9–7. Determine the normal stress and shear stress acting on the inclined plane \( AB \). Solve the problem using the stress transformation equations. Show the result on the sectioned element.

**Stress Transformation Equations:**

\[
\theta = +135^\circ \text{ (Fig. a)} \quad \sigma_x = 80 \, \text{MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 45 \, \text{MPa}
\]

we obtain,

\[
\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos \theta + \tau_{xy} \sin \theta
\]

\[
= \frac{80 + 0}{2} + \frac{80 - 0}{2} \cos 270^\circ + 45 \sin 270^\circ
\]

\[
= -5 \, \text{MPa} \quad \text{Ans.}
\]

\[
\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin \theta + \tau_{xy} \cos \theta
\]

\[
= -\frac{80 - 0}{2} \sin 270^\circ + 45 \cos 270^\circ
\]

\[
= 40 \, \text{MPa} \quad \text{Ans.}
\]

The negative sign indicates that \( \sigma_x' \) is a compressive stress. These results are indicated on the triangular element shown in Fig. \( b \).
9–15. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

\[ \sigma_x = 45 \text{ MPa} \quad \sigma_y = -60 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa} \]

(a) Principal Stress:

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ = \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2} \]

\[ \sigma_1 = 53.0 \text{ MPa} \]

\[ \sigma_2 = -68.0 \text{ MPa} \]

Orientation of principal stress:

\[ \tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714 \]

\[ \theta_p = 14.87^\circ, \quad -75.13^\circ \]

Use Eq. 9–1 to determine the principal plane of \( \sigma_1 \) and \( \sigma_2 \):

\[ \sigma_y = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta, \quad \text{where} \ \theta = 14.87^\circ \]

\[ = \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2} \cos 29.74^\circ + 30 \sin 29.74^\circ = 53.0 \text{ MPa} \]

Therefore \( \theta_{p1} = 14.9^\circ \) \quad Ans. and \quad \( \theta_{p2} = -75.1^\circ \) \quad Ans.

(b) Maximum In-plane Shear Stress:

\[ \tau_{\text{max,in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa} \] \quad Ans.

\[ \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa} \] \quad Ans.

Orientation of maximum in-plane shear stress:

\[ \tan 2\theta_y = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{45 - (-60)}{30} = -1.75 \]

\[ \theta_y = -30.1^\circ \quad \text{Ans. and} \quad \theta_{y} = 59.9^\circ \quad \text{Ans.} \]

By observation, in order to preserve equilibrium along \( AB \), \( \tau_{\text{max}} \) has to act in the direction shown.
9–18. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.

For element $a$:
\[ \sigma_x = \sigma_y = 85 \text{ MPa} \quad \tau_{xy} = 0 \quad \theta = -45^\circ \]
\[ (\sigma_x)_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \]
\[ = \frac{85 + 85}{2} + \frac{85 - 85}{2} \cos (-90^\circ) + 0 = 85 \text{ MPa} \]
\[ (\sigma_y)_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \]
\[ = \frac{85 + 85}{2} - \frac{85 - 85}{2} \cos (-90^\circ) - 0 = 85 \text{ MPa} \]
\[ (\tau_{xy})_a = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \]
\[ = \frac{85 - 85}{2} \sin (-90^\circ) + 0 = 0 \]

For element $b$:
\[ \sigma_x = \sigma_y = 0 \quad \tau_{xy} = 60 \text{ MPa} \quad \theta = -60^\circ \]
\[ (\sigma_x)_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \]
\[ = 0 + 0 + 60 \sin (-120^\circ) = -51.96 \text{ MPa} \]
\[ (\sigma_y)_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \]
\[ = 0 - 0 - 60 \sin (-120^\circ) = 51.96 \text{ MPa} \]
\[ (\tau_{xy})_b = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \]
\[ = \frac{85 - 85}{2} \sin (-120^\circ) + 60 \cos (-120^\circ) = -30 \text{ MPa} \]
\[ \sigma_x = (\sigma_x)_a + (\sigma_x)_b = 85 + (-51.96) = 33.0 \text{ MPa} \]
\[ \sigma_y = (\sigma_y)_a + (\sigma_y)_b = 85 + 51.96 = 137 \text{ MPa} \]
\[ \tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 0 + (-30) = -30 \text{ MPa} \]

Ans:
\[ \sigma_x = 33.0 \text{ MPa}, \sigma_y = 137 \text{ MPa}, \tau_{xy} = -30 \text{ MPa} \]
9-25. The wooden block will fail if the shear stress acting along the grain is 550 psi. If the normal stress \( \sigma_x = 400 \text{ psi} \), determine the necessary compressive stress \( \sigma_y \) that will cause failure.

\[
\tau'_{x'y'} = -\left( \frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta
\]

\[
550 = -\left( \frac{400 - \sigma_y}{2} \right) \sin 296^\circ + 0
\]

\( \sigma_y = -824 \text{ psi} \)

Ans:

\( \sigma_y = -824 \text{ psi} \)
9–27. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point B on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.

**Internal Loadings:** Consider the equilibrium of the free-body diagram of the bracket’s left cut segment, Fig. a.

\[ \Delta \sum F_x = 0; \quad N - 3 = 0 \quad N = 3 \text{kip} \]
\[ \sum M_y = 0; \quad 3(4) - M = 0 \quad M = 12 \text{kip} \cdot \text{in} \]

**Normal and Shear Stresses:** The normal stress is the combination of axial and bending stress. Thus,

\[ \sigma = \frac{N}{A} - \frac{M_y}{I} \]

The cross-sectional area and the moment of inertia about the z axis of the bracket’s cross section is

\[ A = 1(2) - 0.75(1.5) = 0.875 \text{ in}^2 \]
\[ I = \frac{1}{12} (1)(2^3) - \frac{1}{12} (0.75)(1.5^3) = 0.45573 \text{ in}^4 \]

For point B, \( y = -1 \text{ in.} \) Then

\[ \sigma_B = \frac{3}{0.875} - \frac{(-12)(-1)}{0.45573} = -22.90 \text{ ksi} \]

Since no shear force is acting on the section,

\[ \tau_B = 0 \]

The state of stress at point A can be represented on the element shown in Fig. b.

**In - Plane Principal Stress:** \( \sigma_x = -22.90 \text{ ksi}, \sigma_y = 0, \) and \( \tau_{xy} = 0. \) Since no shear stress acts on the element,

\[ \sigma_1 = \sigma_y = 0 \quad \sigma_2 = \sigma_x = -22.90 \text{ ksi} \quad \text{Ans.} \]

The state of principal stresses can also be represented by the elements shown in Fig. b.

**Maximum In - Plane Shear Stress:**

\[ \tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-22.90 - 0}{2}\right)^2 + 0^2} = 11.5 \text{ ksi} \quad \text{Ans.} \]

**Orientation of the Plane of Maximum In - Plane Shear Stress:**

\[ \tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-22.90 - 0)/2}{0} = -\infty \]

\[ \theta_s = 45^\circ \text{ and } 135^\circ \quad \text{Ans.} \]
9–27. Continued

Substituting \( \theta = 45^\circ \) into

\[
\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta
\]

\[
= \frac{-22.9 - 0}{2} \sin 90^\circ + 0
\]

\[
= 11.5 \text{ ksi} = \tau_{\text{max in-plane}}
\]

This indicates that \( \tau_{\text{max in-plane}} \) is directed in the positive sense of the \( y' \) axes on the element defined by \( \theta_x = 45^\circ \).

Average Normal Stress:

\[
\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-22.9 + 0}{2} = -11.5 \text{ ksi}
\]

The state of maximum in-plane shear stress is represented by the element shown in Fig. c.

**Ans:**

\( \sigma_1 = 0, \sigma_2 = -22.90 \text{ ksi}, \tau_{\text{max in-plane}} = 11.5 \text{ ksi}, \theta_x = 45^\circ \text{ and } 135^\circ \)
9–33. The clamp bears down on the smooth surface at \( E \) by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stress at points \( A \) and \( B \) and show the results on elements located at each of these points. The cross-sectional area at \( A \) and \( B \) is shown in the adjacent figure.

**Support Reactions:** As shown on FBD(a).

**Internal Forces and Moment:** As shown on FBD(b).

**Section Properties:**

\[
I = \frac{1}{12} (0.03) (0.05^3) = 0.3125 \left(10^{-6}\right) \text{m}^4
\]

\[
Q_A = 0
\]

\[
Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right) \text{m}^3
\]

**Normal Stress:** Applying the flexure formula \( \sigma = \frac{My}{I} \).

\[
\sigma_A = \frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192 \text{ MPa}
\]

\[
\sigma_B = \frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0
\]

**Shear Stress:** Applying the shear formula \( \tau = \frac{VQ}{It} \).

\[
\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0
\]

\[
\tau_B = \frac{24.0(10^3)[9.375(10^{-6})]}{0.3125(10^{-6})(0.03)} = 24.0 \text{ MPa}
\]

**In-Plane Principal Stresses:** \( \sigma_x = 0, \sigma_y = -192 \text{ MPa} \), and \( \tau_{xy} = 0 \) for point \( A \). Since no shear stress acts on the element.

\[
\sigma_1 = \sigma_x = 0 \quad \text{ Ans.}
\]

\[
\sigma_2 = \sigma_y = -192 \text{ MPa} \quad \text{ Ans.}
\]

\( \sigma_x = \sigma_y = 0 \) and \( \tau_{xy} = -24.0 \text{ MPa} \) for point \( B \). Applying Eq. 9.5

\[
\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}
\]

\[
= 0 \pm \sqrt{0 + (-24.0)^2}
\]

\[
= 0 \pm 24.0
\]

\[
\sigma_1 = 24.0 \text{ MPa} \quad \sigma_2 = -24.0 \text{ MPa} \quad \text{ Ans.}
\]
9–33. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point B.

\[
\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty
\]

\[\theta_p = -45.0^\circ \quad \text{and} \quad 45.0^\circ\]

Substituting the results into Eq. 9-1 yields

\[
\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta
\]

\[= 0 + 0 + [-24.0 \sin (-90.0^\circ)]\]

\[= 24.0 \text{ MPa} = \sigma_1\]

Hence,

\[\theta_{p1} = -45.0^\circ \quad \theta_{p2} = 45.0^\circ\]

\[\text{Ans.}\]

\[\text{Ans:}\]

Point A: \(\sigma_1 = 0, \sigma_2 = -192 \text{ MPa},\)

\[\theta_{p1} = 0, \theta_{p2} = 90^\circ\]

Point B: \(\sigma_1 = 24.0 \text{ MPa}, \sigma_2 = -24.0 \text{ MPa},\)

\[\theta_{p1} = -45.0^\circ, \theta_{p2} = 45.0^\circ\]
9–42. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stresses and (b) the maximum in-plane shear stress at a point on its surface at section \( a \).

**Internal Forces and Torque:** As shown on FBD(a).

**Section Properties:**

\[
A = \frac{\pi}{4} \left( 3^2 - 2.5^2 \right) = 0.6875 \pi \text{ in}^2
\]

\[
J = \frac{\pi}{2} \left( 1.5^4 - 1.25^4 \right) = 4.1172 \text{ in}^4
\]

**Normal Stress:**

\[
\sigma = \frac{N}{A} = \frac{-2500}{0.6875 \pi} = -1157.5 \text{ psi}
\]

**Shear Stress:** Applying the torsion formula.

\[
\tau = \frac{T c}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}
\]

a) **In-Plane Principal Stresses:** \( \sigma_x = 0, \sigma_y = -1157.5 \text{ psi} \) and \( \tau_{xy} = 3497.5 \text{ psi} \) for any point on the shaft’s surface. Applying Eq. 9-5,

\[
\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}
\]

\[
= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left( 0 - (-1157.5) \right)^2 + (3497.5)^2}
\]

\[
= -578.75 \pm 3545.08
\]

\( \sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi} \)  

\( \sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi} \)  

Ans.

b) **Maximum In-Plane Shear Stress:** Applying Eq. 9–7,

\[
\tau_{\text{max in-plane}} = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}
\]

\[
= \sqrt{\left( 0 - (-1157.5) \right)^2 + (3497.5)^2}
\]

\[
= 3545 \text{ psi} = 3.55 \text{ ksi}
\]

Ans.
9–58. Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.

\[ R = CA = CB = 550 \]

\[ \sigma_x = -550 \sin 50^\circ = -421 \text{ MPa} \]

\[ \tau_{x'y'} = -550 \cos 50^\circ = -354 \text{ MPa} \]

\[ \sigma_y = 550 \sin 50^\circ = 421 \text{ MPa} \]

Ans:
\[ \sigma_x = -421 \text{ MPa}, \tau_{x'y'} = -354 \text{ MPa}, \sigma_y = 421 \text{ MPa} \]
9-61. Draw Mohr’s circle that describes each of the following states of stress.

(a) 5 MPa and 5 MPa

(b) 20 ksi and 20 ksi

(c) 18 MPa

A (5,0) B (5,0) C (5,0)

A (-20,0) B (20,0) C (0,0)

A (0, 18) B (0, 18) C (0, 0)
9–65. The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.

**Section Properties:**

\[ A = \pi \left( 0.275^2 - 0.25^2 \right) = 0.013125 \pi \text{ in}^2 \]

\[ J = \frac{\pi}{2} \left( 0.275^4 - 0.25^4 \right) = 2.84768 \left( 10^{-3} \right) \text{ in}^4 \]

**Normal Stress:** Since \( \frac{r}{t} = \frac{0.25}{0.025} = 10 \), thin wall analysis is valid.

\[ \sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{200}{0.013125 \pi} + \frac{500(0.25)}{2(0.025)} = 7.350 \text{ ksi} \]

\[ \sigma_{\text{hoop}} = \frac{pr}{t} = \frac{500(0.25)}{0.025} = 5.00 \text{ ksi} \]

**Shear Stress:** Applying the torsion formula,

\[ \tau = \frac{Tc}{J} = \frac{20(12)(0.275)}{2.84768 \left( 10^{-3} \right)} = 23.18 \text{ ksi} \]

**Construction of the Circle:** In accordance with the sign convention \( \sigma_x = 7.350 \text{ ksi} \), \( \sigma_y = 5.00 \text{ ksi} \), and \( \tau_{xy} = -23.18 \text{ ksi} \). Hence,

\[ \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5.00}{2} = 6.175 \text{ ksi} \]

The coordinates for reference points \( A \) and \( C \) are

\[ A(7.350, -23.18) \quad C(6.175, 0) \]

The radius of the circle is

\[ R = \sqrt{(7.350 - 6.175)^2 + 23.18^2} = 23.2065 \text{ ksi} \]

**In-Plane Principal Stress:** The coordinates of point \( B \) and \( D \) represent \( \sigma_1 \) and \( \sigma_2 \), respectively.

\[ \sigma_1 = 6.175 + 23.2065 = 29.4 \text{ ksi} \quad \text{Ans.} \]

\[ \sigma_2 = 6.175 - 23.2065 = -17.0 \text{ ksi} \quad \text{Ans.} \]

Ans:

\( \sigma_1 = 29.4 \text{ ksi}, \sigma_2 = -17.0 \text{ ksi} \)
9–75. If the box wrench is subjected to the 50 lb force, determine the principal stress and maximum in-plane shear stress at point B on the cross section of the wrench at section a–a. Specify the orientation of these states of stress and indicate the results on elements at the point.

**Internal Loadings:** Considering the equilibrium of the free-body diagram of the wrench’s cut segment, Fig. a,

\[ \Sigma F_y = 0; \quad V_y + 50 = 0 \quad V_y = -50 \text{ lb} \]

\[ \Sigma M_x = 0; \quad T + 50(12) = 0 \quad T = -600 \text{ lb} \cdot \text{in} \]

\[ \Sigma M_z = 0; \quad M_z - 50(2) = 0 \quad M_z = 100 \text{ lb} \cdot \text{in} \]

**Section Properties:** The moment of inertia about the \( z \) axis and the polar moment of inertia of the wrench’s cross section are

\[ I_z = \frac{\pi}{4}(0.5^4) = 0.015625 \pi \text{ in}^4 \]

\[ J = \frac{\pi}{2}(0.5^4) = 0.03125 \pi \text{ in}^4 \]

Referring to Fig. b,

\[ (Q_y)_B = 0 \]

**Normal and Shear Stress:** The normal stress is caused by the bending stress due to \( M_z \).

\[ (\sigma_x)_B = -\frac{M_z y_B}{I_z} = \frac{-100(0.5)}{0.015625 \pi} = -1.019 \text{ ksi} \]

The shear stress at point B along the \( y \) axis is \( (\tau_{xy})_B = 0 \) since \( (Q_y)_B \). However, the shear stress along the \( z \) axis is caused by torsion.

\[ (\tau_{xz})_B = \frac{T \cdot z}{J} = \frac{600(0.5)}{0.03125 \pi} = 3.056 \text{ ksi} \]

The state of stress at point B is represented by the two-dimensional element shown in Fig. c.
Construction of the Circle: \( \sigma_x = -1.019 \) ksi, \( \sigma_z = 0 \), and \( \tau_{xz} = -3.056 \) ksi. Thus,

\[
\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ ksi}
\]

The coordinates of reference point \( A \) and the center \( C \) of the circle are

\[
A(-1.019, -3.056) \quad \quad C(-0.5093, 0)
\]

Thus, the radius of the circle is

\[
R = CA = \sqrt{(-1.019 - (-0.5093))^2 + (-3.056)^2} = 3.0979 \text{ ksi}
\]

Using these results, the circle is shown in Fig. \( d \).

In-Plane Principal Stress: The coordinates of reference points \( B \) and \( D \) represent \( \sigma_1 \) and \( \sigma_2 \), respectively.

\[
\sigma_1 = -0.5093 + 3.0979 = 2.59 \text{ ksi} \quad \quad \text{Ans.}
\]

\[
\sigma_2 = -0.5093 - 3.0979 = -3.61 \text{ ksi} \quad \quad \text{Ans.}
\]

Maximum In-Plane Shear Stress: The coordinates of point \( E \) represent the maximum in-plane stress, Fig. \( a \).

\[
\tau_{max}^{in-plane} = R = 3.10 \text{ ksi} \quad \quad \text{Ans.}
\]

\[\sigma_1 = 2.59 \text{ ksi}, \sigma_2 = -3.61 \text{ ksi}, \theta_{p1} = -40.3^\circ, \theta_{p2} = 49.7^\circ, \tau_{max}^{in-plane} = 3.10 \text{ ksi}, \theta_r = 4.73^\circ\]
9-90. The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15 \text{ rad/s}$ when the engine develops 900 kW of power. This causes a thrust of $F = 1.23 \text{ MN}$ on the shaft. If the shaft has a diameter of 250 mm, determine the maximum in-plane shear stress at any point located on the surface of the shaft.

**Power Transmission:** Using the formula developed in Chapter 5,

\[ P = 900 \text{ kW} = 0.900 \left(10^6\right) \text{ N} \cdot \text{m/s} \]

\[ T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0 \left(10^3\right) \text{ N} \cdot \text{m} \]

**Internal Torque and Force:** As shown on FBD.

**Section Properties:**

\[ A = \frac{\pi}{4} \left(0.25^2\right) = 0.015625\pi \text{ m}^2 \]

\[ J = \frac{\pi}{2} \left(0.125^4\right) = 0.3835 \left(10^{-3}\right) \text{ m}^4 \]

**Normal Stress:**

\[ \sigma = \frac{N}{A} = -\frac{1.23 \left(10^6\right)}{0.015625\pi} = -25.06 \text{ MPa} \]

**Shear Stress:** Applying the torsion formula.

\[ \tau = \frac{T_c}{J} = \frac{60.0 \left(10^3\right) \left(0.125\right)}{0.3835 \left(10^{-3}\right)} = 19.56 \text{ MPa} \]

**Maximum In-Plane Principal Shear Stress:** $\sigma_x = -25.06 \text{ MPa}$, $\sigma_y = 0$, and $\tau_{xy} = 19.56 \text{ MPa}$ for any point on the shaft’s surface. Applying Eq. 9-7,

\[ \tau_{\text{in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ = \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2} \]

\[ = 23.2 \text{ MPa} \quad \text{Ans.} \]
9–93. Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr’s circle.

\[ A(6, 0) \quad B(-10, 0) \quad C(-2, 0) \]

\[ R = CA = CB = 8 \]

\[ \sigma_x = -2 + 8 \cos 80° = -0.611 \text{ ksi} \]

\[ \tau_{xy} = 8 \sin 80° = 7.88 \text{ ksi} \]

\[ \sigma_y = -2 - 8 \cos 80° = -3.39 \text{ ksi} \]

Ans:

\[ \sigma_x = -0.611 \text{ ksi}, \quad \tau_{xy} = 7.88 \text{ ksi}, \quad \sigma_y = -3.39 \text{ ksi} \]
9-94. The crane is used to support the 350-lb load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 6 in. and a thickness of 3 in. Use Mohr’s circle.

\[ A = 6(3) = 18 \text{ in}^2 \quad I = \frac{(3)(6^3)}{12} = 54 \text{ in}^4 \]

\[ Q_B = (1.5)(3)(3) = 13.5 \text{ in}^3 \]

\[ Q_A = 0 \]

For point A:

\[ \sigma_A = -\frac{P}{A} - \frac{M_y}{I} = -\frac{597.49}{18} - \frac{1750(12)(3)}{54} = -1200 \text{ psi} \]

\[ \tau_A = 0 \]

\[ \sigma_1 = 0 \]

\[ \sigma_2 = -1200 \text{ psi} = -1.20 \text{ ksi} \]

For point B:

\[ \sigma_B = -\frac{P}{A} = -\frac{597.49}{18} = -33.19 \text{ psi} \]

\[ \tau_B = \frac{VQ_B}{It} = \frac{247.49(13.5)}{54(3)} = 20.62 \text{ psi} \]

A(−33.19, −20.62) \quad B(0, 20.62) \quad C(−16.60, 0)

\[ R = \sqrt{16.60^2 + 20.62^2} = 26.47 \]

\[ \sigma_1 = -16.60 + 26.47 = 9.88 \text{ psi} \]

\[ \sigma_2 = -16.60 - 26.47 = -43.1 \text{ psi} \]

Ans:

Point A: \( \sigma_1 = 0, \sigma_2 = -1.20 \text{ ksi} \),
Point B: \( \sigma_1 = 9.88 \text{ psi}, \sigma_2 = -43.1 \text{ psi} \)