

1 INTRODUCTION

1.1 Degrees of freedom

The *number of degrees of freedom* (DOF) of a system is equal to the number of independent parameters (measurements) that are needed to uniquely define its position in space at any instant of time. The number of DOF is defined with respect to a reference frame.

Figure 1.1 shows a rigid body (RB) lying in a plane. The rigid body is assumed to be incapable of deformation and the distance between two particles on the RB is constant at any time. If this rigid body will always remain in the plane, three parameters (three DOF) are required to completely define its position: two linear coordinates (x, y) to define the position of any one point on the RB, and one angular coordinate θ to define the angle of the body with respect to the axes. The minimum number of measurements needed to define its position are shown in the figure as x, y , and θ . A rigid body in a plane then has three DOF. Note that the particular parameters chosen to define its position are not unique. Any alternate set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must be three parameters per set, such as two lengths

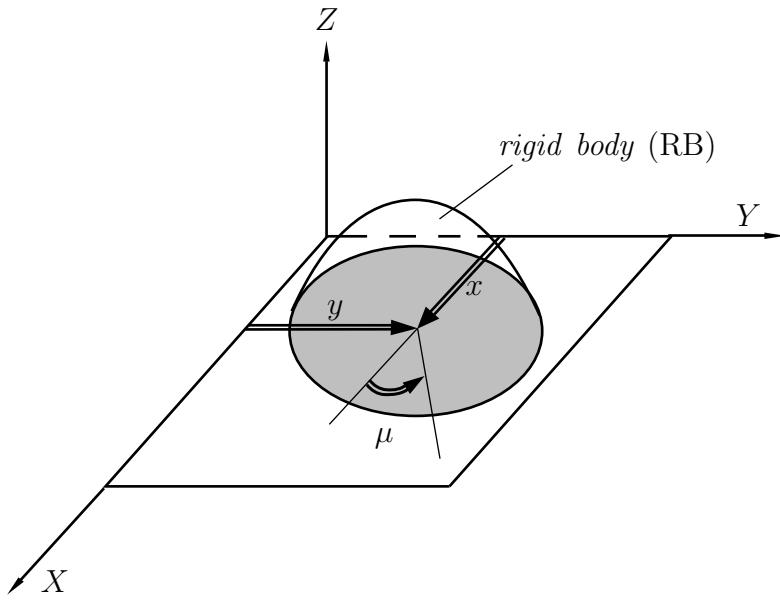


Figure 1.1

and an angle, to define the position because a rigid body in plane motion has three DOF.

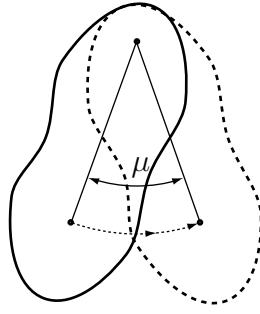
Six parameters are needed to define the position of a free rigid body in a three-dimensional (3-D) space. One possible set of parameters which could be used are three lengths, (x, y, z) , plus three angles $(\theta_x, \theta_y, \theta_z)$. Any free rigid body in three-dimensional space has six degrees of freedom.

1.2 Motion

A rigid body free to move in a reference frame will, in the general case, have complex motion, which is simultaneously a combination of rotation and translation. For simplicity, only the two-dimensional (2-D) or planar case will be presented. For the planar motion the following terms will be defined, Fig. 1.2:

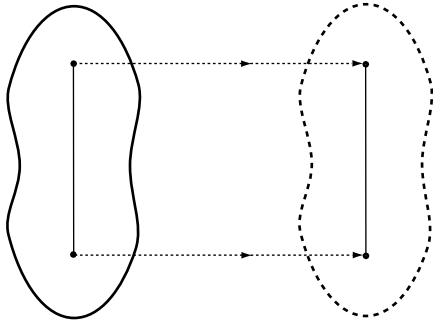
- pure rotation in which the body possesses one point (center of rotation) which has no motion with respect to a “fixed” reference frame, Fig. 1.2(a). All other points on the body describe arcs about that center.
- pure translation in which all points on the body describe parallel paths, Fig. 1.2(b).
- complex motion which exhibits a simultaneous combination of rotation and

rotation

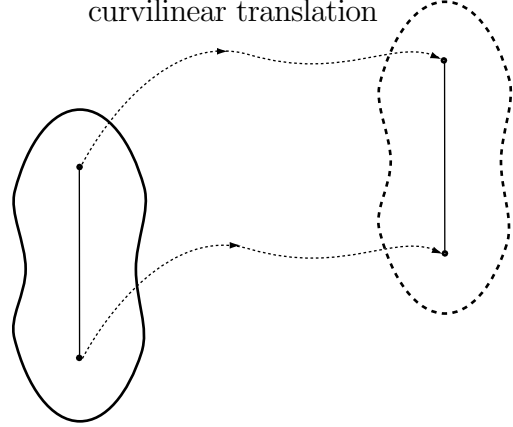


(a)

rectilinear translation

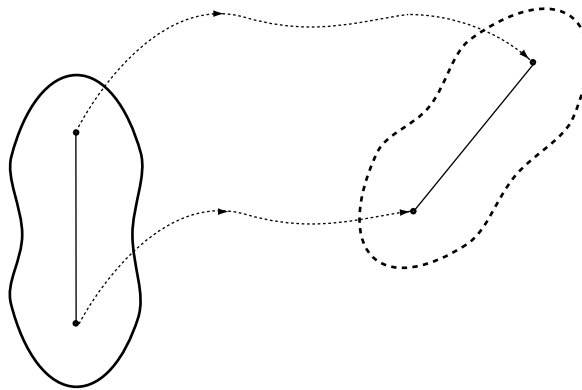


curvilinear translation



(b)

general plane motion



(c)

Figure 1.2

translation, Fig. 1.2(c). Points on the body will travel non-parallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Translation and rotation represent independent motions of the body. Each can exist without the other. For a 2-D coordinate system, as shown in Fig. 1.1, the x and y terms represent the translation components of motion, and the θ term represents the rotation component.

1.3 Links and joints

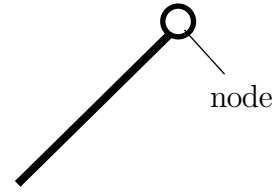
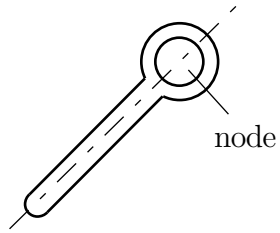
Linkages are the basic elements of all mechanisms. Linkages are made up of links and joints. A *link* or an element or a member is an (assumed) rigid body which possesses nodes. *Nodes* are defined as points at which links can be attached. A link connected to its neighbouring elements by s nodes is an element of *degree* s . An link of degree 1 is also called unary, Fig. 1.3(a), of degree 2, binary, Fig. 1.3(b), and of degree 3, ternary Fig. 1.3(c), etc.

A *joint* is a connection between two or more links (at their nodes), which connection allows some motion between the connected links. Joints are also called *kinematic pairs*.

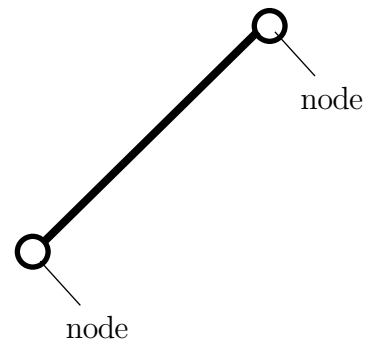
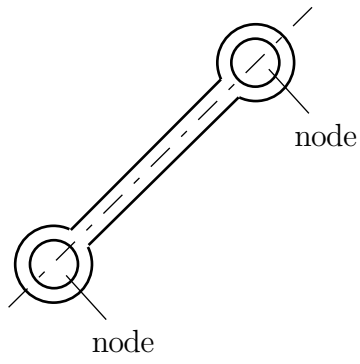
For the number of independent coordinates uniquely determining the rel-

Link

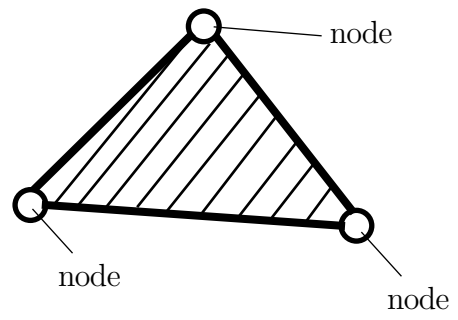
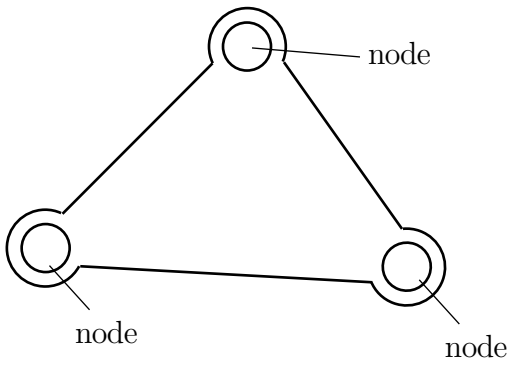
Schematic representation



(a)



(b)



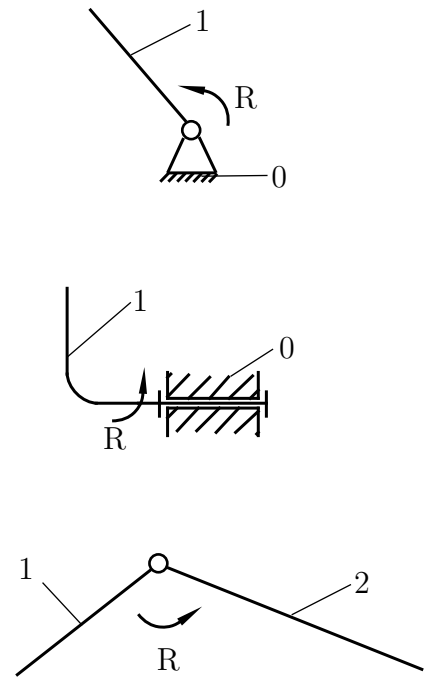
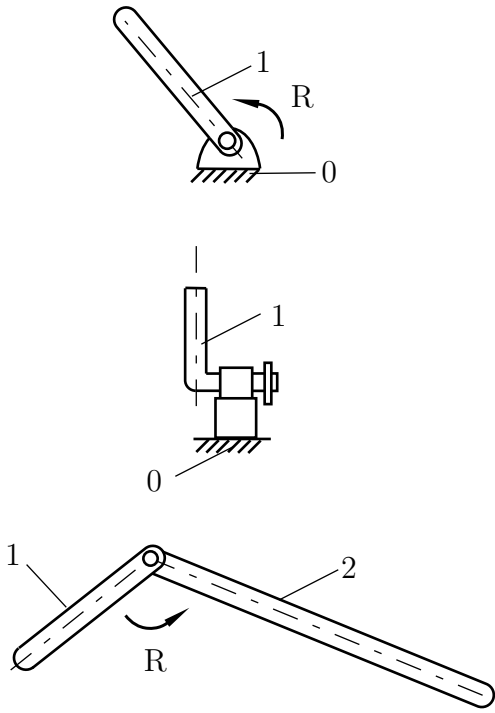
(c)

Figure 1.3

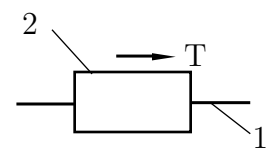
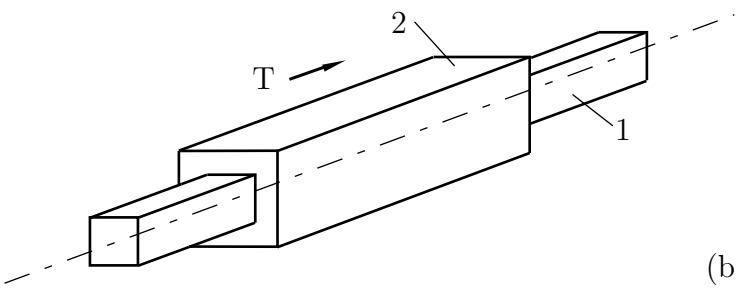
ative position of two constrained links, the term *degree of freedom* of a given joint is used. Alternatively the term *joint class* is introduced. A kinematic pair is of the j -th class if it diminishes the relative motion of linked bodies by j degrees of freedom, i.e. j scalar constraint conditions correspond to the given kinematic pair. It follows that such a joint has $(6j)$ independent coordinates. The degree of freedom is the fundamental characteristic quantity of joints. One of the links of a system is usually considered to be the reference link, and the position of other RB is determined in relation to this reference body. If the reference link is stationary, the term *frame* or *ground* is used. The coordinates in the definition of degree of freedom can be linear or angular, absolute (measured with regard to the frame) or relative. Figures 1.4-1.9 show examples of joints commonly found in mechanisms. Figures 1.4(a) and 1.4(b) show two forms of a planar, one degree of freedom joint, namely a rotating pin joint and a translating slider joint. These are both typically referred to as *full joints* and are of the 5-th class. The pin joint allows one rotational (R) DOF, and the slider joint allows one translational (T) DOF between the joined links. These are both special cases of another common, one degree of freedom joint, the screw and nut, Fig. 1.5(a). Motion of either the nut or the screw with respect to the other results in helical motion. If

Type of full joint

Schematic representation



(a)

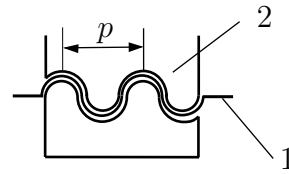
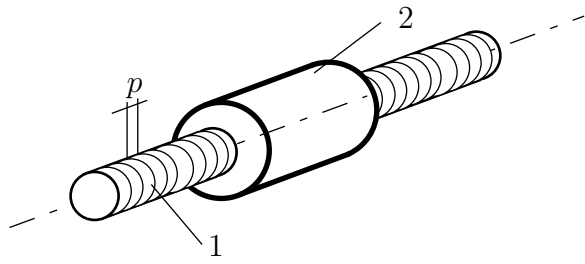


(b)

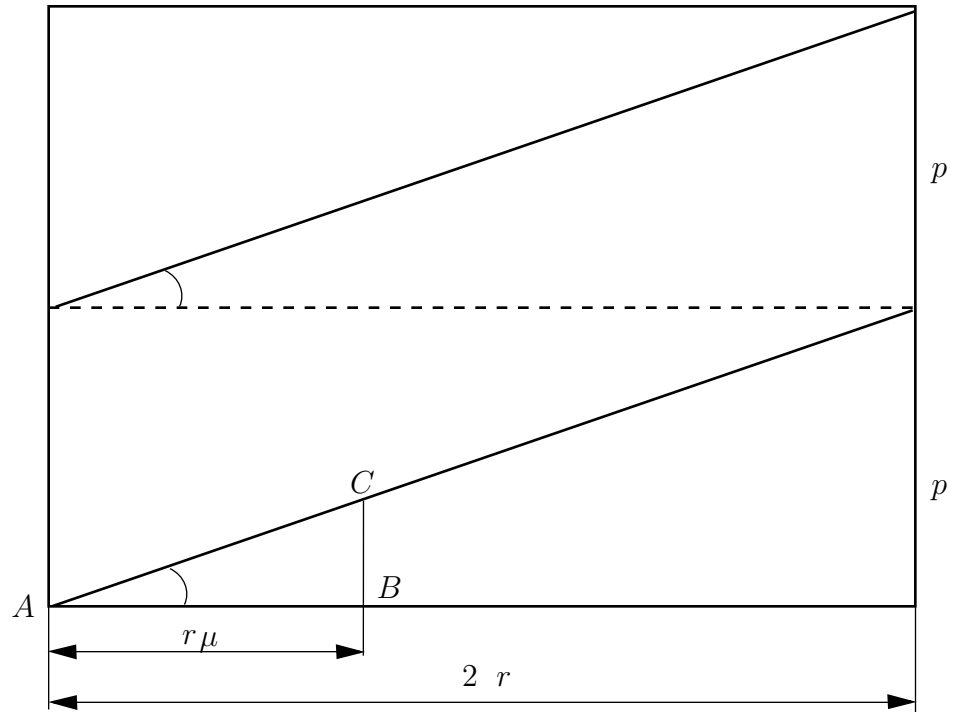
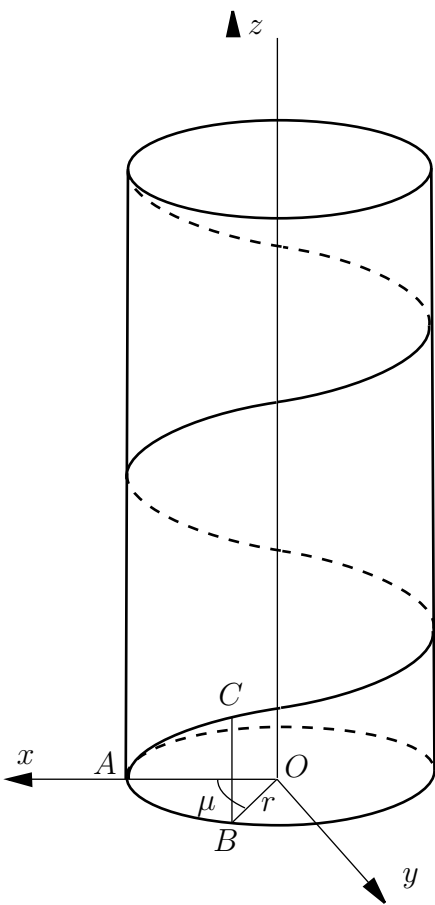
Figure 1.4

Type of full joint

Schematic representation



(a)



μ = helix angle
 p = pitch

(b)

Figure 1.5

the helix angle is made zero, Fig. 1.5(b), the nut rotates without advancing and it becomes the pin joint. If the helix angle is made 90° , the nut will translate along the axis of the screw, and it becomes the slider joint.

Figure 1.6 shows examples of two degrees of freedom joints, which simultaneously allow two independent, relative motions, namely translation (T) and rotation (R), between the joined links. The two degrees of freedom joint is usually referred to as a *half joint* and is of the 4-th class. The half joint is sometimes also called a roll-slide joint because it allows both rotation (rolling) and translation (sliding).

A joystick, ball-and-socket joint, or sphere joint, Fig. 1.7(a), is an example of a three degrees of freedom joint (3-rd class) which allows three independent angular motions between the two links joined. This ball joint would typically be used in a three-dimensional mechanism, one example being the ball joints in an automotive suspension system. The plane joint, Fig. 1.7(b), is also an example of a three degrees of freedom joint with two translations and one rotation.

Note that to visualize the degree of freedom of a joint in a mechanism, it is helpful to “mentally disconnect” the two links which create the joint from the rest of the mechanism. It is easier to see how many degrees of freedoms

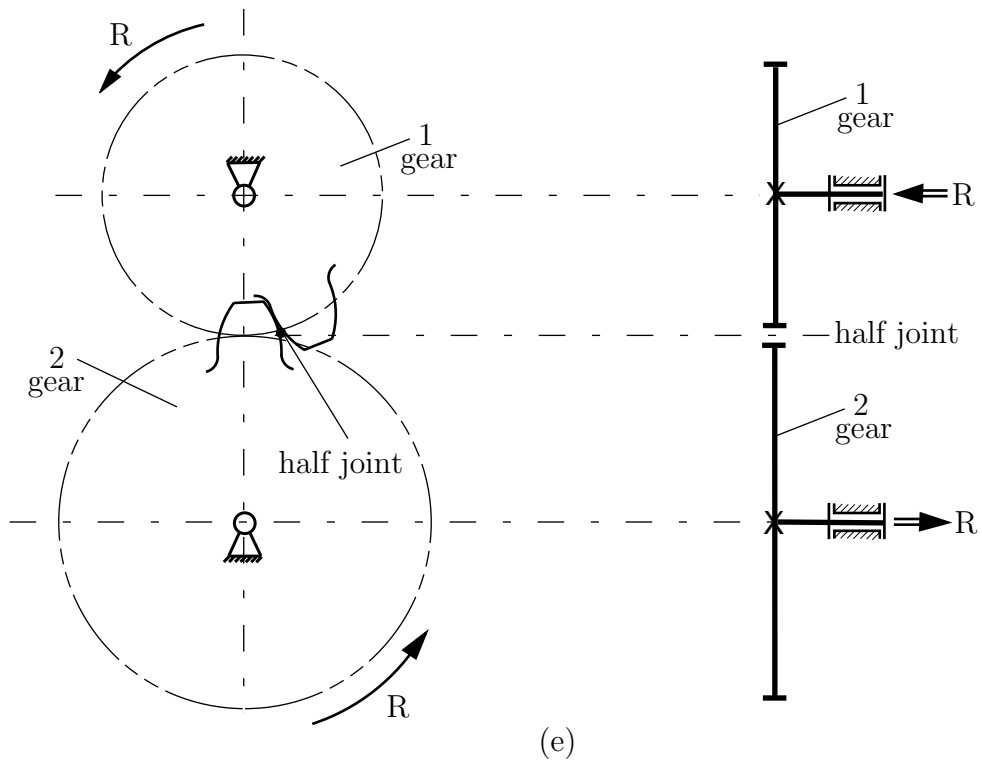
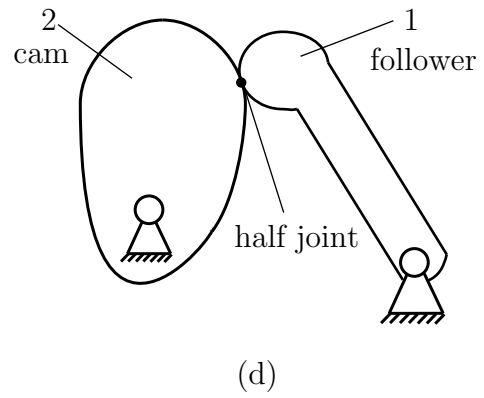
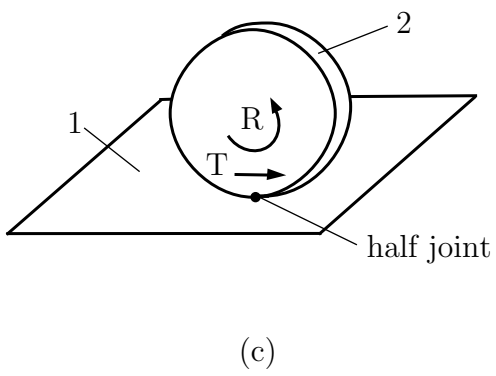
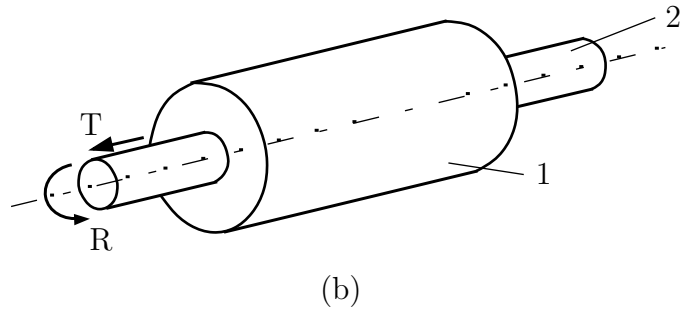
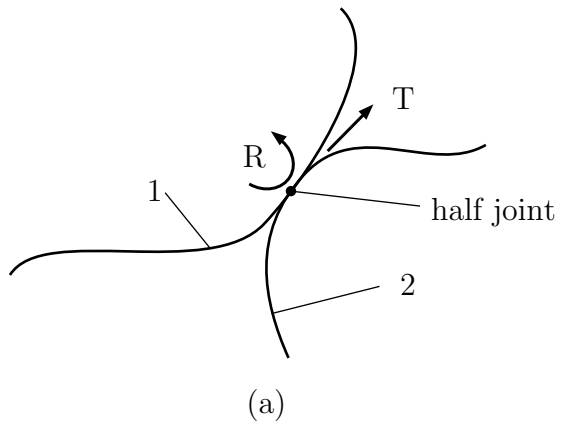
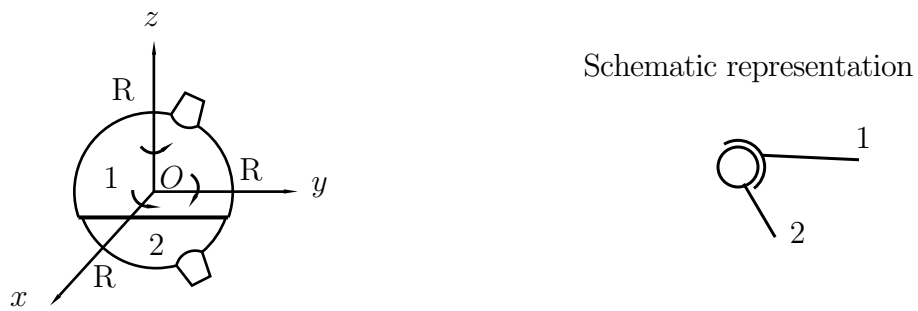
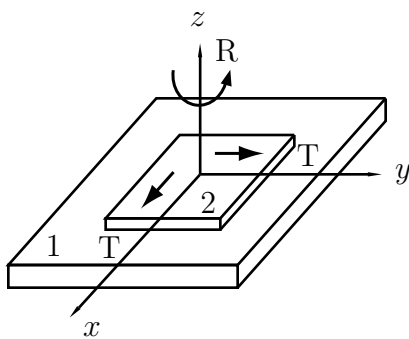


Figure 1.6



(a)



(b)

Figure 1.7

the two joined links have with respect to one another. Figure 1.8 shows an example of a 2-nd class joint (cylinder on plane) and Figure 1.9 represents a 1-st class joint (sphere on plane).

The type of contact between the elements can be point (P), curve (C), or surface (S). The term *lower joint* was coined by Reuleaux to describe joints with surface contact. He used the term *higher joint* to describe joints with point or curve contact. The main practical advantage of lower joint over higher joints is their better ability to trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint. The lubricant is more easily squeezed out of a higher joint, non-enveloping joint.

A *closed joint* is a joint that is kept together or closed by its geometry. A pin in a hole or a slider in a two-sided slot are form closed joints. A *force closed joint*, such as a pin in a half-bearing or a slider on a surface, requires some external force to keep it together or closed. This force could be supplied by gravity, by a spring, or by some external means. In linkages, closed joint is usually preferred, and it is easy to accomplish. For cam-follower systems force closure is often preferred.

The *order of a joint* is defined as the number of links joined minus one. The simplest joint combination of two links has order one, Fig. 1.10(a), and

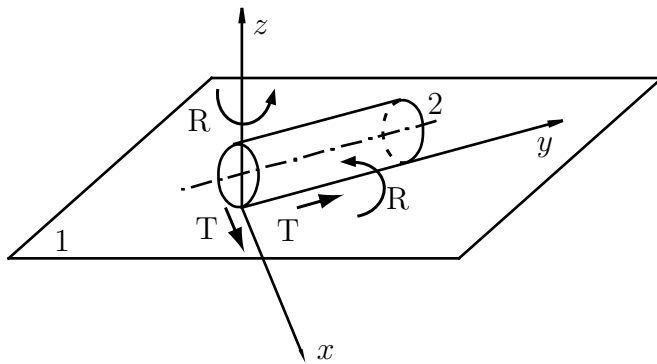


Figure 1.8

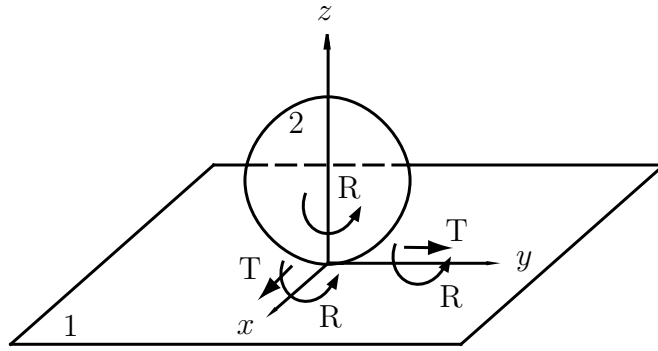
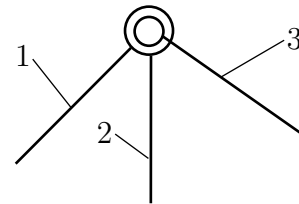
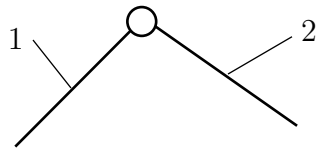
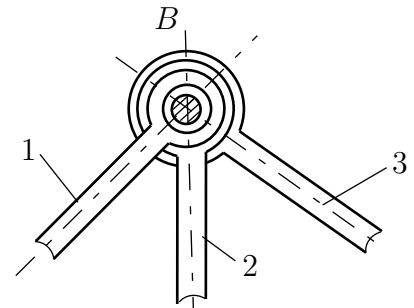
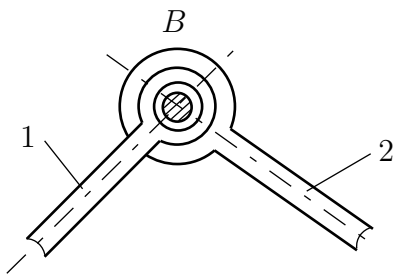
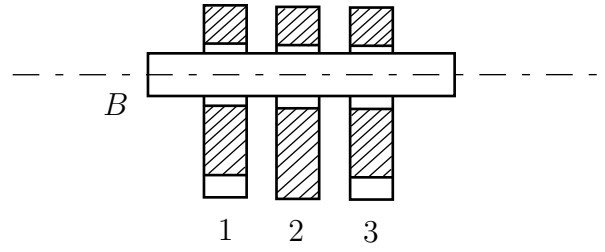
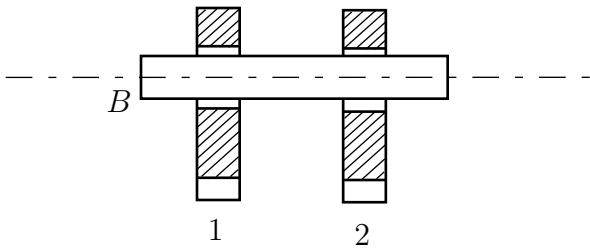


Figure 1.9

joint of order one
(one pin joint)

joint of order two
(two pin joints)



(a)

(b)

Figure 1.10

it is a single joint. As additional links are placed on the same joint, the order is increased on a one for one basis, Fig. 1.10(b). Joint order has significance in the proper determination of overall degrees of freedom for the assembly.

A number of bodies linked by joints form a *kinematic chain*. The simplest description of its structure is by a chart where an abscissa represents a link, Fig. 1.11.

A *contour* or *loop* is a configuration described by a polygon, Fig. 1.11(a). On the basis of the presence of loops in a mechanical structure it can be distinguished:

- *closed kinematic chains*, if there are one or more loops so that each link and each joint is contained in at least one of them, Fig. 1.11(a). A closed kinematic chain have no open attachment point.
- *open kinematic chains*, if they contain no loop, Fig. 1.11(b). A common example of an open mechanism is an industrial robot.
- *mixed kinematic chains*, if they are a combination of the above chains.

Another classification may distinguish:

- *simple chains* containing only binary elements,
- *complex chains* containing at least one element of degree 3 or higher.

A *mechanism* is defined as a kinematic chain in which at least one link

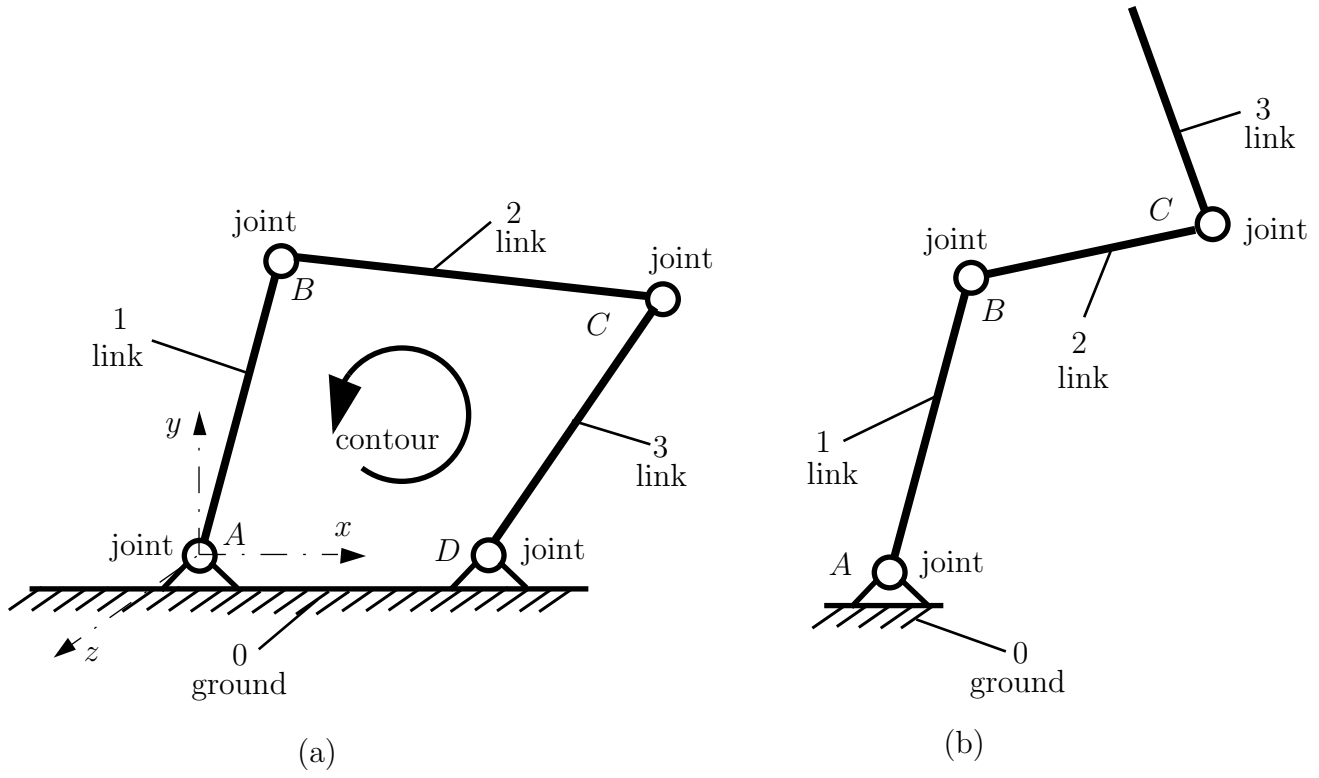


Figure 1.11

has been “grounded” or attached to the frame, Figs. 1.11(a) and 1.12. By Reuleaux’s definition a *machine* is a collection of mechanisms arranged to transmit forces and do work. He viewed all energy or force transmitting devices as machines which utilize mechanisms as their building blocks to provide the necessary motion constraints.

The following terms are defined, Fig. 1.12:

- a crank is a link which makes a complete revolution about a fixed grounded pivot,
- a rocker is a link which has oscillatory (back and forth) rotation and is fixed to a grounded pivot,
- a coupler or connecting rod is a link which has complex motion and is not fixed to ground.

Ground is defined as any link or links that are fixed (non-moving) with respect to the reference frame. Note that the reference frame may in fact itself be in motion.

1.4 Number of degrees of freedom

The concept of *number of degrees of freedom* is fundamental to analysis of mechanisms. It is usually necessary to be able to determine quickly the num-

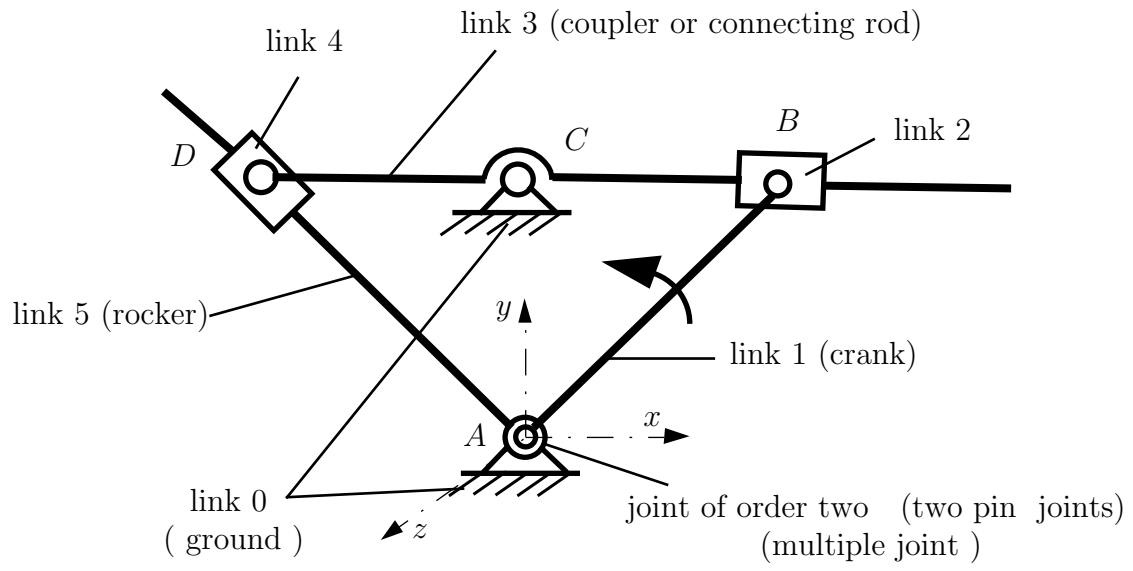


Figure 1.12

ber of DOF of any collection of links and joints which may be suggested as a solution to a problem. The number of degrees of freedom or the *mobility* of a system can be defined as:

- the number of inputs which need to be provided in order to create a predictable system output, or
- the number of independent coordinates required to define the position of the system.

The *family* f of a mechanism is the number of DOF eliminated from all the links of the system.

Every free body in space has six degrees of freedom. A system of family f consisting of n movable links has $(6 - f)n$ degrees of freedom. Each joint of class j diminishes the freedom of motion of the system by $j - f$ degrees of freedom. Designating the number of joints of class k as c_k , it follows that the number of degrees of freedom of the particular system is

$$M = (6 - f)n - \sum_{j=f+1}^5 (j - f)c_j. \quad (1.1)$$

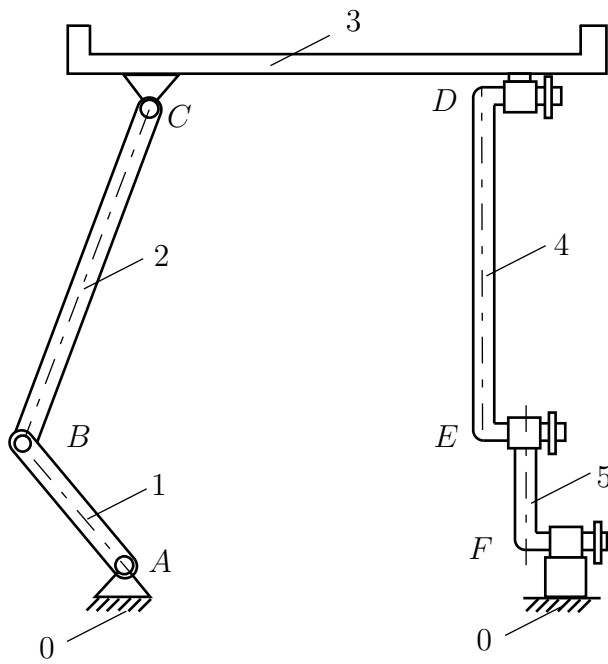
This is referred to in literature as the Dobrovolski formula.

A *driver* link is that part of a mechanism which causes motion, such as the crank. The number of driver links is equal with the number of DOF of the mechanism. A *driven* link or *follower* is that part of a mechanism whose

motion is affected by the motion of the driver.

Mechanisms of family $f=1$

The family of a mechanism can be computed with the help of a mobility table. Assume the mechanism in Fig. 1.13 which is used to measure the weight of postal envelopes. The translation along the i axis is denoted with T_i , and the rotation about the i axis is denoted by R_i , where $i = x, y, z$. Every link in the mechanism is analysed in terms of translation and rotation about reference frame xyz . For example the link 0 (ground) has no translations, $T_i=\text{No}$, and no rotations, $R_i=\text{No}$. The link 1 has a rotation motion about z axis, $R_z=\text{Yes}$. The link 2 has a planar motion (xy the plane of motion) with a translation along x , $T_x=\text{Yes}$, a translation along y , $T_y=\text{Yes}$, and a rotation about z , $R_z=\text{Yes}$. The link 3 has a translation along y , $T_y=\text{Yes}$. The link 4 has a planar motion (yz the plane of motion) with a translation along y , $T_y=\text{Yes}$, a translation along z , $T_z=\text{Yes}$, and a rotation about x , $R_x=\text{Yes}$. The link 5 has a rotation about x axis, $R_x=\text{Yes}$. The results of this analysis are presented with the help of the mobility table



Schematic representation

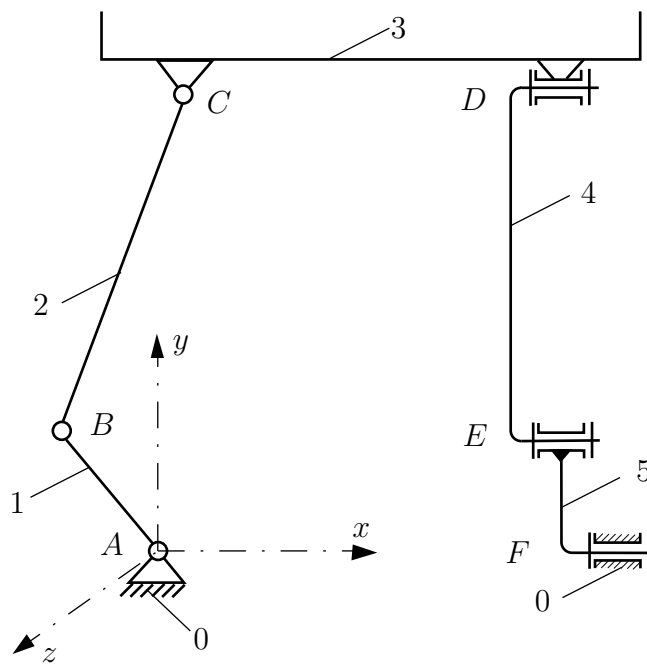


Figure 1.13

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	Yes	No	No	No	No
4	No	Yes	Yes	Yes	No	No
5	No	No	No	Yes	No	No
					No	

for all links $R_y = \text{No} \implies f = 1$

Table 1.1 Mobility table for the mechanism in Fig. 1.13

From the mobility table it can be noticed that link i , $i = 0, 1, 2, 3, 4, 5$ has no rotation about y axis, i.e. there is no rotation about y axis for all the links of the mechanism ($R_y = \text{No}$). The family of the mechanism is $f = 1$ because there is one DOF, rotation about y , eliminated from all the links.

There are six joints of class 5 (rotational joints) in the system at A, B, C, D, E , and F .

The number of DOF for the mechanism in Fig. 1.11 of $f = 1$ family is

$$M = 5n - \sum_{j=2}^5 (j-1)c_j = 5n - 4c_5 - 3c_4 - 2c_3 - c_2 = 5(5) - 4(6) = 1.$$

The mechanism has one DOF and one driver link.

Mechanisms of family $f=2$

A mechanism of family $f=2$ with the mobility table given below is depicted

in Fig. 1.14

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	Yes	No	Yes
3	Yes	Yes	No	No	No	Yes
4	No	No	No	No	No	Yes
			No		No	

for all links $T_z=\text{No} \ \& \ R_y=\text{No} \implies f=2$

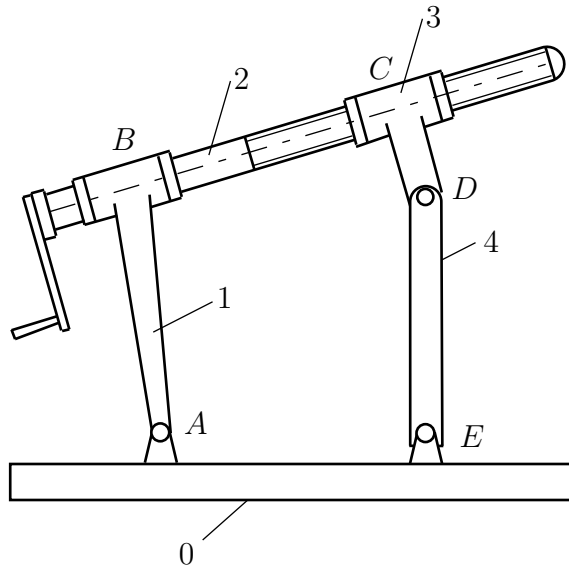
Table 1.2 Mobility table for the mechanism in Fig. 1.14

The number of DOF for the mechanisms of $f=2$ family is

$$M = 4n - \sum_{j=3}^5 (j-2)c_j = 4n - 3c_5 - 2c_4 - c_3.$$

The mechanism in Fig. 1.14 has four moving links ($n = 4$), four rotational joints (A, B, D, E) and one screw and nut joint (C) i.e. there are five joints of class 5 ($c_5 = 5$). The number of DOF for this mechanism is

$$M = 4n - 3c_5 - 2c_4 - c_3 = 4(4) - 3(5) = 1.$$



Schematic representation

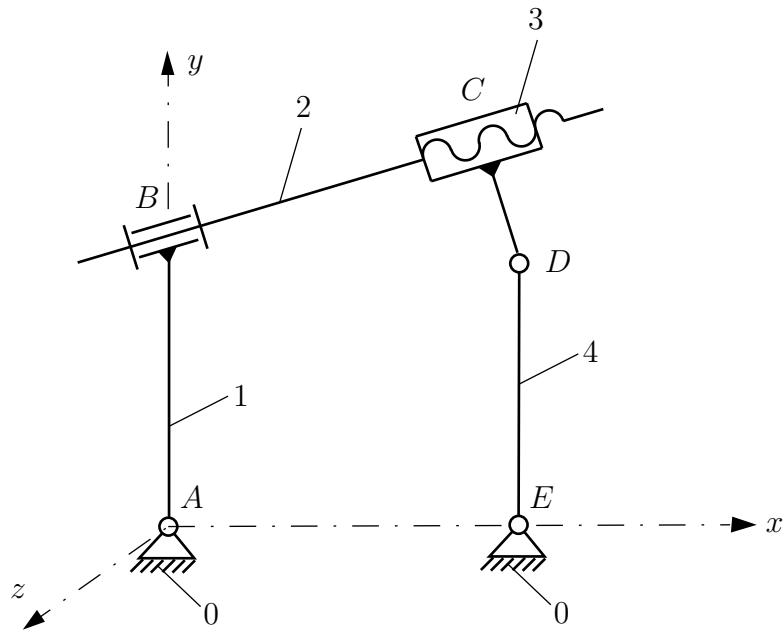


Figure 1.14

Mechanisms of family $f=3$

The number of DOF for the mechanisms of $f=3$ family is

$$M = 3n - \sum_{j=4}^5 (j-3) c_j = 3n - 2c_5 - c_4.$$

For the mechanism in Fig. 1.11(a) the mobility table is

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	No	No	No	No	Yes
			No	No	No	

for all links $T_z=\text{No}$ & $R_x=\text{No}$ & $R_y=\text{No} \implies f=3$

Table 1.3 Mobility table for the mechanism in Fig. 1.11(a)

The mechanism in Fig. 1.11(a) has three moving links ($n = 3$) and four rotational joints at A , B , C , and D , ($c_5 = 4$). The number of DOF for this mechanism is

$$M = 3n - 2c_5 - c_4 = 3(3) - 2(4) = 1.$$

For the mechanism in Fig. 1.12 the mobility table is

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	No	No	No	No	Yes
4	Yes	Yes	No	No	No	Yes
5	No	No	No	No	No	Yes
			No	No	No	

for all links $T_z=\text{No} \ \& \ R_x=\text{No} \ \& \ R_y=\text{No} \ \implies \ f=3$

Table 1.4 Mobility table for the mechanism in Fig. 1.12

There are seven joints of class 5 ($c_5 = 7$) in the system:

- at A one rotational joint between link 0 and link 1;
- at B one rotational joint between link 1 and link 2;
- at B one translational joint between link 2 and link 3;
- at C one rotational joint between link 0 and link 3;
- at D one rotational joint between link 3 and link 4;
- at D one translational joint between link 4 and link 5;
- at A one rotational joint between link 5 and link 0.

The number of moving links is five ($n = 5$). The number of DOF for this

mechanism is

$$M = 3n - 2c_5 - c_4 = 3(5) - 2(7) = 1,$$

and this mechanism has one driver link.

Mechanisms of family $f=4$

The number of DOF for the mechanisms of $f=4$ family is

$$M = 2n - \sum_{j=5}^5 (j - 4) c_j = 2n - c_5.$$

For the mechanism in Fig. 1.15 the mobility table is

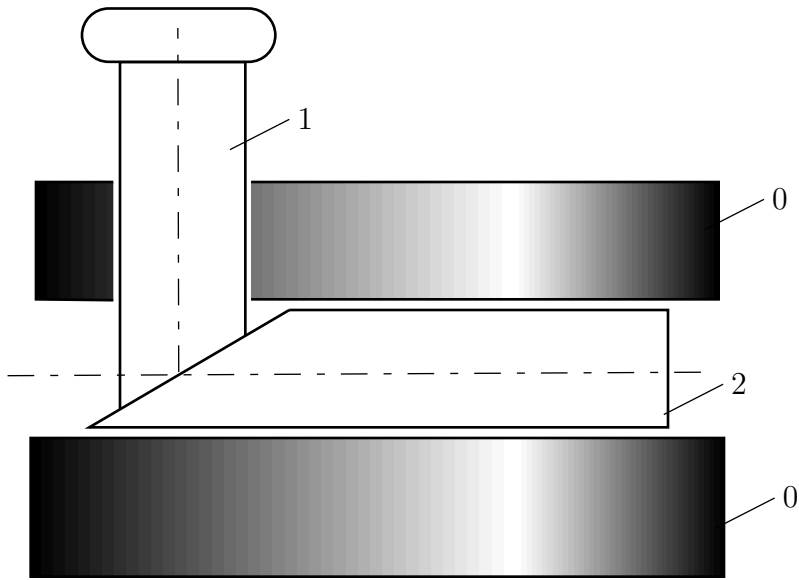
Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	Yes	No	No	No	No
2	Yes	No	No	No	No	No
		No	No	No	No	No

for all links $T_z=\text{No} \ \& \ R_x=\text{No} \ \& \ R_y=\text{No} \ \& \ R_z=\text{No} \implies f=4$

Table 1.5 Mobility table for the mechanism in Fig. 1.15

There are three translational joints of class 5 ($c_5 = 3$) in the system:

- at B one translational joint between link 0 and link 1;
- at C one translational joint between link 1 and link 2;



Schematic representation

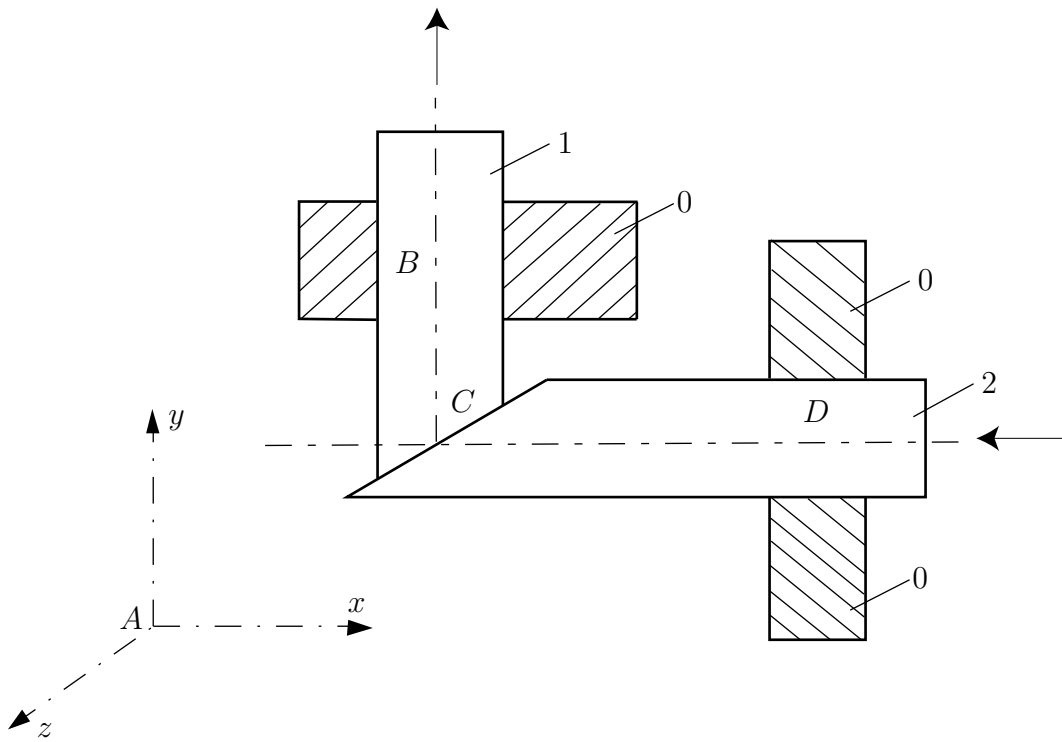


Figure 1.15

- at D one translational joint between link 2 and link 0.

The number of DOF for this mechanism with two moving links ($n = 2$) is

$$M = 2n - c_5 = 2(2) - (3) = 1.$$

Mechanisms of family $f=5$

The number of DOF for the mechanisms of $f=5$ family is equal with the number of moving links

$$M = n.$$

In this category there are the driver link with rotational motion, Fig. 1.16(a), and the driver link with translational motion, Fig. 1.16(b).

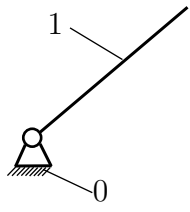
1.5 Planar mechanisms

For the special case of planar mechanisms ($f=3$) the formula (1.1) has the form

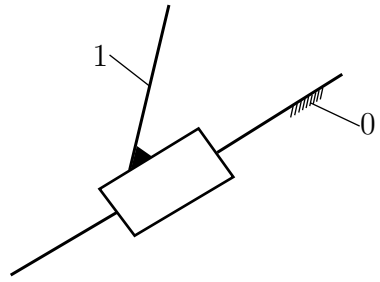
$$M = 3n - 2c_5 - c_4, \quad (1.2)$$

where c_5 is the number of full joints and c_4 is the number of half joints.

There is a special significance to kinematic chains which do not change their mobility after being connected to an arbitrary system. Kinematic chains defined in this way are called system groups. Connecting them to or discon-



(a)



(b)

Figure 1.16

necting them from a given system enables given systems to be modified or structurally new systems to be created while maintaining the original freedom of motion. The term “system group” has been introduced for the classification of planar mechanisms used by Assur and further investigated by Artobolevskij. Limiting to planar systems containing only kinematic pairs of class 2, from Eq.(1.2) it can be obtained

$$3n - 2c_5 = 0, \quad (1.3)$$

according to which the number of system group links n is always even. The simplest system is the binary group with two links ($n=2$) and three full joints ($c_5 = 3$). The binary group is called also *dyad*. The sets of links shown in Fig. 1.17 are dyads and one can distinguished the following types:

- rotation rotation rotation (dyad RRR), Fig. 1.17(a);
- rotation rotation translation (dyad RRT), Fig. 1.17(b);
- rotation translation rotation (dyad RTR), Fig. 1.17(c);
- translation rotation translation (dyad TRT), Fig. 1.17(d);
- translation translation rotation (dyad TTR), Fig. 1.17(e).

The advantage of the group classification of a system lies in its simplicity. The solution of the whole system can then be obtained by composing partial solutions.

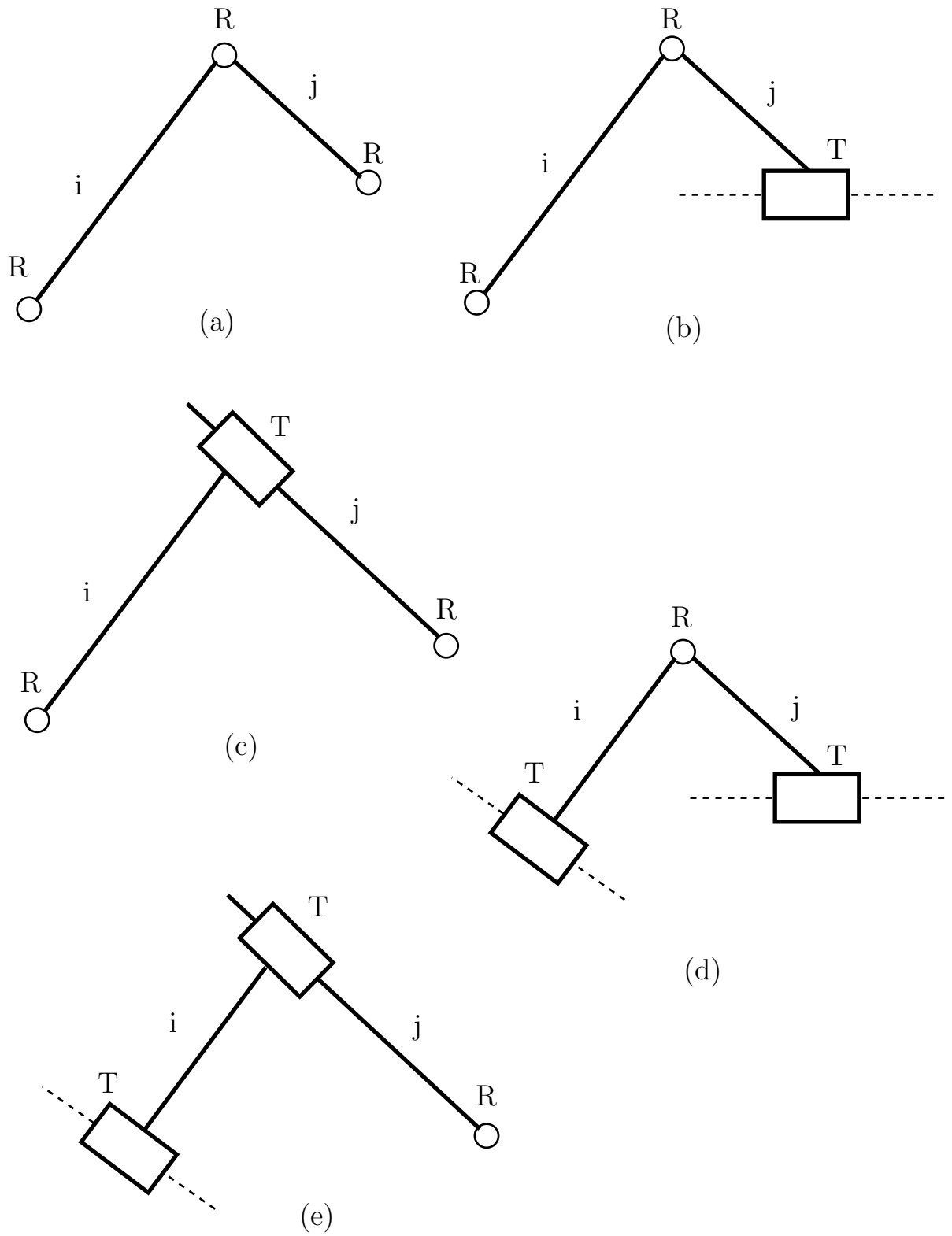


Figure 1.17

The mechanism in Fig. 1.11(a) has one driver, link 1, with rotational motion and one dyad RRR, link 2 and link 3.

The mechanism in Fig. 1.12 is obtained by composing:

- the driver link 1 with rotational motion;
- the dyad RTR: links 2 and 3, and the joints B rotation (B_R), B translation (B_T), C rotation (C_R);
- the dyad RTR: links 4 and 5, and the joints D rotation (D_R), D translation (D_T), A rotation (A_R).

Figure captions:

Figure 1.1. Rigid body in planar motion with three DOF: translation along the x axis, translation along the y axis, and rotation, θ , about the z .

Figure 1.2. Rigid body in motion: (a) pure rotation, (b) pure translation, and (c) general motion

Figure 1.3. Types of links: (a) unary, (b) binary, and (c) ternary elements

Figure 1.4. One degree of freedom joint, full joint (5-th class): (a) pin joint, and (b) slider joint

Figure 1.5. (a) Screw and nut joint; (b) helical motion

Figure 1.6. Two degrees of freedom joint, half joint (4-th class): (a) general joint, (b) cylinder joint, (c) roll and slide disk, and (d) cam-follower joint

Figure 1.7. Three degrees of freedom joint (3-rd class): (a) ball and socket joint, and (b) plane joint

Figure 1.8. Four degrees of freedom joint (2-nd class) cylinder on a plane

Figure 1.9. Five degrees of freedom joint (1-st class) sphere on a plane

Figure 1.10. Order of a joint: (a) joint of order one, and (b) joint of order two (multiple joints)

Figure 1.11. Kinematic chains: (a) closed kinematic chain, and (b) open

kinematic chain

Figure 1.12. Complex mechanism with five moving links

Figure 1.13. Spatial mechanism of family $f=1$

Figure 1.14. Spatial mechanism of family $f=2$

Figure 1.15. Spatial mechanism of family $f=4$

Figure 1.16. Spatial mechanism of family $f=5$: (a) driver link with rotational motion, and (b) driver link with translational motion

Figure 1.17. Types of dyades: (a) RRR, (b) RRT, (c) RTR, (d) TRT, and (e) TTR

1. Determine the mobility (number of DOF) of the planar elipsograph mechanism in Fig. P.1. Find the analytical expression of any point P on the link 2.
2. Find the mobility of the planar mechanism represented in Fig. P.2.
3. For the mechanism depicted in Fig. P.3 determine the family and the number of degrees of freedom.
4. The roll 2 of the mechanism in Fig. P.4 has an independent rotation about its axis and does not influence the motion of the link 3. The purpose of element 2 is to substitute the sliding friction with a rolling friction. From a kinematical point of view the roll 2 is a passive element. Find the mobility of the mechanism.
5. Find the family and the mobility of the mechanism in Fig. P.5.

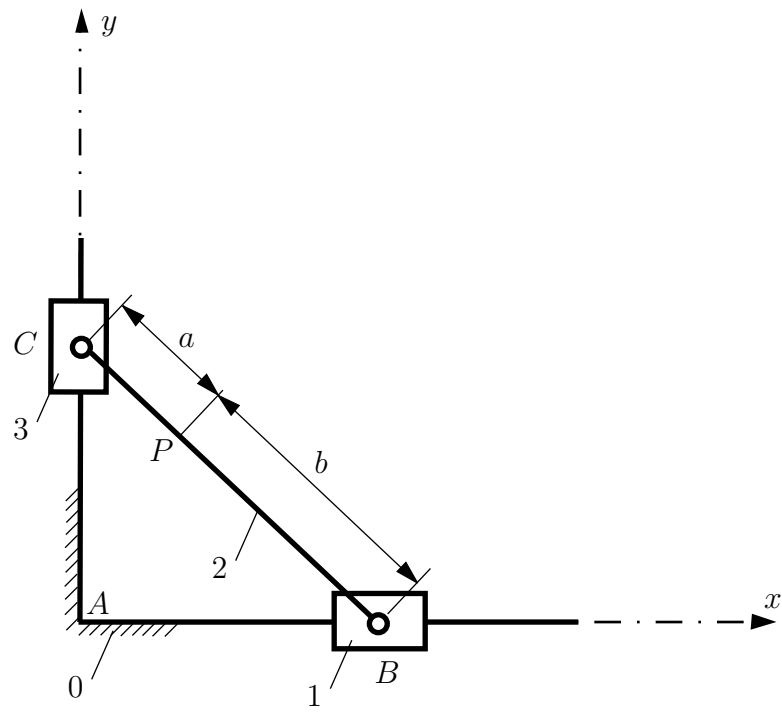


Figure P.1

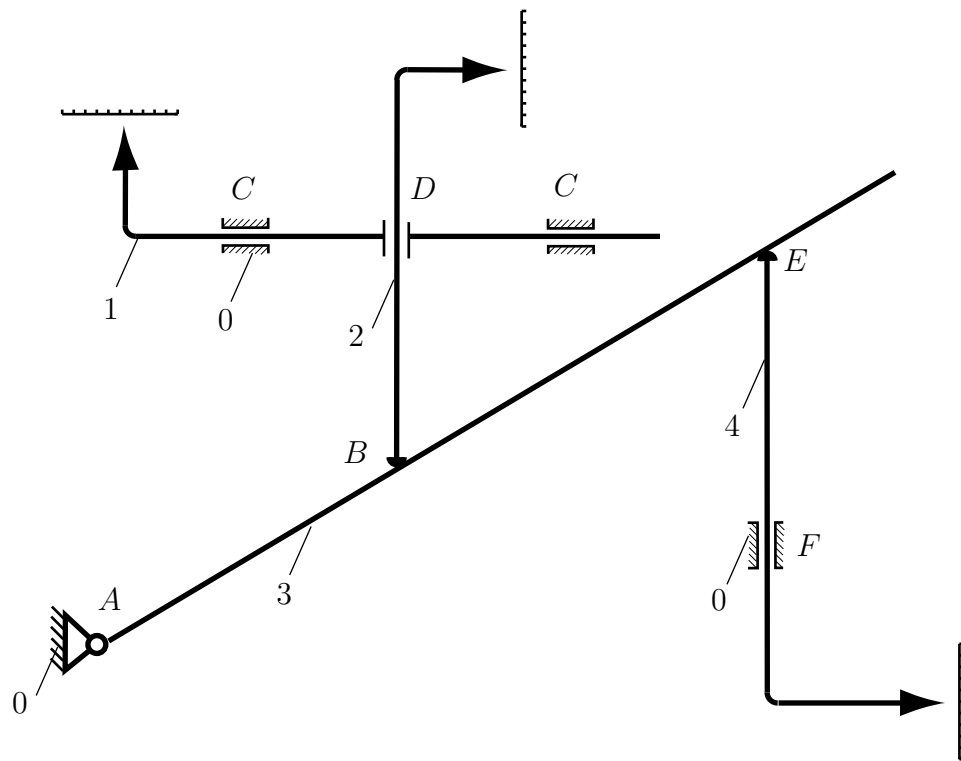


Figure P.2

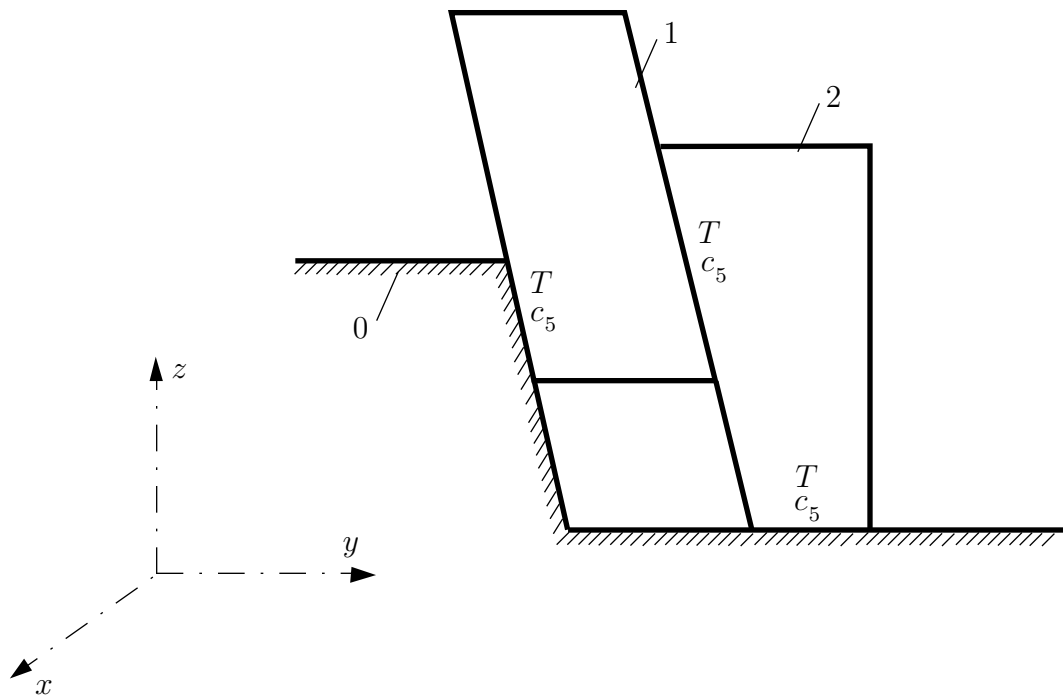


Figure P.3

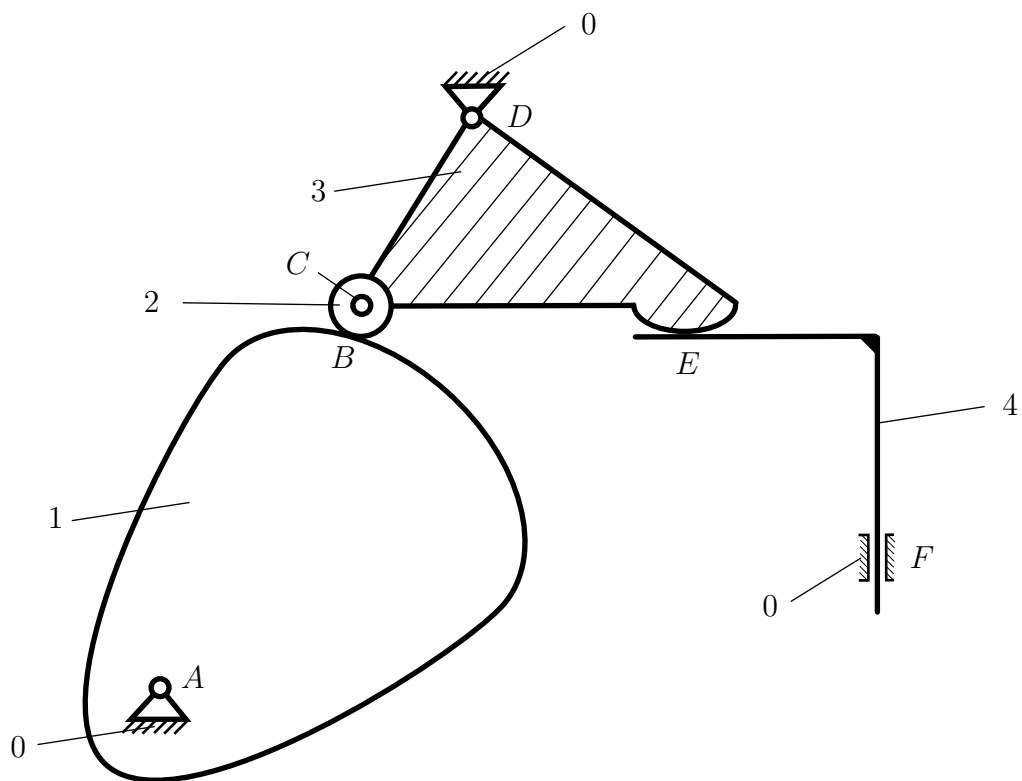


Figure P.4