9 PROBLEMS

9.1 Introduction

1. Determine the number of degrees of freedom of the planar elipsograph mechanism in Fig. 9.1. Find the analytical expression of any point P on the link 2.

2. Find the mobility of the planar mechanism represented in Fig. 9.2.

3. Determine the family and the number of degrees of freedom for the mechanism depicted in Fig. 9.3.

4. The roller 2 of the mechanism in Fig. 9.4 undergoes an independent rotation about its axis which does not influence the motion of the link 3. The purpose of the element 2 is to substitute the sliding friction with a rolling friction. From a kinematical point of view the roller 2 is a passive element. Find the number of degrees of freedom of the mechanism.

5. Find the family and the number of degrees of freedom of the mechanism in Fig. 9.5.

9.2 Kinematics and force analysis

6. The following data are given for the mechanism shown in Fig. 9.6: $AB = CD = 0.04$ m and $AD = BC = 0.09$ m. Find the trajectory of the point M located on the link BC, for the case a) $BM = MC$, and b) $MC = 2\,BM$.

7. The planar mechanism depicted in Fig. 9.7 has dimensions $AB =$ 0.03 m, $BC = 0.065$ m, $CD = 0.05$ m, $BM = 0.09$ m, and $CM =$ 0.12 m. Find the trajectory described by the point M.

8. The mechanism shown in Fig. 9.8 has dimensions $AB = 0.03$ m, $BC =$ 0.12 m, $CD = 0.12$ m, $ED = 0.0$ m, $CF = 0.17$ m, $R_1 = 0.04$ m, $R_4 = 0.08$ m, $L_a = 0.025$ m, and $L_b = 0.105$ m. Find the trajectory of the joint C.

9. I. The length of the links are known for the mechanism shown in Fig. 9.9. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.9. Determine: a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints B, C, D, and E;

c) the linear velocities of the joints B, C, D, E, and the angular velocities of the links 2, 3, and 4;

d) the linear accelerations of the joints B, C, D, E, and the angular accelerations of the links 2, 3, and 4;

II. The links 1, 2, and 4 are homogeneous bars made of steel, each with a mass density ρ=7 800 kg m^3, and each with a constant cross section A_s=1 cm^2. The sliders 3 and 5 have negligible dimensions and can be considered as material points with masses $m_3 = m_5$=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external vertical force of magnitude $|F_e|$=250 N acts on link 5 at the point D.

10. I. The mechanism shown in Fig. 9.10 has links of known length. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.10. Determine:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints B, D, E, F and the trajectory of C;

the trajectory of C for a complete rotation of the driver link 1, $\phi \in [0°, ..., 360°]$;

c) the linear velocities of the joints B, E, F, and the angular velocities of the links 2, 3, and 4;

d) the linear accelerations of the joints B, E, F, and the angular accelerations of the links 2, 3, and 4;

II. The links 1, 2, and 4 are homogeneous bars made of steel having a mass density ρ=7 800 kg m^3, with a constant cross section A_s=2 cm^2. The sliders 3 and 5 have negligible dimensions and can be considered as material points with masses $m_3 = m_5$=0.2 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and equilibrium moment if an external vertical force of magnitude $|F_e|$=500 N acts on link 5 at the point E .

11. I. The dimensions of the links are given for the mechanism shown in Fig. 9.11. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.11. Determine:

a) the family f of the mechanism and the number of degrees of freedom;

b) the positions of the joints B and D;

c) the linear velocities of the joints B, D, and the angular velocities of the links 2, 3, 4, and 5;

d) the linear accelerations of the joints B, D, and the angular accelerations of the links 2, 3, 4, and 5;

II. The links 1, 3, and 5 are homogeneous bars made of aluminum having a constant cross section A_s=1 cm^2. The sliders 2 and 4 have negligible dimensions and can be considered as material points with masses $m_2 = m_4$=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external torque of magnitude $|M_e|$=500 N m acts on link 5 at the point E.

12. I. The lengths of the links are known for the mechanism shown in Fig. 9.12 the dimensions of the links are given. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.12. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints B, C, D and F. The link 2 is a plate.

c) the linear velocities of the joints B, C, D, F and the angular velocities of the links 2, 3, 4, and 5;

d) the linear accelerations of the joints B, C, D, F and the angular accelerations of the links 2, 3, 4, and 5;

II. The links 1, 3, and 4 are homogeneous bars made of steel with a constant cross section A_s=1 cm^2. The link 2 is a homogeneous plate made of steel with a width of 1 cm. The slider 5 has negligible dimensions and can be considered as a material point with a mass m_5=0.2 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at the point F.

13. I. The lengths of the links are known for the mechanism shown in Fig. 9.13. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.13. Determine:

a) the family f and the number of degrees of freedom;

b) the positions of the joints; the trajectory of the center of mass of the link 2 for a complete rotation of the driver link 1, $\phi \in [0°, ..., 360°]$;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 4, and 5 are homogeneous bars made of aluminum having a constant cross section A_s=0.5 cm^2. The slider 3 has negligible dimensions and can be considered as a material point with a mass m_3=0.01 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external torque of magnitude $|M_e|$=500 N m acts on link 5 at the point F.

14. I. The dimensions of the links are given for the mechanism shown in Fig. 9.14. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.14. Determine:

a) the family f and the number of degrees of freedom;

b) the positions of the joints; the trajectory of the center of mass of the link 2 for a complete rotation of the driver link 1;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 3, and 4 are homogeneous bars made of steel each having a constant cross section A_s=1 cm^2. The slider 5 has negligible dimensions and can be considered as a material point with a mass m_5=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and equilibrium moment if an external force $|F_e|$=800 N acts on link 5 at the point F.

15. I. The lengths of the links are known for the mechanism shown in Fig. 9.15. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.15. Determine:

a) the family f and the number of degrees of freedom;

b) the positions of the joints; the trajectory of the center of mass of the link 2 for a complete rotation of the driver link 1, $\phi \in [0°, ..., 360°]$;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, and 5 are homogeneous bars made of steel each with a constant cross section A_s=0.5 cm^2. The sliders 3 and 4 have negligible dimensions and can be considered as material points each with masses $m_3 = m_4$=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external torque of magnitude $|M_e|$=500 N m acts on link 5 at the point E.

16. I. The dimensions of the links are given for the mechanism shown in Fig. 9.16. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.16. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, and 4 are homogeneous bars made of aluminum each with a constant cross section A_s=1 cm^2. The sliders 3 and 5 have negligible dimensions and can be considered as material points each with masses $m_3 = m_5$=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at the point D.

17. I. The lengths of the links are known for the mechanism shown in Fig. 9.17. The angle of the driver link 1 with the horizontal is ϕ and the angular speed of the driver link 1 is n=constant. The input data for ten cases are given in the table in Fig. 9.17. Determine:

a) the family f and the number of degrees of freedom;

b) the positions of the joints; the trajectory of the center of mass of the link 4 for a complete rotation of the driver link 1;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 4, and 5 are homogeneous bars made of steel each with a constant cross section A_s=1.5 cm^2. The slider 3 has negligible dimensions and can be considered as a material point with mass m_3=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and equilibrium moment if an external torque of magnitude $|M_e|$=900 N m acts on link 5 at the point F.

18. I. The dimensions of the links are given for the mechanism in Fig. 9.18. The angle of the driver link 1 with the horizontal is ϕ and the angular speed of the driver link 1 is n=constant. The input data for ten cases are given in the table in Fig. 9.18. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 3, and 4 are homogeneous bars made of aluminum each with a constant cross section A_s=0.1 cm^2. The slider 5 has negligible dimensions and can be considered as a material point with mass m_5=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=600 N acts on link 5 at F.

19. I. The dimensions of the links are given for the mechanism in Fig. 9.19. The angle of the driver link 1 with the horizontal is ϕ and the angular speed of the driver link 1 is n=constant. The input data for ten cases are given in the table in Fig. 9.19. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 3, and 4 are homogeneous bars made of steel each with a constant cross section A_s=1 cm^2. The slider 5 has negligible dimensions and can be considered as a material point with a mass m_5=0.3 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=600 N acts on link 5 at F.

20. I. The dimensions of the links are given for the mechanism in Fig. 9.20. The angle of the driver link 1 with the horizontal is ϕ and the angular speed of the driver link 1 is n=constant. The input data for ten cases are given in the table in Fig. 9.20. Find:

a) the family f and the number of degrees of freedom;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, 3, and 4 are homogeneous bars made of steel each with a constant cross section A_s=0.5 cm^2. The slider 5 has negligible dimensions and can be considered as a material point with mass m_5=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at E.

21. I. The dimensions of the links are given for the mechanism in Fig. 9.21. The angle of the driver link 1 with the horizontal is ϕ and the angular speed of the driver link 1 is n=constant. The input data for ten cases are given in the table in Fig. 9.21. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, and 4 are homogeneous bars made of aluminum each with a constant cross section A_s=1 cm^2. The sliders 3 and 5 have negligible dimensions and can be considered as material points each with masses $m_3 = m_5$=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 3 at the point C.

22. I. The dimensions of the links are given for the mechanism in Fig. 9.22. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.22. Find:

a) the family f of the mechanism and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear acceleration of the joints and the angular accelerations of the links;

II. The links 1, 3, and 4 are homogeneous bars made of steel each having a constant cross section A_s=1 cm^2. The link 2 is a homogeneous plate made of steel with a width of 0.5 cm. The slider 5 has negligible dimensions and can be considered as a material point with mass m_5=0.1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at the point F.

23. I. The dimensions of the links are given for the mechanism in Fig. 9.23. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.23. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 3, and 4 are homogeneous bars made of aluminum with a constant cross section A_s=1 cm^2. The sliders 2 and 5 have negligible dimensions and can be considered as material points with masses $m_2 = m_5$=0.5 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at E.

24. I. The dimensions of the links are given for the mechanism in Fig. 9.24. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.24. Find:

    a) the family f and the number of degrees of freedom of the mechanism;

    b) the positions of the joints;

    c) the linear velocities of the joints and the angular velocities of the links;

    d) the linear accelerations of the joints and the angular accelerations of the links;

    II. The links 1, 3, and 4 are homogeneous bars made of steel each having a constant cross section A_s=0.5 cm^2. The sliders 2 and 5 have negligible dimensions and can be considered as material points each having masses $m_2 = m_5$=0.1 kg. Find:

    e) the forces and moments of inertia for each link;

    f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at the point E.

25. I. The dimensions of the links are given for the mechanism in Fig. 9.25. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the

angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.25. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 3, and 4 are homogeneous bars made of steel each with a constant cross section A_s=1 cm^2. The sliders 2 and 5 have negligible dimensions and can be considered as material points each with masses $m_2 = m_5$=1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=700 N acts on link 5 at the point E.

26. I. The dimensions of the links are given for the mechanism in Fig. 9.26. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.26. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 3, and 5 are homogeneous bars made of steel each with a constant cross section A_s=1 cm^2. The sliders 2 and 4 have negligible dimensions and can be considered as material points each with masses $m_2 = m_4$=0.5 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external torque of magnitude $|M_e|$=700 N m acts on link 5 at the point A.

27. I. The dimensions of the links are given for the mechanism in Fig. 9.27. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.27. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, and 4 are homogeneous bars made of steel with a constant cross section A_s=0.6 cm^2. The sliders 3 and 5 have negligible dimensions and can be considered as material points with masses $m_3 = m_5$=0.5 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=500 N acts on link 5 at E.

28. I. The dimensions of the links are given for the mechanism in Fig. 9.28. The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.28. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 2, and 5 are homogeneous bars made of steel each having a constant cross section A_s=0.5 cm^2. The sliders 3 and 4 have negligible dimensions and can be considered as material points with masses $m_3 = m_4$=0.5 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=900 N acts on link 5 at the point G.

29. I. The dimensions of the links are given for the mechanism shown in Fig. 9.29 . The angle of the driver link 1 with the horizontal is $\phi = \phi_1$ and the angular speed of the driver link 1 is $n = n_1$=constant. The input data for ten cases are given in the table in Fig. 9.29. Find:

a) the family f and the number of degrees of freedom of the mechanism;

b) the positions of the joints;

c) the linear velocities of the joints and the angular velocities of the links;

d) the linear accelerations of the joints and the angular accelerations of the links;

II. The links 1, 3, and 5 are homogeneous bars made of steel with a constant cross section A_s=1 cm^2. The sliders 2 and 4 have negligible dimensions and can be considered as material points with masses $m_2 = m_4$=1 kg. Find:

e) the forces and moments of inertia for each link;

f) the joint forces and the equilibrium moment if an external force of magnitude $|F_e|$=700 N acts on link 5 at the point G.

9.3 Gears

30. The number of teeth of the gears in contact are given for the planetary gear train shown in Fig. 9.30. The gear 1 has N_1 external gear teeth, the gear 2 has N_2 external gear teeth, and the gear $2'$ has $N_{2'}$ external gear teeth. The gears 2 and $2'$ are fixed on the same shaft CC'. The gear 3 has N_3 external gear teeth, and the gear $3'$ has $N_{3'}$ external gear teeth . The gears 3 and $3'$ are fixed on the same shaft EE'. The planet gear 4 has N_4 external gear teeth, and the planet gear 5 has N_5 external gear teeth. The gear 1 rotates with a constant input angular speed n_1. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.30. a) Find the absolute angular velocity of the output planet arm 6. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=500 N m acts on the planet arm 6 at the point L.

31. The planetary gear train considered in Fig. 9.31 has gears with the same module m. The sun gear 1 has N_1 external gear teeth, the planet gear 2 has N_2 external gear teeth, the gear 3 has N_3 external gear teeth, and the gear 4 has N_4 external gear teeth. The gears 3 and $3'$ are fixed

on the same shaft. The sun gear 1 rotates with an input angular speed n_1 rpm, and the arm 5 rotates with n_5 rpm. The input data for ten cases are given in the table in Fig. 9.31. a) Find the number of DOF for the planetary gear train. b) Find the absolute angular velocity of the output gear 4. c) Find the equilibrium torque if an external torque of magnitude $|M_e|$=600 N m acts on gear 4.

32. The number of teeth of the gears in contact are given for the planetary gear train considered in Fig. 9.32 The gear 1 has N_1 external gear teeth, the gear 2 has N_2 external gear teeth, the gear $2'$ has $N_{2'}$ external gear teeth, the gear 3 has N_3 external gear teeth, and the gear $3'$ has $N_{3'}$ external gear teeth. The planet gears 2 and $2'$ are fixed on the same link and the planet gears 3 and $3'$ are fixed on the same shaft. The gear 1 rotates with the input angular speed n_1 rpm, and the arm 5 rotates at n_5 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.32. a) Find the absolute angular velocity of the output ring gear 4. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=400 N m acts on the ring gear 4.

33. A planetary gear train is shown in Fig. 9.33. The gear 1 has N_1 external

gear teeth, the planet gear 2 has N_2 external gear teeth, the gear $2'$ has $N_{2'}$ external gear teeth, the gear 3 has N_3 internal gear teeth, the sun gear $3'$ has $N_{3'}$ external gear teeth, and the planet gear 4 has N_4 external gear teeth. The gears 2 and $2'$ are fixed on the same shaft and the gears 3 and $3'$ are fixed on the same shaft. The sun gear 1 rotates with an input angular speed of n_1 rpm. The module of the gears is m mm. The input data for ten cases are given in the table in Fig. 9.33. a) Find the absolute angular velocity of the output arm 5. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=700 N m acts on arm 5.

34. A planetary gear train is shown in Fig. 9.34. The ring gear 1 has N_1 internal gear teeth, the planet gear 2 has N_2 external gear teeth , the planet gear $2'$ has $N_{2'}$ external gear teeth, the planet gear 3 has N_3 internal gear teeth, and the ring gear 4 has N_4 internal gear teeth. The gears 2 and $2'$ are fixed on the same shaft and the gears 3 and $3'$ are fixed on the same shaft. The gear 1 rotates with the input angular speed n_1 rpm, and the arm 5 rotates at n_5 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.34. a) Find the absolute angular velocity of the output ring gear

4. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=800 N m acts on the output ring gear 4.

35. The number of teeth of the gears in contact are given for the planetary gear train shown in Fig. 9.35. The sun gear 1 has N_1 external gear teeth, the planet gear 2 has N_2 external gear teeth, the gear 3 has N_3 internal gear teeth, the gear $3'$ has $N_{3'}$ external gear teeth. the gear 4 has N_4 external gear teeth, and the gear 5 has N_5 external gear teeth. The gear 1 rotates with a constant input angular speed n_1 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.35. a) Find the absolute angular velocity of the output ring arm 6. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=900 N m acts on the ring gear 6.

36. A planetary gear train is shown in Fig. 9.36. The sun ring gear 1 has N_1 internal gear teeth, the planet gear 2 has N_2 external gear teeth, and the planet gear $2'$ has $N_{2'}$-tooth external. The gears 2 and $2'$ are fixed on the same shaft. The gear 1 rotates with the input angular speed n_1 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.36. a) Find the absolute angular

velocity of the output ring gear 4. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=800 N m acts on the output ring gear 4.

37. A planetary gear train is shown in Fig. 9.37. The ring gear 1 has N_1 internal gear teeth, the planet gear 2 has N_2 external gear teeth, the planet gear $2'$ has $N_{2'}$-tooth external, the gear 3 has N_3 external gear teeth, the gear 4 has N_4 external gear teeth, and the ring gear 5 has N_5 internal gear teeth. The gears 2 and $2'$ are fixed on the same shaft. The gear 1 rotates with the input angular speed n_1 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.37. a) Find the absolute angular velocity of the output ring gear 5. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=500 N m acts on the output ring gear 5.

38. A planetary gear train is shown in Fig. 9.38. The gear 1 has N_1 external gear teeth, the planet gear 2 has N_2 external gear teeth, the gear 3 has N_3 external gear teeth, the gear 4 has N_4 external gear teeth, the gear $4'$ has $N_{4'}$ external gear teeth, and the ring gear 5 has N_5 internal gear teeth. The gears 4 and $4'$ are fixed on the same shaft. The gear 1

rotates with the input angular speed n_1. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.38. a) Find the absolute angular velocity of the output ring gear 5. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=500 N m acts on the output ring gear 5.

39. The planetary gear train considered is shown in Fig. 9.39. The sun gear 1 has N_1 external gear teeth, the planet gear 2 has N_2 external gear teeth, the planet gear $2'$ has $N_{2'}$ external gear teeth, the sun gear 4 has N_4 external gear teeth, the planet gear 5 has N_5 external gear teeth, and the ring gear 6 has N_6 internal gear teeth. The gears 2 and $2'$ are fixed on the same shaft.

    The gear 1 rotates with the input angular speed n_1 rpm and the ring gear 6 rotates at the input angular speed n_6 rpm. The module of the gears is m. The input data for ten cases are given in the table in Fig. 9.39. a) Find the absolute angular velocity of each gear. b) Find the equilibrium torque if an external torque of magnitude $|M_e|$=250 N m acts on the output ring gear 6.

9.4 Open kinematic chains

40. The dimensions of the links, for the planar manipulator in Fig. 9.40, are AB =3 cm, BC =2 cm, and CD =4 cm. The input data are:

    $q_1 = 45°$, $q_2 = 195°$, $q_3 = 165°$,

    $\dot{q}_1$ =1 rad/s, $\dot{q}_2$ =2 rad/s, $\dot{q}_3$ =-3 rad/s,

    $\ddot{q}_1$ =0.1 rad/s^2, $\ddot{q}_2$ =-0.2 rad/s^2, $\ddot{q}_3$ =0.4 rad/s^2.

    Find the velocity and acceleration of point D.

41. The link AB, for the planar manipulator shown in Fig. 9.41, has a length AB =1 m. The input data are:

    $q_1 = 35°$, $q_2 = 95°$, $q_3 = 1$ m,

    $\dot{q}_1$ =0.1 rad/s, $\dot{q}_2$ =-0.2 rad/s, $\dot{q}_3$ =0.5 m/s,

    $\ddot{q}_1$ =0.01 rad/s^2, $\ddot{q}_2$ =0.05 rad/s^2, $\ddot{q}_3$ =0.4 m/s^2.

    Find the velocity and acceleration of point C.

42. The link BC, for the planar manipulator shown in Fig. 9.42, has a length BC =0.5 m. The input data are:

    $q_1 = 45°$, $q_2 = 1$ m, $q_3 = 90°$,

    $\dot{q}_1$ =1 rad/s, $\dot{q}_2$ =-2 m/s, $\dot{q}_3$ =2 rad/s,

    $\ddot{q}_1$ =0 rad/s^2, $\ddot{q}_2$ =0.5 m/s^2, $\ddot{q}$ =0.4 rad/s^2.

Find the velocity and acceleration of point C.

43. The dimensions of the links, for the spatial manipulator shown in Fig. 9.43, are AB =3 m and BC =4 m. The links 1 and 2 are homogeneous bars made of steel with a constant cross section of 0.5 cm^2. Links 2 is perpendicular to link 1. Find and solve the equations of motion of the system for the following initial conditions $q_1(0) = 45°$, $q_2(0) = 30°$, $\dot{q}_1(0)$ =1 rad/s, and $\dot{q}_2(0)$ =2 rad/s.

44. The dimensions for the spatial manipulator shown in Fig. 9.44 are BC =0.5 m, CD =0.5 m, and the length of link 1 is 1.5 m. The links are homogeneous bars made of steel with a constant cross section of 1 cm^2. Links 2 is perpendicular to link 1, and links 3 is perpendicular to link 2. The initial conditions are

    $q_1(0) = 45°$, $q_2(0)$ =0.1 m, $q_3(0) = 30°$, $q_4(0) = 60°$,

    $\dot{q}_1(0)$ =0.1 rad/s, $\dot{q}_2(0)$ =0.01 m/s, $\dot{q}_3(0)$ =0.1 rad/s, $\dot{q}_4(0)$ =0.3 rad/s,

    Find and solve the equations of motion of the system.

45. Figure 9.45 is a schematic representation of a robot [Kane], with four links 1, 2, 3, and 4. Link 1 rotates in a "fixed" reference frame (0) about a vertical axis fixed in both (0) and 1. Link 1 supports link 2,

and link 2 rotates relative to 1 about a horizontal axis fixed in both 1 and 2. The link 2, supports link 3, and link 3 performs translational motions relative to 2 while carrying 4. The link 4 rotates relative to 3 about an axis fixed in both 3 and 4. To characterize the instantaneous configuration of the arm, the generalized coordinates q_1, q_2, q_3, q_4 are employed. The first three generalized coordinates are the radian measures of rotation angles, while q_4 is the distance between the mass centers, C_2 and C_3, of 2 and 3, respectively. The dimensions of interest are designated L_1, L_2 in Fig. 9.45 , where C_1, C_2, C_3, C_4, are the mass centers 1, 2, 3, and 4 respectively. Let $\mathbf{\imath}_0$, $\mathbf{\jmath}_0$, $\mathbf{k}_0$ form a set of cartesian perpendicular unit vectors fixed in the reference frame (0) as shown in Fig. 9.45. Let $\mathbf{\imath}_4$, $\mathbf{\jmath}_4$, $\mathbf{k}_4$ be a set of unit vectors fixed in 4 in such a way that $\mathbf{\imath}_0 = \mathbf{\imath}_4$, $\mathbf{\jmath}_0 = \mathbf{\jmath}_4$, $\mathbf{k}_4 = \mathbf{k}_0$ when $q_1 = q_2 = q_3 = 0$. One can introduce the generalized speeds u_1, u_2, u_3, u_4 as

$$u_1 = \boldsymbol{\omega}_{40} \cdot \mathbf{\imath}_4.$$

$$u_2 = \boldsymbol{\omega}_{40} \cdot \mathbf{\jmath}_4,$$

$$u_3 = \boldsymbol{\omega}_{40} \cdot \mathbf{k}_4,$$

$$u_4 = \dot{q}_4, \tag{9.1}$$

where $\boldsymbol{\omega}_{40}$ denotes the angular velocity of 4 in the reference frame (0). One may first verify that

$$\boldsymbol{\omega}_{40} = (\dot{q}_1 s_2 s_3 + \dot{q}_2 c_3)\mathbf{i}_4 + (\dot{q}_1 c_2 + \dot{q}_3)\mathbf{j}_4 + (-\dot{q}_1 s_2 c_3 + \dot{q}_2 s_3)\mathbf{k}_4, \quad (9.2)$$

where s_i and c_i denote $\sin q_i$ and $\cos q_i$, $i = 1, 2, 3$, respectively. Substitution into Eq. (9.1) then yields

$$\begin{aligned} u_1 &= \dot{q}_1 s_2 s_3 + \dot{q}_2 c_3, \\ u_2 &= \dot{q}_1 c_2 + \dot{q}_3, \\ u_3 &= -\dot{q}_1 s_2 c_3 + \dot{q}_2 s_3, \\ u_4 &= \dot{q}_4. \end{aligned} \quad (9.3)$$

Equation (9.3) can be solved uniquely for $\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4$. Specifically,

$$\begin{aligned} \dot{q}_1 &= (u_1 s_3 - u_3 c_3)/s_2, \\ \dot{q}_2 &= (u_1 c_3 + u_3 s_3), \\ \dot{q}_3 &= u_2 + (u_3 c_3 - u_1 s_3) c_2/s_2, \\ \dot{q}_4 &= u_4. \end{aligned} \quad (9.4)$$

with singularities at $q_2 = 0°$ and $q_2 = 180°$ degrees posing no problem. Thus, u_r as defined in Eq. (9.1) form a set of generalized speeds for the robot. The mass of link r is m_r, $r = 1, 2, 3, 4$. The central inertia dyadic of link r, can be expressed as

$$\bar{I}_r = I_{xr}\mathbf{i}_r\mathbf{i}_r + I_{yr}\mathbf{j}_r\mathbf{j}_r + I_{zr}\mathbf{k}_r\mathbf{k}_r, \tag{9.5}$$

where I_{xr}, I_{yr}, I_{zr} denote the central principal moments of inertia of $r = 1, 2, 3, 4$.

In the case of the robot arm, there are two kinds of forces that contribute to the generalized active forces F_r, $r = 1, 2, 3, 4$ namely, contact forces applied in order to drive links 1, 2, 3, 4, and gravitational forces exerted on the links by the Earth.

Considering, first, the contact forces, the set of such forces transmitted from fixed frame to the link 1 (through the bearings and by means of the motor) can be replaced with a couple of torque $\mathbf{T}_{01}$ together with a force $\mathbf{F}_{01}$ applied to the link 1 at C_1. Similarly, the set of contact forces transmitted from 2 to 1 can be replaced with a couple of torque $\mathbf{T}_{21}$ together with a force $\mathbf{F}_{21}$ applied to the link 1 at C_2 (which is a point fixed in 1). The law of action and reaction then guarantees that the set of contact forces transmitted from link 1 to link 2 is equivalent to a couple of torque -$\mathbf{T}_{21}$ together with the force - $\mathbf{F}_{21}$ applied to the link 2 at C_2. Next, the set of contact forces exerted on link 2 by link 3 can be replaced with a couple of torque $\mathbf{T}_{32}$ together with a force $\mathbf{F}_{32}$ applied to link 2 at $(C_3)_2$, the point of link 2 which instantaneously coincides with the point C_3. The set of forces exerted by link 2 on link 3 is, therefore equivalent to a couple torque -$\mathbf{T}_{32}$ together with the force -$\mathbf{F}_{32}$ applied to link 3 at C_3. Similarly, torque $\mathbf{T}_{43}$ and forces $\mathbf{F}_{43}$ come into play in connection with the interactions of link 3 and link 4 at G_4. The gravitational forces exerted on the links of the robot by the Earth, are denoted by $\mathbf{G}_r$, r = 1, 2, 3, 4 respectively.

The following notations are introduced

$$
\begin{aligned}
\tau_1 &= \mathbf{T}_{01} \cdot \mathbf{J}_1 = k_1(q_1^* - q_1) - k_2\dot{q}_1, \\
\tau_2 &= \mathbf{T}_{21} \cdot \mathbf{\imath}_2 = k_3(q_2 - q_2^*) + k_4\dot{q}_2 + g[(m_3 + m_4)q_4 + m_4L_3]s_2, \\
\tau_3 &= \mathbf{T}_{43} \cdot \mathbf{J}_3 = k_5(q_3 - q_3^*) + k_6\dot{q}_3, \\
\sigma &= \mathbf{F}_{32} \cdot \mathbf{J}_3 = k_7(q_4 - q_4^*) + k_8\dot{q}_4 - g(m_3 + m_4)c_2.
\end{aligned}
\tag{9.6}
$$

where $k_1, ..., k_8$ are constant "gains" k_1 = 3.0 N m, k_2 = 5.0 N m s, k_3 = 1.0 N m, k_4 = 3.0 N m s, k_5 = 0.3 N m, k_6 = 0.6 N m s, k_7 = 30 N m, k_8 = 41 N s, and $q_r^* = \pi/3$ rad $r = 1, 2, 3$ while q_4^* = 0.1 m.

The initial numerical data are L_1 = 0.1 m, L_2 = 0.1 m, L_3 = 0.7 m,

$m_1 = 9$ kg, $m_2 = 6$ kg, $m_3 = 4$ kg, $m_4 = 1$ kg, $I_{x1} = 0.01$ kg m^2, $I_{y1} = 0.02$ kg m^2, $I_{z1} = 0.01$ kg m^2, $I_{x2} = 0.06$ kg m^2, $I_{y2} = 0.01$ kg m^2, $I_{z2} = 0.05$ kg m^2, $I_{x3} = 0.4$ kg m^2, $I_{y3} = 0.01$ kg m^2, $I_{z3} = 0.4$ kg m^2, $I_{x4} = 0.0005$ kg m^2, $I_{y4} = 0.001$ kg m^2, and $I_{z4} = 0.001$ kg m^2.

Find and solve the equations of motion of the system.