := 5\mathbf{ft}$

$\mathbf{b} := 2.5\mathbf{ft}$



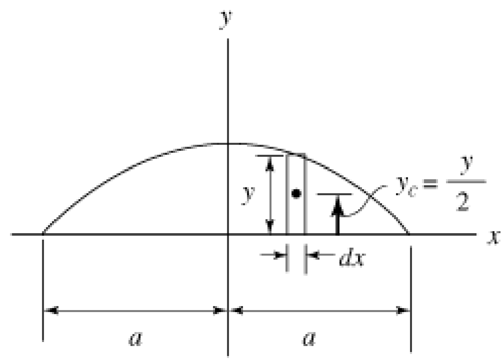
Solution :

a) Using a thickness d*x*.

$$\mathbf{dI_x} = \frac{\mathbf{y}^3}{3}\mathbf{dx} = \frac{\mathbf{b}^3}{3}\left[1-\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2\right]^3\mathbf{dx}$$

$$\mathbf{I_x} := \int_{-\mathbf{a}}^{\mathbf{a}} \frac{\mathbf{b}^3}{3}\cdot\left[1-\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2\right]^3 \mathbf{dx}$$

$\mathbf{I_x} = 23.8\,\mathbf{ft}^4$

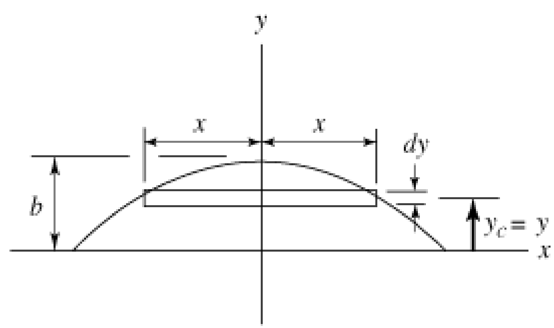


(a)

b) Using a thickness d*y*

$$\mathbf{I_x} := \int_0^{\mathbf{b}} 2\mathbf{y}^2\cdot\mathbf{a}\cdot\sqrt{1-\frac{\mathbf{y}}{\mathbf{b}}}\,\mathbf{dy}$$

$\mathbf{I_x} = 23.8\,\mathbf{ft}^4$

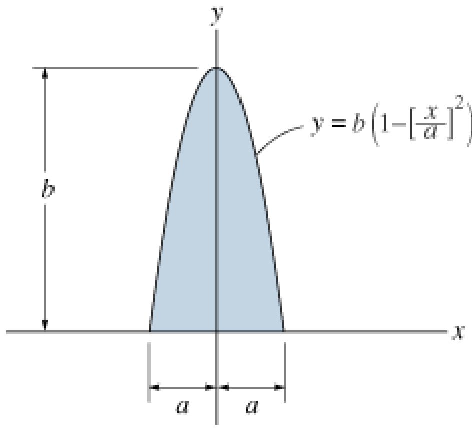


(b)

Problem 10-4

Determine the moment of inertia of the area about the x axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness dx and (b) having a thickness of dy.

Units Used:

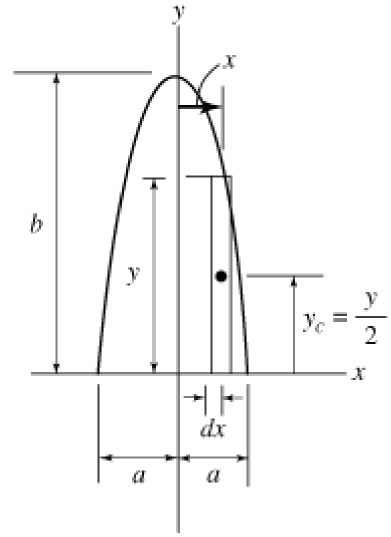


Given:

$\mathbf{a} := 1\mathbf{in}$

$\mathbf{b} := 4\mathbf{in}$

Solution:



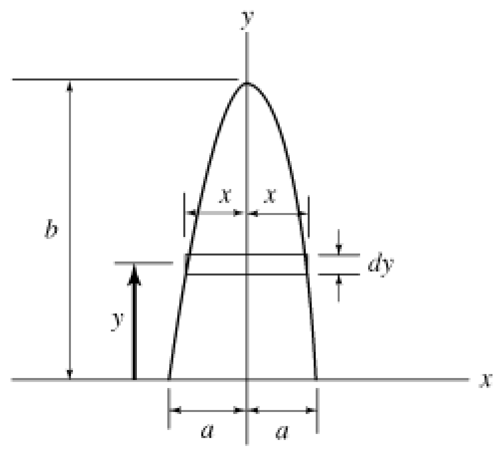
(a)

a) Using a thickness d*x*.

$$\mathbf{dI_x} = \frac{\mathbf{y}^3}{3}\mathbf{dx} = \frac{\mathbf{b}^3}{3}\left[1 - \left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2\right]^3 \mathbf{dx}$$

$$\mathbf{I_x} := \int_{-\mathbf{a}}^{\mathbf{a}} \frac{\mathbf{b}^3}{3}\cdot\left[1 - \left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2\right]^3 \mathbf{dx}$$

$\mathbf{I_x} = 19.5\,\mathbf{in}^4$



(b)

b) Using a thickness d*y*

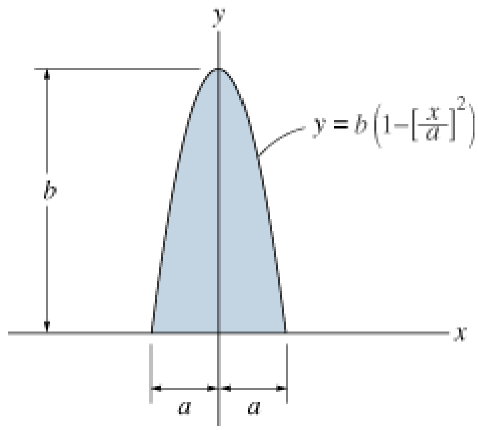
$$\mathbf{I_x} := \int_0^{\mathbf{b}} 2\mathbf{y}^2\cdot\mathbf{a}\cdot\sqrt{1 - \frac{\mathbf{y}}{\mathbf{b}}}\,\mathbf{dy}$$

$\mathbf{I_x} = 19.5\,\mathbf{in}^4$

Problem 10-5

Determine the moment of inertia of the area about the y axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of *dx*, and (b) having a thickness of *dy*.

Given:



$a := 1\text{in}$

$b := 4\text{in}$

$y = b \cdot \left[1 - \left(\frac{x}{a}\right)^2\right]$

Solution:

a) Using d*x*

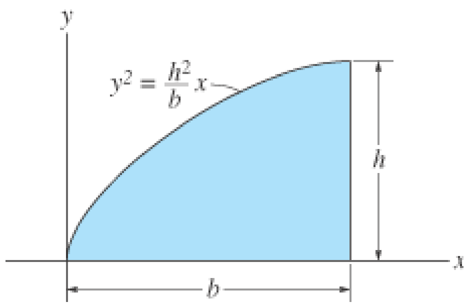
$$I_y := \int_{-a}^{a} x^2 \cdot b \cdot \left[1 - \left(\frac{x}{a}\right)^2\right] dx \qquad I_y = 1.07\ \text{in}^4$$

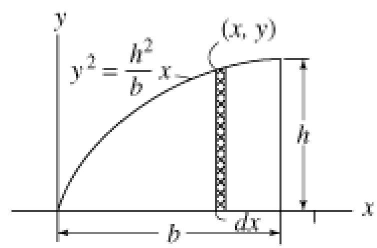
b) Using d*y*

$$I_y := \int_{0}^{b} \frac{2}{3} \cdot \left(a \cdot \sqrt{1 - \frac{y}{b}}\right)^3 dy \qquad I_y = 1.07\ \text{in}^4$$

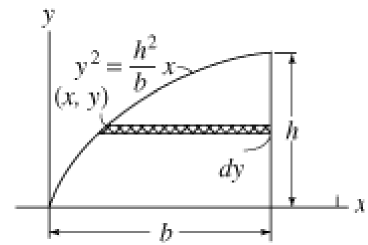
Problem 10-6

Determine the moment of inertia of the shaded area about the *x* axis.





Solution:



$$I_x = \int_0^b \frac{\left(h \cdot \sqrt{\frac{x}{b}}\right)^3}{3}\, dx \rightarrow I_x = \frac{2}{15} \cdot b \cdot h^3$$

$$I_x = \frac{2}{15} \cdot b \cdot h^3$$

Alternatively

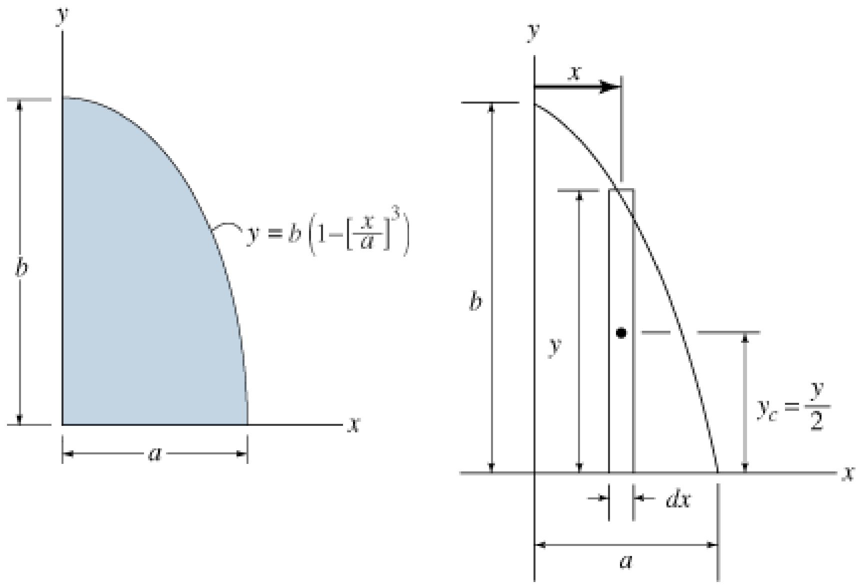
$$I_x = \int_0^h y^2 \cdot \left(b - b \cdot \frac{y^2}{h^2}\right) dy \rightarrow I_x = \frac{2}{15} \cdot b \cdot h^3$$

$$I_x = \frac{2}{15} \cdot b \cdot h^3$$

Problem 10-7

Determine the moment of inertia of the shaded area about the *x* axis.

Given:



$\mathbf{a} := 1\,\mathbf{in}$

$\mathbf{b} := 2\,\mathbf{in}$

Solution:

$$\mathbf{I_x} := \int_0^{\mathbf{a}} \frac{1}{3}\cdot\left[\mathbf{b}\cdot\left[1-\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^3\right]\right]^3 \mathbf{dx}$$

$$\mathbf{I_x} = 1.54\,\mathbf{in}^4$$

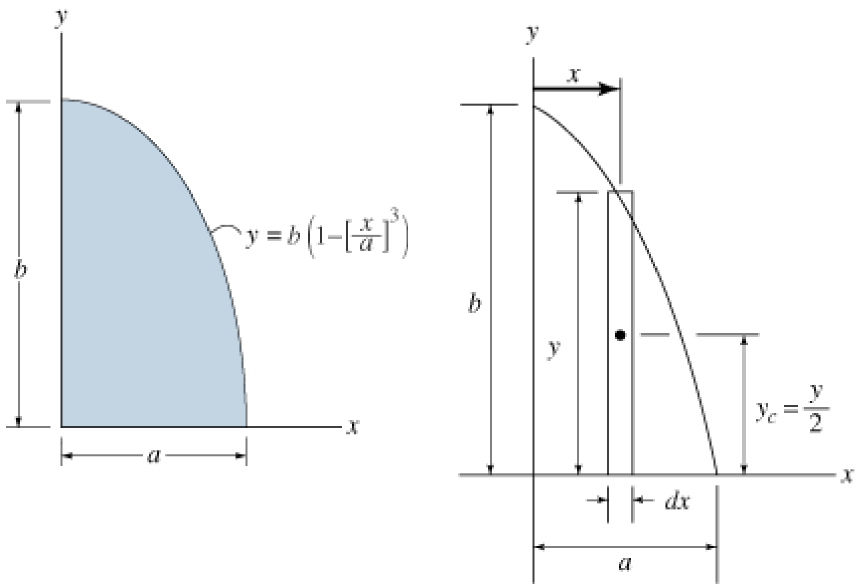
Problem 10-8

Determine the moment of inertia of the shaded area about the *y* axis.

Given:

$\mathbf{a} := 1\,\mathbf{in}$

$\mathbf{b} := 2\,\mathbf{in}$

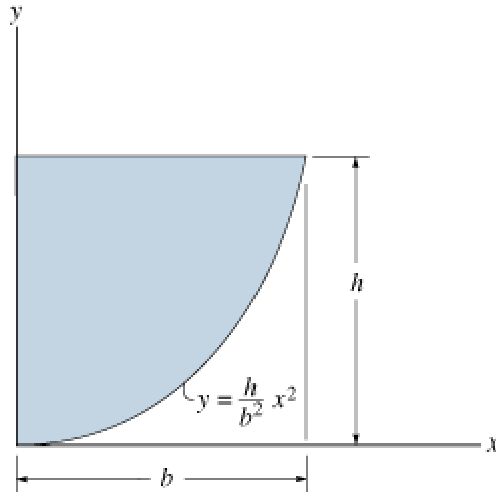


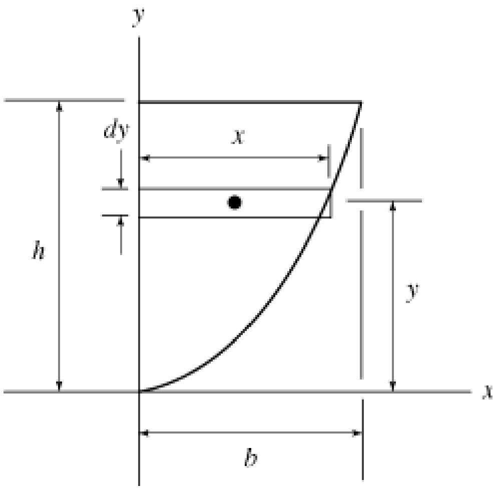
Solution:

$$\mathbf{I_y} := \int_0^{\mathbf{a}} \mathbf{x}^2 \cdot \mathbf{b} \cdot \left[1 - \left(\frac{\mathbf{x}}{\mathbf{a}}\right)^3\right] d\mathbf{x} \qquad \mathbf{I_y} = 0.33\,\mathbf{in}^4$$

Problem 10-9

Determine the moment of inertia of the shaded area about the *x* axis.





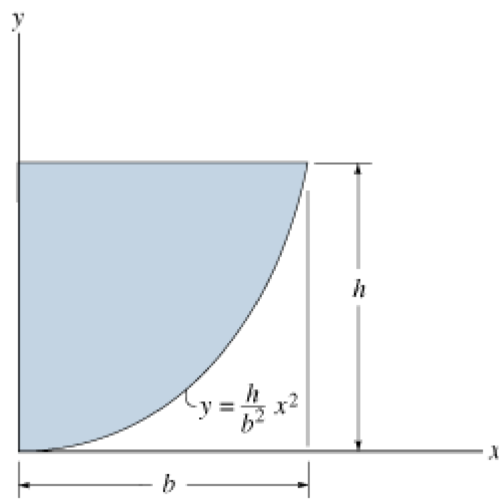
Solution:

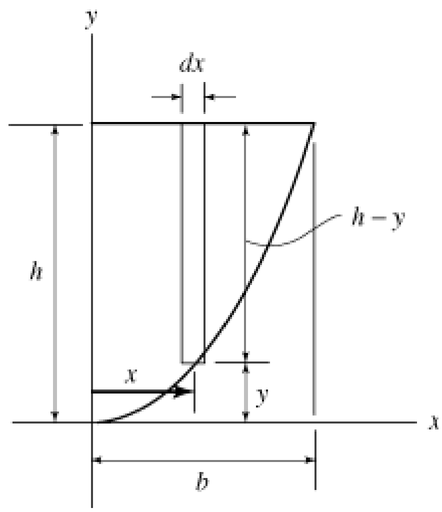
$$I_x = \int_0^h y^2 \cdot b \cdot \sqrt{\frac{y}{h}}\, dy \rightarrow I_x = \frac{2}{7} \cdot h^3 \cdot b$$

$$I_x = \frac{2}{7} \cdot b \cdot h^3$$

Problem 10-10

Determine the moment of inertia of the shaded area about the y axis.





Solution:

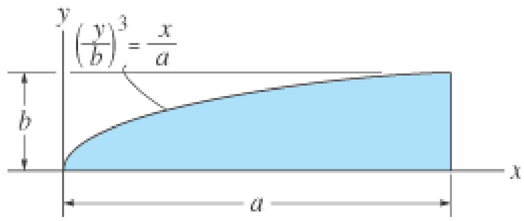
$$I_y = \int_0^b x^2 \cdot \left(h - h \cdot \frac{x^2}{b^2}\right) dx \rightarrow I_y = \frac{2}{15} \cdot h \cdot b^3$$

$$I_y = \frac{2}{15} \cdot h \cdot b^3$$

Problem 10-11

Determine the moment of inertia of the shaded area about the *x* axis

Given:



$a := 8\,in$

$b := 2\,in$

Solution:

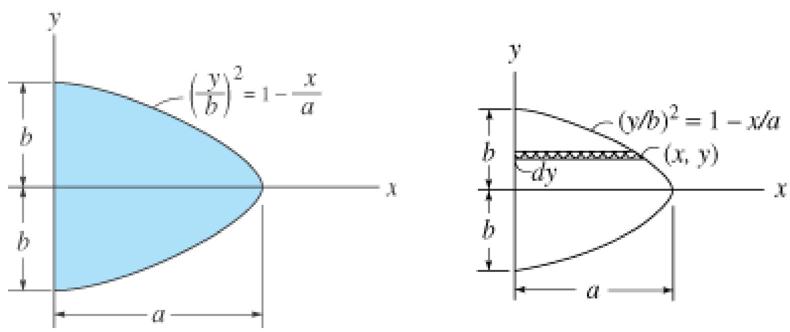
$$I_x := \int_0^b y^2 \cdot \left(a - a \cdot \frac{y^3}{b^3}\right) dy \qquad I_x = 10.667\,in^4$$

$$I_x := \int_0^a \frac{\left[b \cdot \left(\frac{x}{a}\right)^{\frac{1}{3}}\right]^3}{3}\,dx \qquad I_x = 10.667\,in^4$$

Problem 10-12:

Determine the moment of inertia of the shaded area about the *x* axis

Given:



$\mathbf{a} := 2\mathbf{m}$

$\mathbf{b} := 1\mathbf{m}$

Solution:

$$\mathbf{I_x} := \int_{-\mathbf{b}}^{\mathbf{b}} \mathbf{y}^2 \cdot \mathbf{a} \cdot \left(1 - \frac{\mathbf{y}^2}{\mathbf{b}^2}\right) d\mathbf{y}$$

$$\mathbf{I_x} = 0.533\ \mathbf{m}^4$$

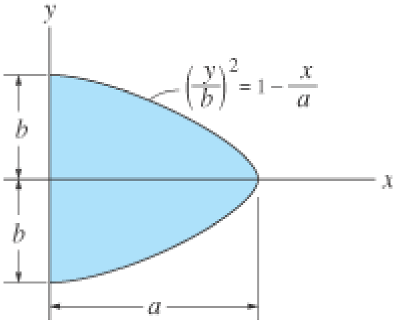
Problem 10-13

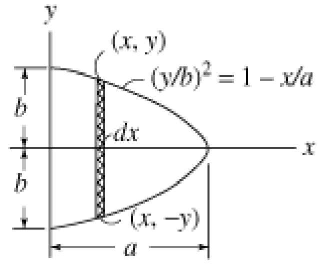
Determine the moment of inertia of the shaded area about the *y* axis

Given:

$\mathbf{a} := 2\mathbf{m}$

$\mathbf{b} := 1\mathbf{m}$



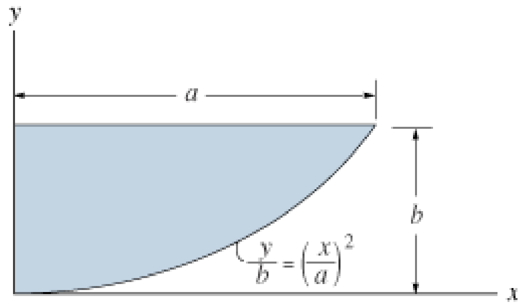


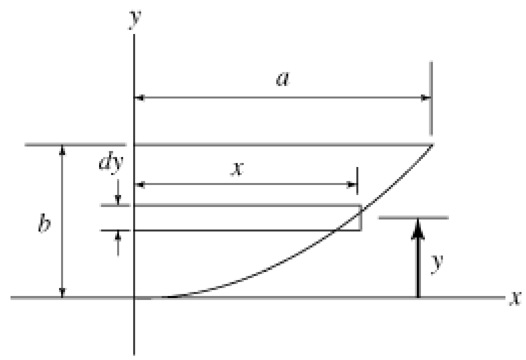
Solution:

$$\mathbf{I_y} := \int_0^{\mathbf{a}} \mathbf{x}^2 \cdot 2 \cdot \mathbf{b} \cdot \sqrt{1 - \frac{\mathbf{x}}{\mathbf{a}}}\, d\mathbf{x} \qquad \mathbf{I_y} = 2.438\ \mathbf{m}^4$$

Problem 10-14

Determine the moment of inertia of the shaded area about the *x* axis





Given:

$a := 2\,\mathrm{in}$

$b := 1\,\mathrm{in}$

$$I_x := \int_0^b y^2 \cdot a \cdot \sqrt{\frac{y}{b}}\, dy$$

$I_x = 0.571\,\mathrm{in}^4$

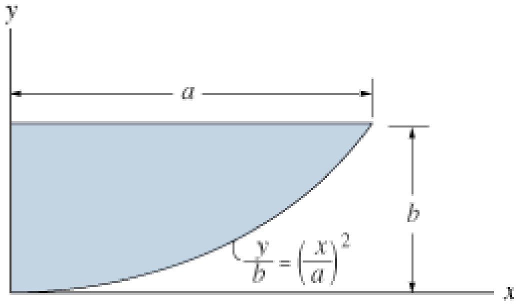
Problem 10-15

Determine the moment of inertia of the shaded area about the *y* axis.

Given:

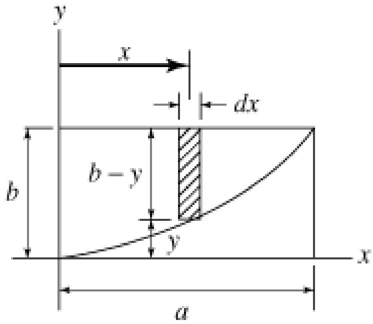
$a := 2in$

$b := 1in$



Solution:

$$I_y := \int_0^a x^2 \cdot \left(b - b \cdot \frac{x^2}{a^2}\right) dx$$



$I_y = 1.067\,in^4$

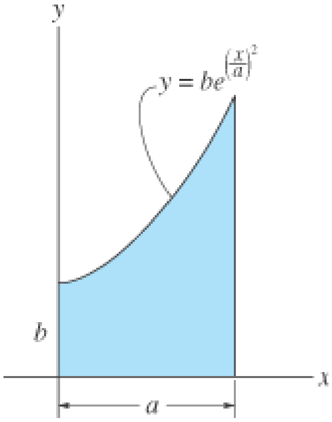
Problem 10-16

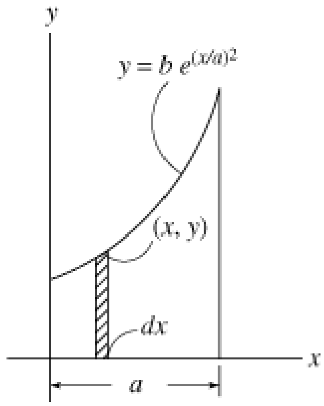
Determine the moment of inertia of the area about the *y* axis. Use Simpson's rule to evaluate the integral.

Given :

$\mathbf{a} := 1\mathbf{m}$

$\mathbf{b} := 0.5\mathbf{m}$





Solution :

$$\mathbf{I_y} := \int_0^{\mathbf{a}} \mathbf{x}^2 \cdot \mathbf{b} \cdot \mathbf{e}^{\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2} \, \mathbf{dx} \qquad \mathbf{I_y} = 0.314\ \mathbf{m}^4$$

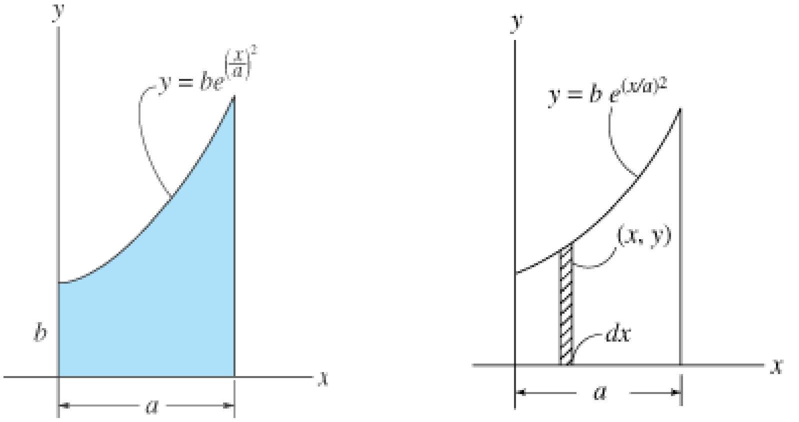
Problem 10-17

Determine the moment of inertia of the area about the *x* axis. Use Simpson's rule to evaluate the integral.

Given :

$\mathbf{a} := 1\mathbf{m}$

$\mathbf{b} := 0.5\mathbf{m}$



Solution :

$$I_x := \int_0^{a} \frac{\left[b \cdot e^{\left(\frac{x}{a}\right)^2}\right]^3}{3}\, dx \qquad I_x = 0.176\ \mathrm{m}^4$$

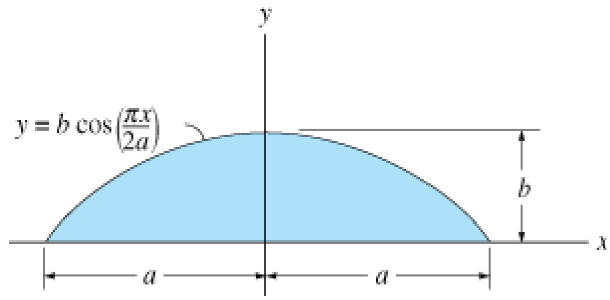
Problem 10-18

Determine the moment of inertia of the shaded area about the *x* axis.

Given :

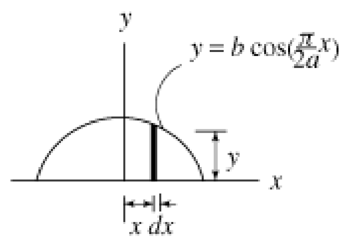
$\mathbf{a} := 4\,\mathbf{in}$

$\mathbf{b} := 2\,\mathbf{in}$



$$\mathbf{I_x} := \int_{-\mathbf{a}}^{\mathbf{a}} \frac{\left(\mathbf{b}\cdot\cos\left(\frac{\pi\cdot\mathbf{x}}{2\cdot\mathbf{a}}\right)\right)^3}{3}\,\mathbf{dx}$$

$\mathbf{I_x} = 9.05\,\mathbf{in}^4$



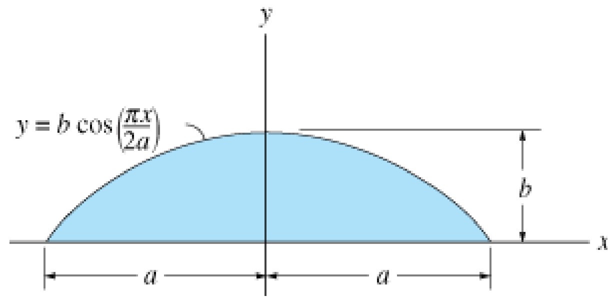
Problem 10-19

Determine the moment of inertia of the shaded area about the *y* axis.

Given :

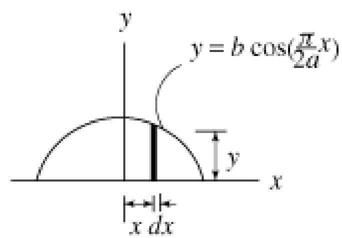
$\mathbf{a} := 4\mathbf{in}$

$\mathbf{b} := 2\mathbf{in}$



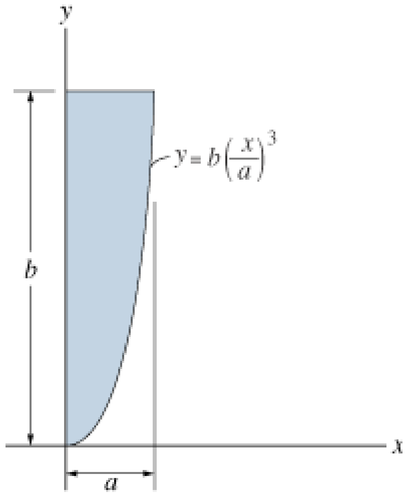
Solution :

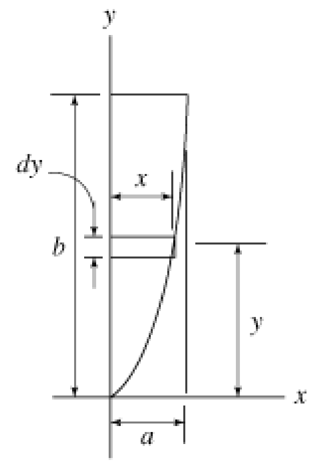
$$\mathbf{I_y} := \int_{-\mathbf{a}}^{\mathbf{a}} \mathbf{x}^2 \cdot \mathbf{b} \cdot \cos\left(\frac{\pi \cdot \mathbf{x}}{2 \cdot \mathbf{a}}\right) d\mathbf{x} \qquad \mathbf{I_y} = 30.87\,\mathbf{in}^4$$



Problem 10-20

Determine the moment of inertia of the shaded area about the *x* axis.





Given :

$\mathbf{a} := 2\mathbf{in}$

$\mathbf{b} := 8\mathbf{in}$

Solution :

$$\mathbf{I_x} := \int_0^{\mathbf{b}} \mathbf{y}^2 \cdot \mathbf{a} \cdot \left(\frac{\mathbf{y}}{\mathbf{b}}\right)^{\frac{1}{3}} \, \mathbf{dy} \qquad \mathbf{I_x} = 307.20\,\mathbf{in}^4$$

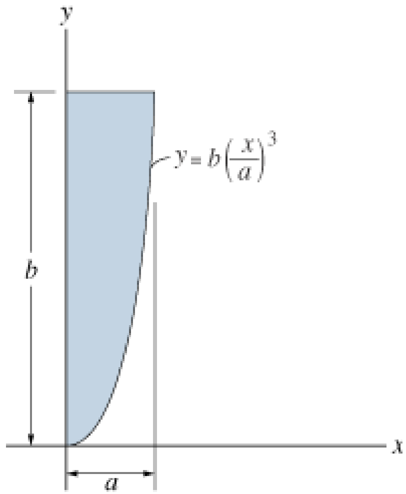
Problem 10-21

Determine the moment of inertia of the shaded area about the y axis.

Given :

$\mathbf{a} := 2\mathbf{in}$

$\mathbf{b} := 8\mathbf{in}$



Solution :

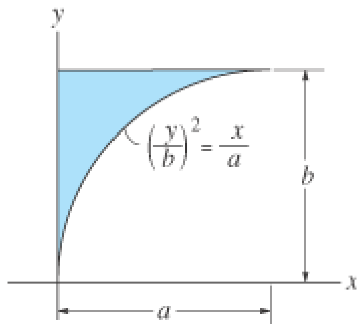
$$\mathbf{I_y} := \int_0^{\mathbf{a}} \mathbf{x}^2 \cdot \left(\mathbf{b} - \mathbf{b} \cdot \frac{\mathbf{x}^3}{\mathbf{a}^3}\right) \mathbf{dx}$$

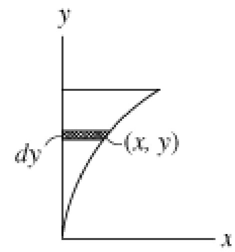
$$\mathbf{I_y} = 10.67\,\mathbf{in}^4$$

Problem 10-22

Determine the moment of inertia of the shaded area about the *x* axis.

Given :





$\mathbf{a} := 2\mathbf{m}$

$\mathbf{b} := 2\mathbf{m}$

Solution :

$$\mathbf{I_x} := \int_0^{\mathbf{b}} \mathbf{y}^2 \cdot \mathbf{a} \cdot \frac{\mathbf{y}^2}{\mathbf{b}^2}\, d\mathbf{y}$$

$$\mathbf{I_x} = 3.20\ \mathbf{m}^4$$

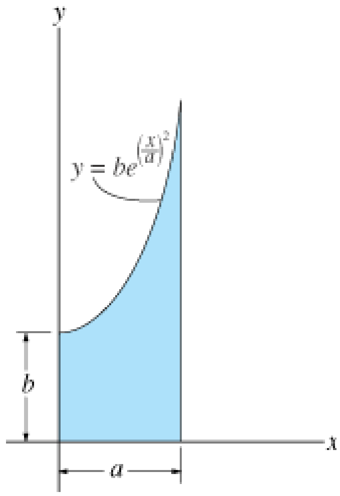
Problem 10-23

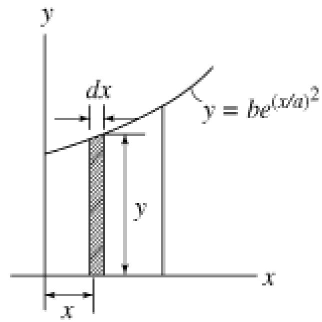
Determine the moment of inertia of the shaded area about the *y* axis. Use Simpson's rule to evaluate the integral.

Given:

$\mathbf{a} := 1\mathbf{m}$

$\mathbf{b} := 1\mathbf{m}$





Solution:

$$\mathbf{I_y} := \int_0^{\mathbf{a}} \mathbf{x}^2 \cdot \mathbf{b} \cdot \mathbf{e}^{\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2} \, \mathbf{dx} \qquad \mathbf{I_y} = 0.628\ \mathbf{m}^4$$

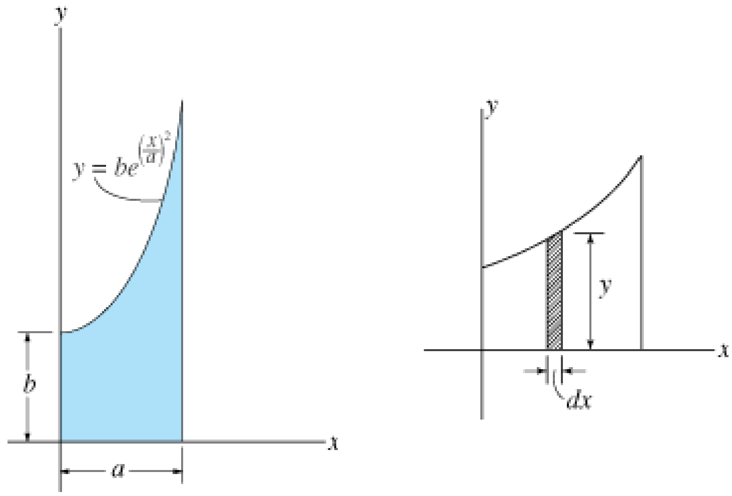
Problem 10-24

Determine the moment of inertia of the shaded area about the *x* axis. Use Simpson's rule to evaluate the integral.

Given:

$\mathbf{a} := 1\mathbf{m}$

$\mathbf{b} := 1\mathbf{m}$

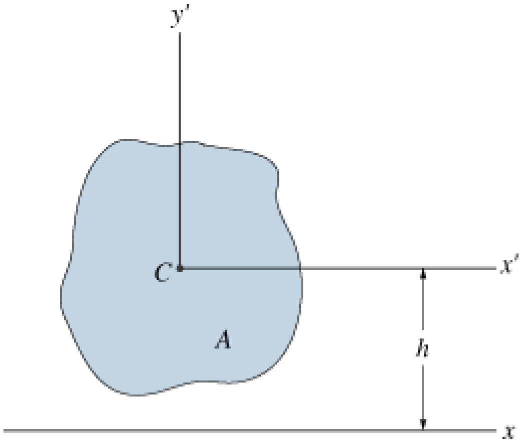


Solution:

$$\mathbf{I_y} := \int_0^{\mathbf{a}} \frac{\left[\mathbf{b}\cdot \mathbf{e}^{\left(\frac{\mathbf{x}}{\mathbf{a}}\right)^2}\right]^3}{3}\,\mathbf{dx} \qquad \mathbf{I_y} = 1.41\ \mathbf{m}^4$$

Problem 10-25

The polar moment of inertia of the area is J_{cc} about the z' axis passing through the centroid C. If the moment of inertia about the y' axis is $I_{cy'}$ and the moment of inertia about the x axis is I_x, determine the area A.



Given:

$J_{cc} := 23\,\mathrm{in}^4$

$I_{cy'} := 5\,\mathrm{in}^4$

$I_x := 40\,\mathrm{in}^4$

$h := 3\,\mathrm{in}$

Solution:

Moment of Inertia:

$J_{cc} = I_{cx'} + I_{cy'}$ $\quad I_{cx'} := J_{cc} - I_{cy'}$ $\quad I_{cx'} = 18.0\,\mathrm{in}^4$

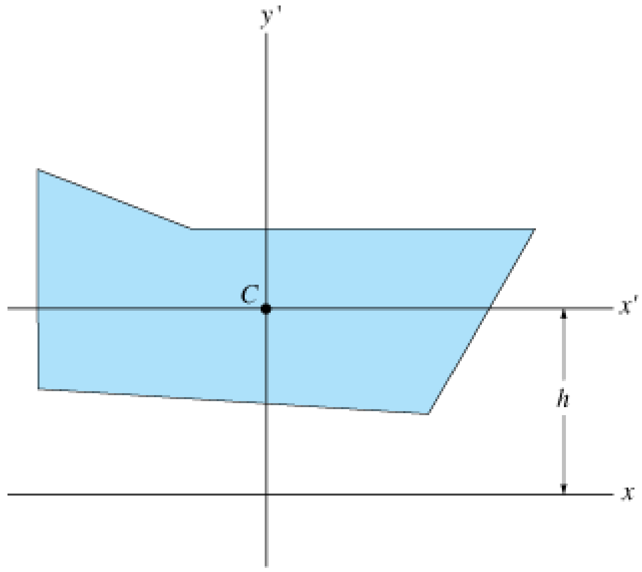
Applying the parallel-axis theorem, Eq. 10-3 we have

$I_x = I_{cx'} + A \cdot h^2$ $\quad A := \dfrac{I_x - I_{cx'}}{h^2}$ $\quad A = 2.44\,\mathrm{in}^2$

Problem 10-26

The polar moment of inertia of the area is J_{cc} about the z' axis passing through the centroid C. If the moment of inertia about the y' axis is $I_{y'}$ and the moment of inertia about the x axis is I_x. Determine the area A.

Given:



$J_{cc} := 548 \times 10^6 \, \mathrm{mm}^4$

$I_{y'} := 383 \times 10^6 \, \mathrm{mm}^4$

$I_x := 856 \times 10^6 \, \mathrm{mm}^4$

$h := 250 \mathrm{mm}$

Solution:

$I_{x'} = I_x - A \cdot h^2$

$J_{cc} = I_{x'} + I_{y'}$

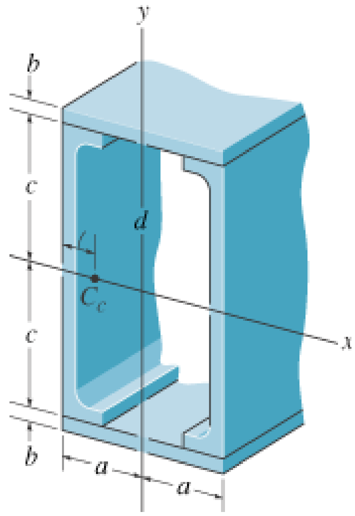
$J_{cc} = I_x - A \cdot h^2 + I_{y'}$

$$A := \frac{I_x + I_{y'} - J_{cc}}{h^2}$$

$A = 11.1 \times 10^3 \, \mathrm{mm}^2$

Problem 10-27

The beam is constructed from the two channels and two cover plates. If each channel has a cross-sectional area A_c and a moment of inertia about a vertical axis passing through its own centroid, C_c, of I_{xcc}, determine the moment of inertia of the beam about the x axis.



Given:

$A_c := 11.8\,\text{in}^2$

$I_{xcc} := 349\,\text{in}^4$

$a := 6\,\text{in}$

$b := 1\,\text{in}$

$c := 10\,\text{in}$

$d := 1.28\,\text{in}$

Solution:

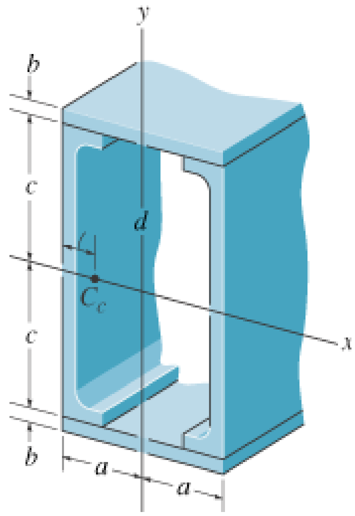
$$I_x := 2\left[\frac{1}{12}\cdot(2\cdot a)\cdot b^3 + b\cdot(2\cdot a)\cdot\left(c + \frac{b}{2}\right)^2\right] + 2\cdot I_{xcc}$$

$$I_x = 3.35\times 10^3\,\text{in}^4$$

Problem 10-28

The beam is constructed from the two channels and two cover plates. If each channel has a cross-sectional area A_c and a moment of inertia about a vertical axis passing through its own centroid, C_c, of I_{ycc}, determine the moment of inertia of the beam about the y axis.

Given:



$A_c := 11.8 in^2$

$I_{ycc} := 9.23 in^4$

$a := 6 in$

$b := 1 in$

$c := 10 in$

$d := 1.28 in$

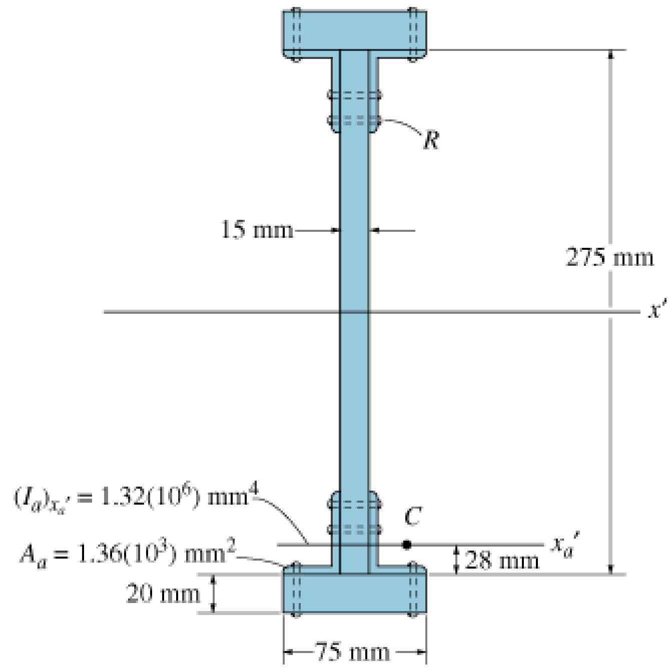
Solution:

$$I_y := 2\left[\frac{1}{12}\cdot b\cdot(2\cdot a)^3\right] + 2\cdot\left[\left(I_{ycc}\right) + A_c\cdot(a-d)^2\right]$$

$I_y = 832\, in^4$

Problem 10-29

Determine the moment of inertia of the beam's cross-sectional area with respect to the x' centroidal axis. Neglect the size of all the rivet heads. R, for the calculation. Handbook values for the area, moment of inertia, and location of the centroid C of one of the angles are listed in the figure.



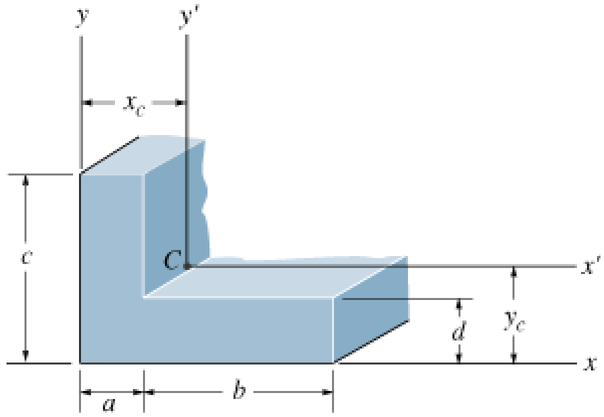
Solution:

$$I_E := \frac{1}{12}(15)\,275^3 + 4\left[1.32\left(10^6\right) + 1.36\left(10^3\right)\left(\frac{275}{2} - 28\right)^2\right] + 2\left[\frac{1}{12}\cdot(75)\,20^3 + (75)\cdot(20)\left(\frac{275}{2} + 10\right)^2\right]$$

$I_E = 161.9 \times 10^6$ $\quad$ $\mathbf{mm}^4$

Problem 10-30

Locate the centroid y_c of the cross-sectional area for the angle. Then find the moment of inertia $I_{cx'}$ about the x' centroidal axis.



Given:

$\mathbf{a} := 2\,\mathbf{in}$

$\mathbf{b} := 6\,\mathbf{in}$

$\mathbf{c} := 6\,\mathbf{in}$

$\mathbf{d} := 2\,\mathbf{in}$

Solution:

Centroid : The area of each segment and its respective centroid are tabulated below.

$$y_c := \frac{a \cdot c \cdot \frac{c}{2} + b \cdot d \cdot \frac{d}{2}}{a \cdot c + b \cdot d} \qquad y_c = 2.00\,\mathbf{in}$$

$$I_{x'} := \frac{1}{12} \cdot a \cdot c^3 + a \cdot c \cdot \left(\frac{c}{2} - y_c\right)^2 + \frac{1}{12} b \cdot d^3 + b \cdot d \cdot \left(y_c - \frac{d}{2}\right)^2 \qquad I_{x'} = 64.00\,\mathbf{in}^4$$

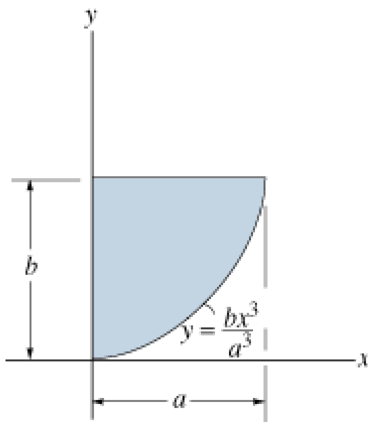
Problem 10-122

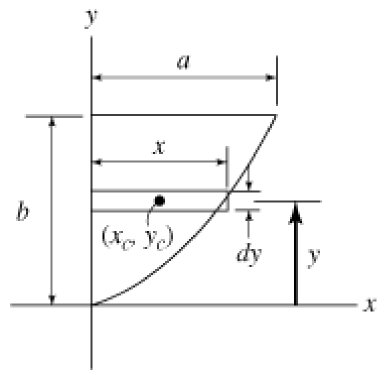
Determine the product of inertia of the shaded area with respect to the x and y axes.

Given :

$\mathbf{a} := 1\mathbf{m}$

$\mathbf{b} := 1\mathbf{m}$





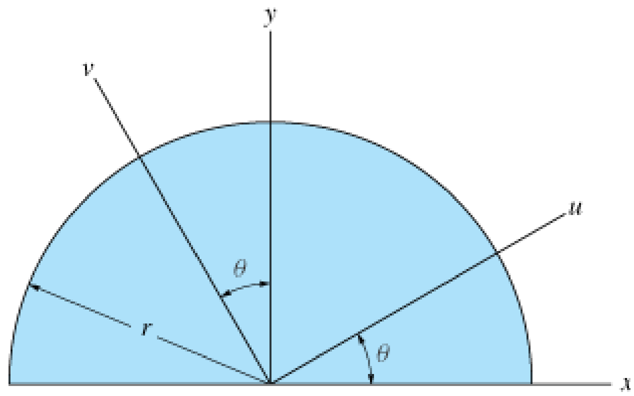
Solution :

$$\mathbf{I_{xy}} := \int_0^{\mathbf{b}} \frac{\mathbf{1}}{2}\mathbf{y}\cdot\mathbf{a}\cdot\left(\frac{\mathbf{y}}{\mathbf{b}}\right)^{\frac{1}{3}}\cdot\mathbf{a}\cdot\left(\frac{\mathbf{y}}{\mathbf{b}}\right)^{\frac{1}{3}}\,\mathbf{dy} \qquad \mathbf{I_{xy}} = 0.1875\ \mathbf{m}^{\mathbf{4}}$$

$I_{xy} = 0.19\ m^4$

Problem 10-117

Determine the area moments of inertia I_u and I_v and the product of inertia I_{uv} for the semicircular area.



Given:

$\mathbf{r} := 60\mathbf{mm}$

$\theta := 30\mathbf{deg}$

Solution:

$I_x := \frac{\pi \cdot r^4}{8} \qquad I_y := I_x$

(Due to symmetry) $\qquad I_{xy} := 0\mathbf{mm}^4$

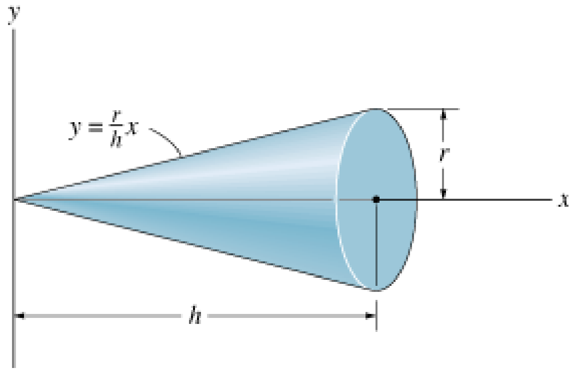
$$I_u := \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cdot \cos(2\cdot\theta) - I_{xy} \cdot \sin(2\cdot\theta) \qquad I_u = 5.09 \times 10^6\ \mathbf{mm}^4$$

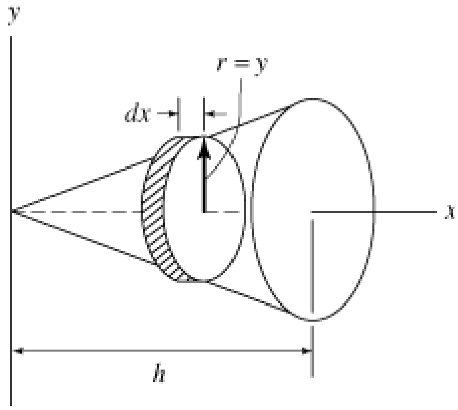
$$I_v := \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cdot \cos(2\cdot\theta) - I_{xy} \cdot \sin(2\cdot\theta) \qquad I_v = 5.09 \times 10^6\ \mathbf{mm}^4$$

$$I_{uv} := \frac{I_x - I_y}{2} \cdot \sin(2\cdot\theta) + I_{xy} \cdot \cos(2\cdot\theta) \qquad I_{uv} = 0\ \mathbf{m}^4$$

Problem 10-92

Determine the moment of inertia I_x of the right circular cone and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.





Solution :

$$m = \rho \cdot \frac{\pi \cdot r^2 \cdot h}{3} \qquad \rho = \frac{3 \cdot m}{\pi \cdot r^2 \cdot h} \qquad dI_x = \frac{1}{2} dm \cdot \left(\frac{r \cdot x}{h}\right)^2$$

$$I_x = \int_0^h \frac{1}{2} \cdot \left(\frac{3 \cdot m}{\pi \cdot r^2 \cdot h}\right) \pi \cdot \left(\frac{r \cdot x}{h}\right)^2 \left(\frac{r \cdot x}{h}\right)^2 dx \rightarrow I_x = \frac{3}{10} \cdot m \cdot r^2$$

$$I_x = \frac{3}{10} \cdot m \cdot r^2$$