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Rough surface electrical contact resistance considering scale dependent properties and quantum effects

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The objective of this work is to evaluate the effect of scale dependent mechanical and electrical properties on electrical contact resistance (ECR) between rough surfaces. This work attempts to build on existing ECR models that neglect potentially important quantum- and size-dependent contact and electrical conduction mechanisms present due to the asperity sizes on typical surfaces. The electrical conductance at small scales can quantize or show a stepping trend as the contact area is varied in the range of the free electron Fermi wavelength squared. This work then evaluates if these effects remain important for the interface between rough surfaces, which may include many small scale contacts of varying sizes. The results suggest that these effects may be significant in some cases, while insignificant for others. It depends on the load and the multiscale structure of the surface roughness. © 2015 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4921110]

I. INTRODUCTION

Electrical contact resistance (ECR) is a phenomenon that occurs when current is driven between two contacting rough surfaces. This is important for electrical connectors that are used in a wide variety of applications such as vehicles, industrial and domestic electronics, microelectromechanical systems (MEMS),1,2 and power generation systems.3,4 In addition, they are under increasing usage due to more vehicles being propelled (at least partially) by electrical current.5–7 Electrical connectors are also very important for growing energy sources such as solar and wind.8–10 It is therefore advantageous to increase our understanding of electrical contacts so that they can be improved.

The roughness of the surfaces causes only small peaks or asperities to be in contact. Since the diameters of the individual contact areas can approach nanometer dimensions, the mechanical and electrical properties can vary significantly from bulk properties. Most previous electrical contact resistance models do not consider the effects of these scale dependent properties; in fact, many classical and widely used ECR models completely neglect the effect of scaled asperities.

A variety of electrical contact resistance models have been developed since rough surface contact models were first considered by Archard in 1955.11 Archard modeled rough surface contact phenomena by stacking smaller asperities on top of larger asperities. Structurally, the asperity scales are in series, while the individual asperities of each scale are in parallel and the load is distributed evenly among them. Archard’s model, however, proved difficult to implement because no technique was presented to calculate the size of the asperities at each scale. The work herein uses the Archard stacked asperity concept, further developed by Ciavarella et al.,12–14 Jackson and Streator,15 and Jackson.16

Many groups have now experimentally observed the phenomena of conductance quantization as a step function.17–19 However, stepped conduction is not usually included in models of contact resistance between rough surfaces,14,20–22 This is because the models do not provide a convenient mathematical structure for multi-scale properties; rather, they are based on the popular statistically based rough surface contact models derived by Greenwood and Williamson.23 Jackson19 included scale effects in a statistical based model but did not predict contact resistance. Holm25 and Greenwood26 derive very similar equations to predict the ECR between rough surfaces, which includes two scales of roughness on the contact resistance. Coutu et al.1 later modified the model to include the effect of scale dependent asperity contact resistance. Kogut and Komvopoulos included the effect of electron tunneling in the contact between fractal surfaces.27,28 Archard’s concept of multiscale contact was first employed for modeling contact resistance by Ciavarella et al.14 for elastic surfaces described by the Weierstrass-Mandelbrot function. Barber29 also defined bounds for the electrical resistance of elastic rough surfaces in contact. Almeida et al.30 used an fast-Fourier transform (FFT) based multi-scale contact model, which included the effect of scale dependent electrical resistance and yield strength to analyze MEMS switches,2,30 but the electrical quantum stepping effects were not included, and the elastic–plastic asperity contact mechanics have been advanced since then. Wilson et al.31 improved the multi-scale methodology by modeling asperities using an elastic-plastic sinusoidal contact model. Wilson et al. showed that the slope of the spectrum of the surface dictated if the predicted contact area was finite or infinitely small as predicted by models using fractal mathematics, but the work did not include scale effects. Simplified closed form versions of the models have also been proposed.16,32,33 In addition, deterministic models (i.e., mathematics are not used to simplify the surface geometry) have been developed that include softening of the
surfaces. Despite the wealth of intense effort in the area, little has been done to determine the influence of scale dependent properties and quantum effects on electrical contact resistance. This is the focus of the current work.

II. CONTACT MECHANICS

A. Multi-scale perfectly elastic contact

The current work uses the multi-scale model derived by Jackson and Streator. A fast Fourier transform is first performed on the surface profile data, resulting in a summation of sine and cosine waves. The complex forms of the sine and cosine terms at each frequency are combined using a complex conjugate to provide the amplitude of the waveform at each scale, used for further calculations. Each frequency is considered a scale or layer of asperities, which are stacked iteratively upon each other. In equation form, this is

\[ A = \left( \prod_{i=1}^{i_{\text{max}}} A_i \eta_i \right) A_n, \]  
\[ F = F_i \eta_i A_{i-1}, \]  

where \( A \) is the real area of contact, \( \eta \) is the areal asperity density, \( P \) is the contact force, \( A_n \) is the nominal contact area, and the subscript \( i \) denotes a scale, with \( i_{\text{max}} \) denoting the highest (i.e., smallest) scale considered. Each scale is modeled using a sinusoidal contact model. Equations derived in Jackson and Streator and fit to the data given by Johnson et al. are used for this

\[ (A_{\text{JGH}})_1 = \frac{2\pi}{f^2} \left( \frac{3}{8\pi p^*} \right)^{2/3}, \]  
\[ (A_{\text{JGH}})_2 = \frac{1}{f^2} \left( 1 - \frac{3}{2\pi} \left[ 1 - \frac{\bar{p}}{p^*} \right] \right). \]  

Then, for \( \frac{\bar{p}}{p^*} < 0.8, \)

\[ A = (A_{\text{JGH}})_1 \left( 1 - \left[ \frac{\bar{p}}{p^*} \right]^{1.51} \right) + (A_{\text{JGH}})_2 \left( \frac{\bar{p}}{p^*} \right)^{1.04}, \]  

and for \( \frac{\bar{p}}{p^*} \geq 0.8, \)

\[ A = (A_{\text{JGH}})_2, \]  

where \( p^* \) is the average pressure for complete contact between the surfaces. It is given by Johnson et al. as

\[ p^* = \sqrt{2\pi E^* \Delta f}. \]  

B. Multi-scale elastic-plastic contact

Many of the asperities at the different frequency scales undergo plastic deformation. An elastic-plastic sinusoidal contact model is needed to consider this effect. The equations used here to calculate the elastic-plastic contact are derived from FEM results by Krithivasan and Jackson. The methodology is similar to the perfectly elastic case but uses a different set of formulas once a calculated critical pressure and area are reached. The critical load and area are given by

\[ F_c = \frac{1}{6\pi} \left( \frac{1}{M^2 E^*} \right)^2 \left( \frac{C}{2S_y} \right)^3, \]  
\[ A_c = \frac{2}{\pi} \left( \frac{CS_y}{8AE^*} \right)^2, \]  

where \( C = 1.295 \cdot \exp(0.736v). \)

At low loads, \( F < F_c, \) and consequently small areas of contact, it is acceptable to assume that the deformation of the contacting asperities will behave perfectly elastically. However, as the load increases to the critical value, plastic deformation will begin to occur within the asperities. To evaluate the plastic deformation, we replace Eq. (3) with

\[ A_P = 2(A_P)^{1/3} \left( \frac{3\bar{p}}{4CS_y^2} \right)^{\frac{2}{3}}, \]  
\[ d = 3.8 \left( \Delta E^* \frac{\Delta f}{2S_y} \right)^{0.11}. \]  

This replacement results in the following equation for contact area:

\[ A = (A_P) \left( 1 - \left[ \frac{\bar{p}}{p^*} \right]^{1.51} \right) + (A_{\text{JGH}})_2 \left( \frac{\bar{p}}{p^*} \right)^{1.04}, \]  

where \( p^*_n \) is the average pressure required to obtain complete contact between sinusoidal surfaces when plasticity occurs and is given by

\[ \frac{p^*_n}{p^*} = \left( \frac{11}{4 \cdot \frac{\Delta}{\Delta c} + 7} \right)^{\frac{1}{3}}, \]  
\[ \Delta c = \frac{\sqrt{2 \cdot S_y \exp(\frac{2v}{3})}}{3\pi E^* f}. \]  

III. ELECTRICAL RESISTIVITY

Our focus here is to predict the electrical contact resistance between surfaces having multiple scales of roughness. Since only a few scattered asperities are actually in contact for any given load, the current is restricted to small contact patches (see Fig. 1). As the current flows through the asperity

FIG. 1. Schematic of “bottlenecked” current flow through asperities.
peaks, it will be effectively “bottlenecked” and result in electrical resistance. Holm\textsuperscript{25} provides a simple formula to calculate the electrical spreading resistance due to asperity contact

\[ ECR_{asp} = \frac{\rho_1 + \rho_2}{4a}, \quad (16) \]

where \( E_{asp} \) refers to the contact resistance, \( a \) is the radius of contact, and \( \rho \) is the specific electrical resistivity of the respective surfaces. This equation is valid only for an asperity in the continuum/macro-scale or well above the atomic scale.

As the size of an asperity decreases, the contact diameter may become similar in magnitude to the electron mean free path, \( l \). Over this size transition, the mechanism changes from a diffusive mechanism (Maxwell\textsuperscript{39}) to a ballistic mechanism (Sharvin\textsuperscript{40}). The transition can therefore be characterized by the Knudsen number, \( K \), given by

\[ K = \frac{l}{a}, \quad (17) \]

where \( a \) is the contact radius and \( l \) is the electron mean free path. For perfect conduction (no material imperfections), Landauer\textsuperscript{41} showed that conduction in the ballistic regime is given by

\[ G = G_o N. \quad (18) \]

\( G_o \) is one quantum unit of conductance for the single perfect mode of transmission, and is given by

\[ G_o = \frac{2e^2}{h}. \quad (19) \]

Here, \( e \) is the electron charge and \( h \) is Planck’s constant. Equation (18) shows the quantization or stepping of the conduction as the contact size approaches the length of the electron mean free path. Since the prediction of \( N \) exactly requires extensive calculations, in the current work, we use the simplification presented by Torres et al.\textsuperscript{42} We also assume that the stepping can be approximated by assuming that the conductance increases as the area increases by one Fermi area. This is acceptable here because we are interested in observing, if the steps are represented in the full rough surface contact model or are smoothed away by the accumulation of asperities on the rough surface. Torres et al.\textsuperscript{42} investigated conduction between the extremes of a cylindrical wire and a circular path between two half spaces. The actual asperity contact for real surfaces is probably somewhere between these two cases; but for simplification, we assume the half spaces geometry, which is similar to the case considered by Holm\textsuperscript{25} using conventional electrical conduction theory. From Torres et al.\textsuperscript{42} this results in \( N \) being given by

\[ N = \text{truncate} \left( \frac{\pi a^2}{\lambda_F^2} - \frac{\pi \cdot a}{2\lambda_F} \right), \quad (20) \]

Substituting Eqs. (19) and (20) into Eq. (18) results in

\[ G_s = \frac{2e^2}{h \cdot \text{truncate} \left( \frac{\pi a^2}{\lambda_F^2} - \frac{\pi \cdot a}{2\lambda_F} \right)}, \quad (21) \]

where \( \lambda_F \) is the free electron Fermi wavelength, (usually known to physicists as simply the “de Broglie wavelength”), given by

\[ \lambda_F = \frac{h}{mv_F}, \quad (22) \]

where \( m \) is the electron mass and \( v_F \) is the Fermi velocity. The Fermi velocity \( v_F \) is the speed of the fastest moving electrons at absolute zero in the free electron model of metal conduction. Typical values of \( v_F \) are \( 1\text{–}2 \times 10^8 \text{cm/s}. \textsuperscript{43} \) Sharvin\textsuperscript{40} derived an equivalent continuous function to Eq. (21) as

\[ G_s = \frac{3\pi a^2}{4pl}. \quad (23) \]

However, since Torres et al.\textsuperscript{42} state that (21) is more accurate, it will be used in the current analysis. Not all of the necessary properties are readily available; so, it is useful to note that the resistivity can be related to fundamental properties by

\[ \rho = \frac{mv_F}{ne^2 l}. \quad (24) \]

To bridge the gap between large scales (the diffusive regime given by Holm as Eq. (16) and small scales (ballistic), Wexler\textsuperscript{45} provided an interpolating equation,

\[ G = G_s \left( 1 + \frac{\gamma(K)}{1 - 8K} \right), \quad (25) \]

where in the current work, Eq. (21) is used for \( G_s \), and \( \gamma(K) \) has limits of 1 when \( K = 0 \) and 0.694, when \( K = \infty \). An approximate integral formulation is given by Mikrajuddin et al.\textsuperscript{46} as

\[ \gamma(K) = \frac{2}{\pi} \int_0^\infty e^{-ks} \sin(c(x)) dx. \quad (26) \]

The equations used to predict \( \gamma(K) \) in Wexler are quite extensive. Here, we provide an approximate closed-form fit to the data presented in Wexler.\textsuperscript{45}

\[ \gamma(K) = \frac{9\pi^2}{128} + \left[ 1 - \frac{9\pi^2}{128} \right] \text{ERFC} \left[ \frac{8K}{3\pi} \right]^{1/3}. \quad (27) \]

Note that this appears to differ from Wexler’s equation by no more than \( \sim 2.5\% \); however, it does not predict the minimum value of 0.6828 located at approximately \( K \approx 8.95 \). The Holm resistance (Eq. (16)), Sharvin Resistance (Eq. (23)), Quantized conduction as approximated by Eq. (21), and the modified Wexler’s (Eq. (25)) are shown in Figs. 2 and 3 for tin and gold (Table I). As shown in Fig. 2, at low values of \( K \) the Sharvin and Quantized models agree, but do not agree
As $K$ increases, the Quantized model deviates from the Sharvin model and starts to show the stepping behavior. The Modified Wexler model asymptotically bridges between the Holm and Quantized model, as expected. The curves for gold and tin are also very similar except that the axes are offset by several orders of magnitude.

IV. SCALE DEPENDENT STRENGTH

It is well known that the strength of a material can vary by several orders of magnitude over different scales. Specifically, the effective yield strength, $S_y$, of a contacting asperity is known to increase as the scale of contact is decreased. In the current work, the fit given in Almeida et al.\textsuperscript{2} to the experimental results of Greer and Nix\textsuperscript{47} is used to predict the yield strength of gold as

$$S_y = S_0 \left( \frac{a}{R^*} \right)^{-4} \left( \frac{1}{R^*} \right)^{1.6},$$

(28)

where $a$ is the contact radius, $R^*$ is a characteristic length (0.35 $\mu$m), and $S_0$ is 50 MPa. The predicted yield strength is also limited by the bulk yield strength of gold and the theoretical limit (approximately $E/10^4$) listed in Table I and plotted in Fig. 4. As shown, the yield strength varies by over two orders of magnitude as the contact diameter changes. Brenner also measured the strength of gold “micro-scale” whiskers and found it to be stronger than the bulk property.\textsuperscript{49}

Similarly, the scale dependent yield strength of tin is also considered. Here, the work by Burek et al.\textsuperscript{50} is used to characterize the yield strength as a function of the contact diameter

$$S_y = S_1 (2a)^{-0.572},$$

(29)

where $S_y$ is 25 kPa for tin. Again, the function is limited by the theoretical limit and bulk value of the yield strength as shown in Fig. 5. Researchers have also measured the tensile strength of tin whiskers,\textsuperscript{51} but the yield strength values are

<table>
<thead>
<tr>
<th>Material properties of tin</th>
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<tbody>
<tr>
<td>$E = 41.4 \times 10^9$ Pa</td>
</tr>
<tr>
<td>$\nu = 0.36$</td>
</tr>
<tr>
<td>$v_f = 1.9 \times 10^6$ m/s</td>
</tr>
<tr>
<td>$S_y = 14 \times 10^6$ Pa</td>
</tr>
<tr>
<td>$\rho = 11.5 \times 10^{-8}$ $\Omega$m</td>
</tr>
<tr>
<td>$L = 3.95 \times 10^{-6}$ m</td>
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<table>
<thead>
<tr>
<th>Material properties of gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 77.2 \times 10^9$ Pa</td>
</tr>
<tr>
<td>$\nu = 0.42$</td>
</tr>
<tr>
<td>$v_f = 1.4 \times 10^6$ m/s</td>
</tr>
<tr>
<td>$S_y = 70 \times 10^5$ Pa</td>
</tr>
<tr>
<td>$\rho = 2.21 \times 10^{-8}$ $\Omega$m</td>
</tr>
<tr>
<td>$l = 3.8 \times 10^{-8}$ m</td>
</tr>
</tbody>
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[FIG. 2. A comparison of several models of single point contact resistance for tin.]

[FIG. 3. A comparison of several models of single point contact resistance for gold.]

TABLE I. Material properties used in this work.

[FIG. 4. The effect of contact radius scale on the yield strength of gold.]

[FIG. 5. The effect of contact radius scale on the yield strength of tin.]
different than those measured by Burek et al.\textsuperscript{50} These are also shown in Fig. 5. The difference could be due to tin whiskers having a single crystal structure, in contrast to bulk tin. The whiskers should therefore have a higher strength and be less dependent on scale, as shown in Fig. 5.

V. CONTACT RESISTANCE MODEL

The multi-scale sinusoidal method here is an iterative method that calculates area and resistance for each particular scale. The first step is to calculate the average radius of contact at scale \( i \),

\[
a_i = \sqrt{\frac{A_i}{2\pi f^2 A_{i-1}}}.
\]

Once the contact radius is established, either Eqs. (16) or (25) are used to calculate the resistance of each asperity at each scale. Oftentimes, an alleviation factor is used in thermal contact resistance calculations to account for the effect of nearby contact spots alleviating the load of electrical current. The effect increases when the contact area is large and the spots are closer together. Since electrical and thermal contact resistances are similar, it is reasonable to presume that the alleviation factor, \( \Psi \), should also be included for electrical contact resistance. The simplified version of the factor offered by Cooper et al.\textsuperscript{52} is used in this work,

\[
\Psi_i = \left(1 - \sqrt{\frac{A_i}{A_{i-1}}}ight)^{1.5}.
\]

Therefore, the electrical contact resistance per each asperity is (note \( ECR_{non} \) is \( 1/G \))

\[
ECR_i = ECR_{non} \times \Psi_i.
\]

This value is then summed over all the considered scales to find the total resistance for the entire surface in contact

\[
ECR = \sum_i ECR_i/(\eta A_i).
\]

VI. MODEL PREDICTIONS

In order to evaluate the effect of scale dependent yield strength and resistivity, the multi-scale model is implemented with and without multi-scale physical properties. When they are not used, the material properties in Table I are used. The surface profile is measured from an arbitrary machined metal sample using a stylus profilometer with a tip radius of 2 \( \mu m \) and a vertical resolution of less than 1 nm. The lateral resolution of the measurement was therefore limited to 1 \( \mu m \) and above (see the resulting surface spectrum in Fig. 6(a). A characterization of the surface profile results in a root mean square roughness of 0.0557 \( \mu m \), a skewness of 0.134 and a kurtosis of 3.10. Therefore, the surface is relatively close to being Gaussian.

Since the lateral resolution of the profilometer is limited, an atomic force microscope (AFM) profile scan of the same surface in different location was also conducted to characterize the smaller scales on the surface. The length of the AFM scan was 3.122 \( \mu m \), and the lateral resolution was 6.1 nm. Because the AFM scan has a limited scan length, this scan was also combined with a second scan using the profilometer described above. The surface spectra of both measurements were combined for use in the contact resistance model. The resulted surface spectrum is shown in Fig. 6(b). The plotted values with wavelengths (>\( \lambda \)) in the micrometer range and below are from the AFM, while the values in the micrometer range and above are from the profilometer. Note that the surface profile and the resulting spectrum can vary significantly even on the same surface due to the location, direction, resolution, and length of the scan. Initially, the data from the surface spectrum shown in Fig. 6(a) is used to analyze the results of the model; but in a second analysis, the surface shown in Fig. 6(b) is also used.

The geometric properties (i.e., roughness) of the surface will be also varied artificially in Sec. VII in order to characterize the effect of the scale dependent properties on different surfaces.

Using the surface spectrum data shown in Fig. 6(a), the predicted ECR for both tin and gold are shown in Fig. 7. The effects of “quantum” conductivity (Eq. (25)) and scale
dependent strength (Eqs. (28) and (29)) are plotted. It is seen that the normalized ECR for both tin and gold is within an order of magnitude for this particular rough surface. The same trends may not be true for other surfaces. It should also be noted that this does not include the effect of an oxide layer on the tin that would not be present on the gold, since gold has no native oxide. At high loads, when nearly all the surface area is contacting, the ECR drops off sharply for both tin and gold. Note that the quantum stepping of the resistivity shown in Figs. 2 and 3 are not present in the multiscale model ECR results.

Since the results in Fig. 7 are plotted on a log-log scale, it is difficult to ascertain the quantitative effects of the scale dependent properties. Whence, the ECR of the various versions of the model including the scale dependent properties are normalized by the ECR without any scale dependent properties (labeled ECR-MM). The normalized results are shown in Fig. 8 for gold and Fig. 9 for tin. It is clear that for most of the considered range of contact forces that the difference between the ECR including and not including scale effects is less than 10%. This is probably within the range of accuracy of the theoretical model.

As shown in Fig. 8, the scale dependent properties tend to decrease the ECR of gold from that predicted without considering them. It also appears that “quantum” resistivity effects play a larger role than scale dependent yield strength, although the effect is still relatively small. Contrastingly, for the case of tin (Fig. 9), the scale dependent strength is more important than the “quantum” resistivity. The scale dependent yield strength even appears to increase the ECR in relation to ECR predicted when using bulk properties. This is due to the yield strength effectively increasing at smaller scales, which reduces the contact area between the asperities because they are more resistant to deformation (plastic deformation causes the asperity to conform and have more contact area).

In theory, the scale at which the electrical and mechanical properties begin to change can be related to the characteristic length of each mechanism. For electrical contact resistance, it is the electron mean free path, \( l \), which is on the order of 10 nm for most metals. However, the characteristic length of the mechanical properties is \( R^* \) and appears to be on the order of \( \mu m \). Therefore, these appear to take affect at scales orders of magnitude apart and scale dependent mechanical properties start influencing the surface contact at larger scales. However, this might not always be the case as shown by the results in Figs. 8 and 9.

Next, using the surface spectrum data shown in Fig. 6(b), the predicted ECR for only tin is shown in Fig. 10. For these results, the employed surface spectrum was a combination of an AFM measurement and a profilometer measurement. The effects of “quantum” conductivity (Eq. (25)) and scale dependent strength (Eqs. (28) and (29)) are both plotted, but the results all fall on the same curve. This is interesting that with a more refined profile the scale dependent effects here seem to lose importance. This could actually be due to the profilometer scan and not the AFM scan. As shown in previous works on multiscale contact, the smaller
scales may not reduce the contact area further if their amplitude to wavelength ratios are not larger, although scale dependent properties are not considered in the previous work. That appears to be the case here. However, this may not hold for all surfaces and so, Sec. VII explores the general cases by artificially varying the structure of the surface.

VII. PARAMETRIC ANALYSIS

Although for the measured surface the scale dependent properties do not appear to be significant, the results could be deceiving due to the particular surface considered and the limitations of the measuring profilometer. Next, two parametric analyses on the roughness of the surface are performed by scaling the profile height. In the first analysis, just the height is varied by a scaling factor, but the profile length is left the same. This alters the roughness, but also changes the spectral properties of the surface by shifting the amplitudes. In a second version, the scale of the height and length of the profile are varied proportionally, which varies the roughness but maintains the slope and average magnitude of the surface spectrum. These analyses were performed for the case of tin, again using the properties listed in Table I.

The results for scaling the height without changing the length are shown in Fig. 11. The ECR considering the scale dependent properties normalized by the without these properties (ECR-MM) is shown as a function of the surface roughness. Here, the ECR still does not change more than approximately 20%, and is for most roughness values is only effecting the ECR by less than 5%. Using this scaling method, the change is only seen for rougher surfaces starting at around 10 nm, which for typical engineering surfaces is very smooth. There is an interesting peak at 10 nm of roughness as well. This peak can be explained by the competing effects of roughness and scale dependent properties. In the absence of scale dependent properties, as the roughness decreases, the electrical contact resistance will typically also decrease. However, for the current analysis, as the roughness occurs on a smaller scale, the scale dependent strength and resistivity cause the contact resistance to increase. This causes the peak in ECR but is eventually simply overcome by the decreasing roughness of the surface.

Next, the surface profile was varied by scaling the height and lateral length by the same factor (see Fig. 12). This also causes the roughness to change but preserves the magnitude of the surface’s spectrum. The trend here is quite different from the previous case. As shown in Fig. 12, the effect of scale dependent properties actually becomes more important at smaller roughnesses. Here, the reason for this is that the surface is actually being scaled down, and therefore, the size of the asperity contact areas decreases. As this size decreases, the strength of the asperities increases further, which reduces the contact area and increases the contact resistance. The resistivity also increases, further increasing the contact resistance. This results in the ECR increasing by up to 70%. In Figures 11 and 12, the curves for the different
loads actually cross and change order. This is due to the scale dependent strength and resistivity taking effect at different scales. The ECR including the scale dependence also appears to plateau and reach a limit for very small scales. This may be due to the scale dependent strength reaching the theoretical limit of approximately $E/10$ and the resistivity reaching a quantum step.

VIII. CONCLUSIONS

This work uses an established multi-scale model of electrical contact resistance to examine the relative importance of scale dependent material properties. An analytical model for including quantum effects on the resistance ofasperity contacts is provided, based on the previous works of multiple authors. The final results suggest that the scale dependent properties do not always play a major role in the overall ECR occurring between rough surfaces in contact, although this might not be true for different surfaces of varying roughness and asperity geometries. For the measured surface under consideration, the scale dependent effects do not appear important; however, when the properties were artificially varied, they did become important. This occurred for rougher surfaces and surface containing more detail at smaller scales. Both of these cases essentially result in the asperity contact areas being smaller. However, it appears that having roughness features at smaller scales, even if the overall roughness is less, is the most important characteristic to cause the scale dependent properties to be significant. So, for real surfaces that are characterized effectively, scale dependent properties may indeed be important for some cases, but less important for other cases.

ACKNOWLEDGMENTS

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