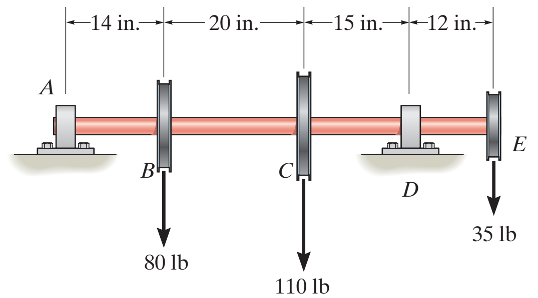
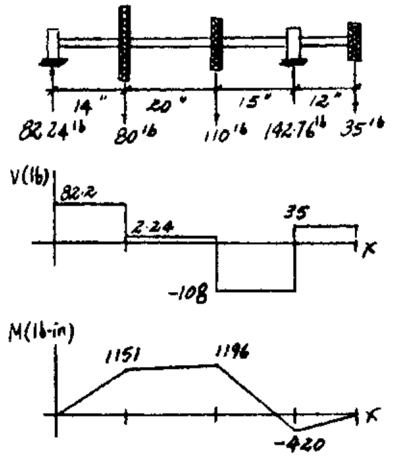
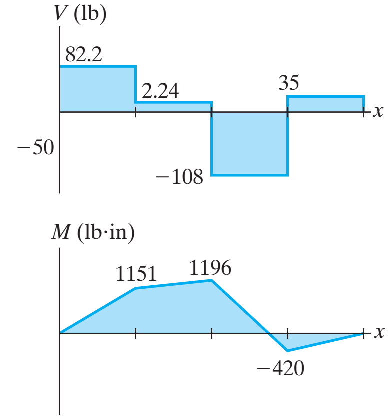
6–2. Draw the shear and moment diagrams for the shaft. The bearings at A and D exert only vertical reaction on the shaft. The loading is applied to the pulleys at B and C and E.

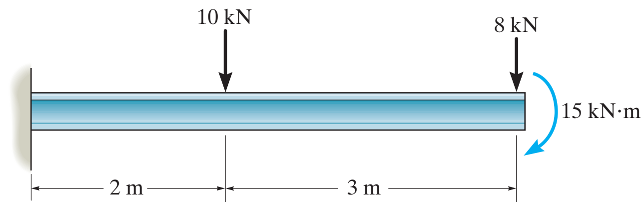


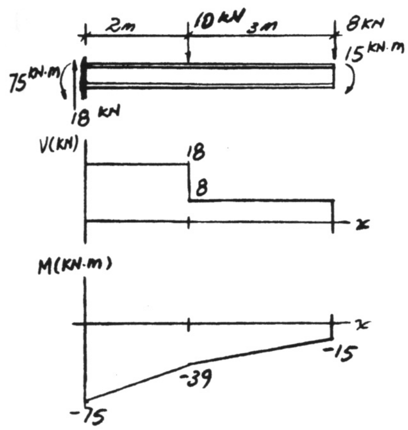


Ans:

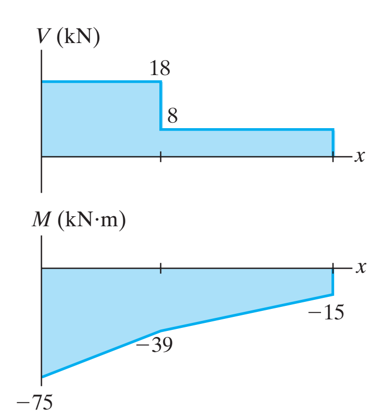


6–5. Draw the shear and moment diagrams for the beam.

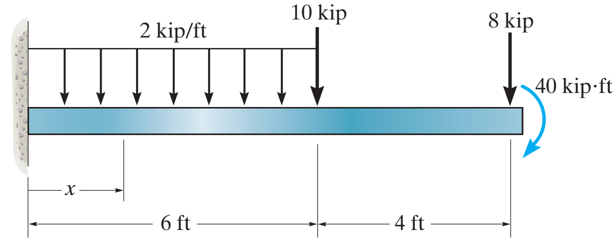




Ans:

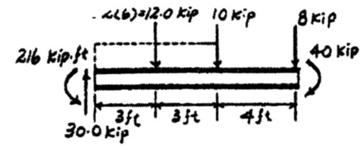


6–18. Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x.



Support Reactions: As shown on FBD.

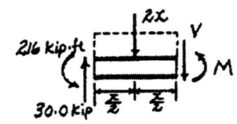
Shear and Moment Function:



For $0 \le x < 6$ ft:

$$+\uparrow \Sigma F_y = 0; \qquad 30.0 - 2x - V = 0$$

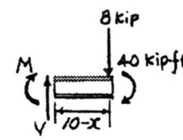
$$V = \{30.0 - 2x\} \text{ kip} \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

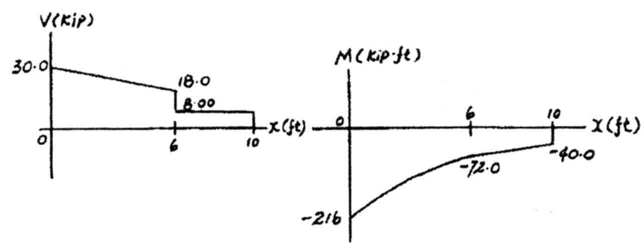
For 6 ft $< x \le 10$ ft:



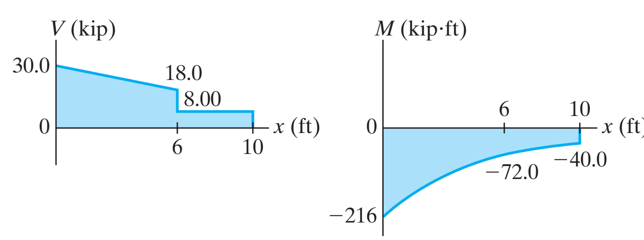
$$+\uparrow \Sigma F_y = 0; \qquad V - 8 = 0 \qquad V = 8.00 \text{ kip} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M - 8(10 - x) - 40 = 0$$

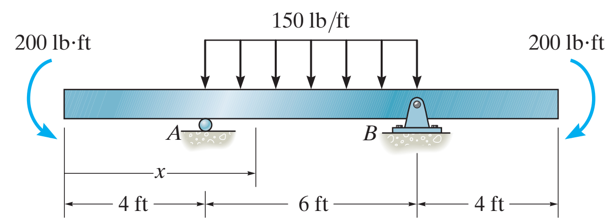
$$M = \{8.00x - 120\} \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$



Ans:
For $0 \le x < 6$ ft: $V = \{30.0 - 2x\}$ kip,
$M = \{-x^2 + 30.0x - 216\}$ kip · ft,
For 6 ft $< x \le 10$ ft: $V = 8.00$ kip,
$M = \{8.00x - 120\}$ kip · ft



6–25. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where 4 ft < x < 10 ft.

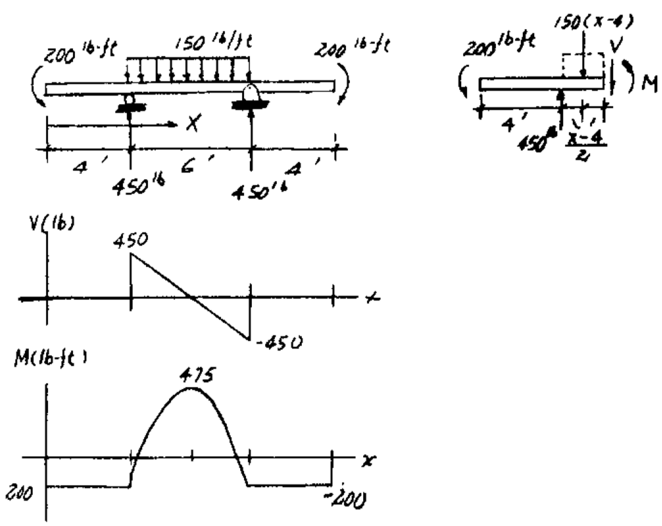


$+\uparrow \Sigma F_y = 0; \qquad -150(x-4) - V + 450 = 0$

$V = 1050 - 150x$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad -200 - 150(x-4)\dfrac{(x-4)}{2} - M + 450(x-4) = 0$

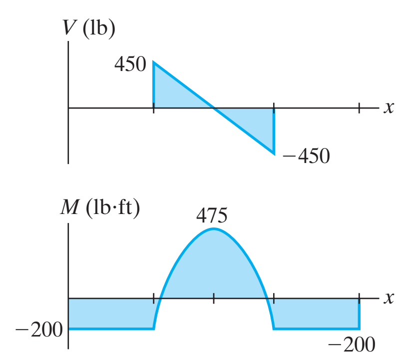
$M = -75x^2 + 1050x - 3200$ **Ans.**



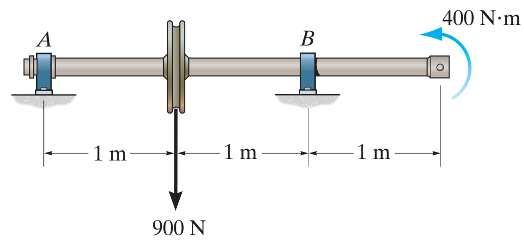
Ans:

$V = 1050 - 150x$

$M = -75x^2 + 1050x - 3200$



6–33. The shaft is supported by a smooth thrust bearing at A and smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



Equations of Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. *a*,

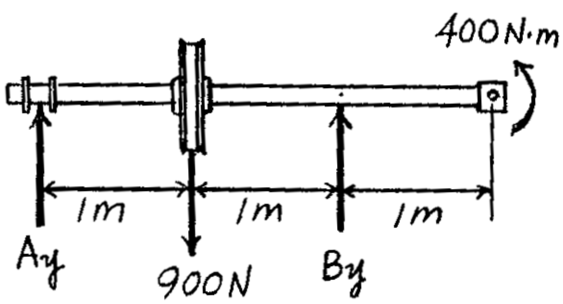
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) + 400 - 900(1) = 0$

$B_y = 250\text{ N}$

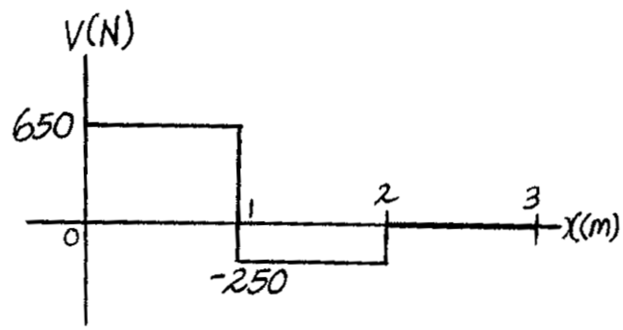
$+\uparrow \Sigma F_y = 0; \qquad A_y + 250 - 900 = 0$

$A_y = 650\text{ N}$

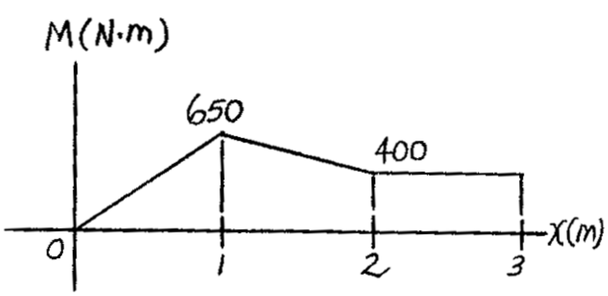
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

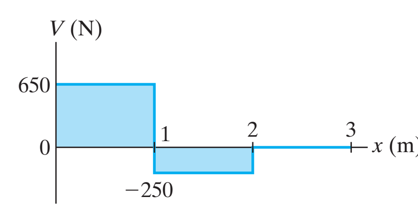


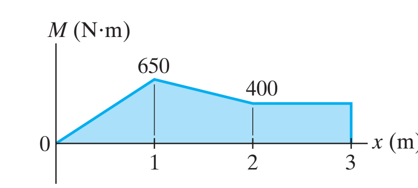
(b)



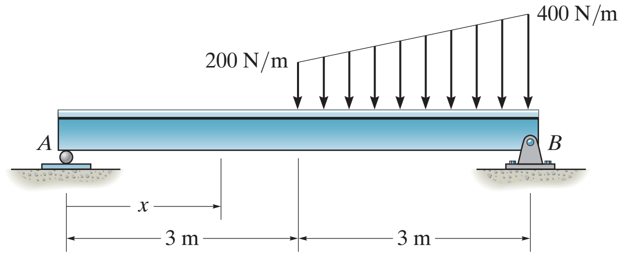
(c)

Ans:



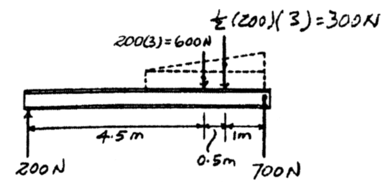


6–35. Draw the shear and moment diagrams for the beam and determine the shear and moment as functions of x.



Support Reactions: As shown on FBD.

Shear and Moment Functions:

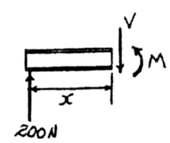


For $0 \leq x < 3$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - V = 0 \qquad V = 200 \text{ N}$$ **Ans.**

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M - 200\,x = 0$$

$$M = \{200\,x\} \text{ N} \cdot \text{m}$$ **Ans.**

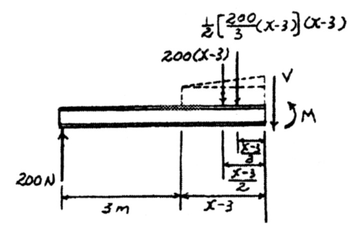


For $3 \text{ m} < x \leq 6$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - 200(x - 3) - \frac{1}{2}\left[\frac{200}{3}(x - 3)\right](x - 3) - V = 0$$

$$V = \left\{-\frac{100}{3}x^2 + 500\right\} \text{ N}$$ **Ans.**

Set $V = 0$, $x = 3.873$ m

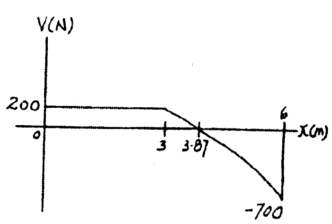


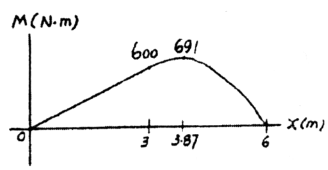
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left[\frac{200}{3}(x - 3)\right](x - 3)\left(\frac{x - 3}{3}\right)$$

$$+ 200(x - 3)\left(\frac{x - 3}{2}\right) - 200x = 0$$

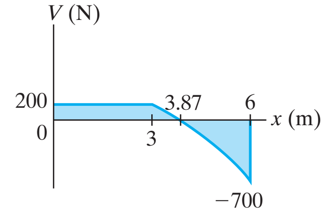
$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\} \text{ N} \cdot \text{m}$$ **Ans.**

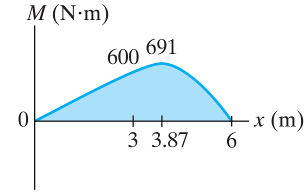
Substitute $x = 3.87$ m, $M = 691$ N·m



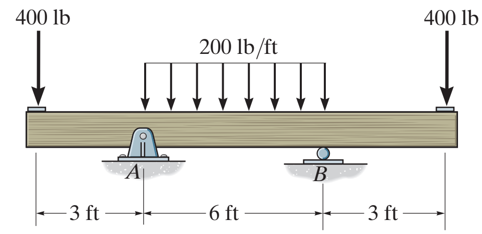


Ans:
For $0 \leq x < 3$ m: $V = 200$ N, $M = (200x)$ N·m,
For $3 \text{ m} < x \leq 6$ m: $V = \left\{-\frac{100}{3}x^2 + 500\right\}$ N,
$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}$ N·m





6–39. Draw the shear and moment diagrams for the double overhanging beam.



Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

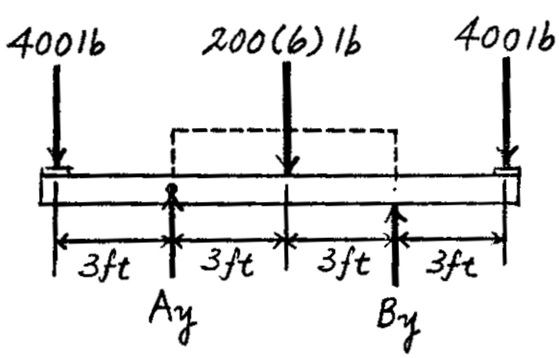
$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(6) + 400(3) - 200(6)(3) - 400(9) = 0$

$B_y = 1000 \text{ lb}$

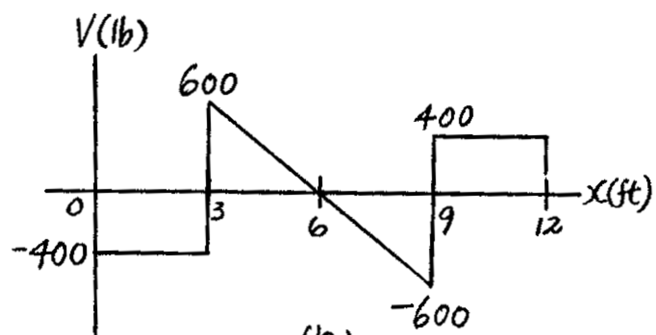
$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 1000 - 400 - 200(6) - 400 = 0$

$A_y = 1000 \text{ lb}$

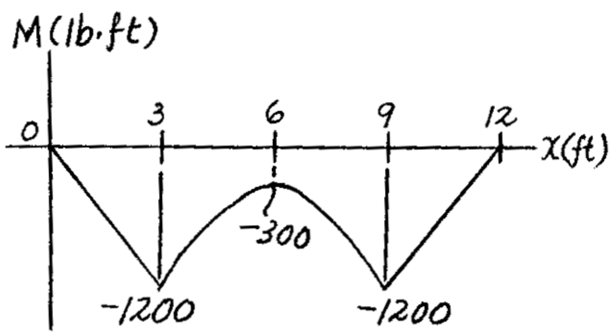
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

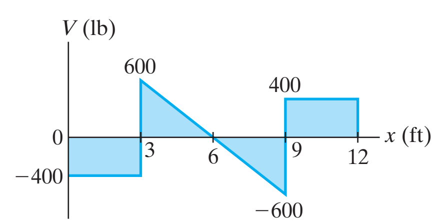


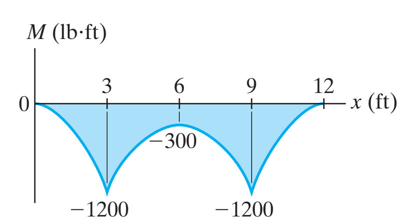
(b)



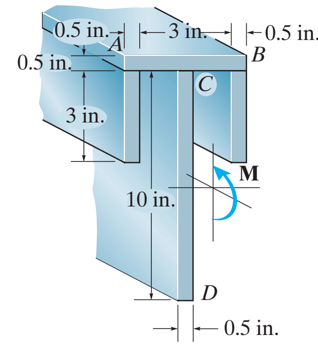
(c)

Ans:





6–49. Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 4\ \text{kip}\cdot\text{ft}$.



Section Properties:

$$\bar{y} = \frac{\Sigma\,\tilde{y}A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40\ \text{in.}$$

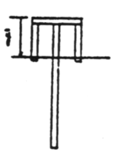
$$I_{NA} = \frac{1}{12}(4)(0.5^3) + 4(0.5)(3.40 - 0.25)^2 + 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right] + \frac{1}{12}(0.5)(10^3) + 0.5(10)(5.5 - 3.40)^2$$

$$= 91.73\ \text{in}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

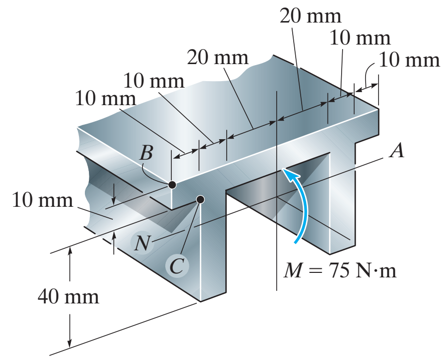
$$(\sigma_t)_{\max} = \frac{4(10^3)(12)(10.5 - 3.40)}{91.73} = 3715.12\ \text{psi} = 3.72\ \text{ksi}$$ **Ans.**

$$(\sigma_c)_{\max} = \frac{4(10^3)(12)(3.40)}{91.73} = 1779.07\ \text{psi} = 1.78\ \text{ksi}$$ **Ans.**



Ans:
$(\sigma_t)_{\max} = 3.72\ \text{ksi},\ (\sigma_c)_{\max} = 1.78\ \text{ksi}$

6–59. The aluminum machine part is subjected to a moment of $M = 75\ \text{kN}\cdot\text{m}$. Determine the maximum tensile and compressive bending stresses in the part.



$$\bar{y} = \frac{0.005(0.08)(0.01) + 2[0.03(0.04)(0.01)]}{0.08(0.01) + 2(0.04)(0.01)} = 0.0175\ \text{m}$$

$$I = \frac{1}{12}(0.08)(0.01^3) + 0.08(0.01)(0.0125^2)$$

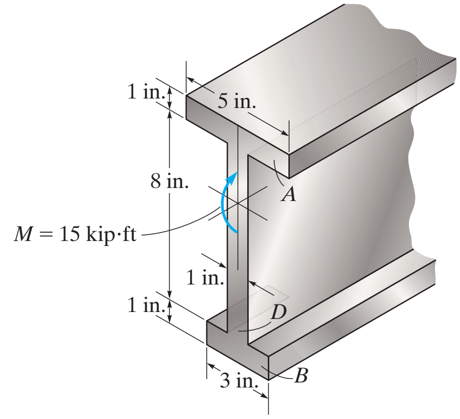
$$+ 2\left[\frac{1}{12}(0.01)(0.04^3) + 0.01(0.04)(0.0125^2)\right] = 0.3633(10^{-5})\ \text{m}^4$$

$$(\sigma_{\max})_t = \frac{Mc}{I} = \frac{75(0.050 - 0.0175)}{0.3633(10^{-6})} = 6.71\ \text{MPa}$$ **Ans.**

$$(\sigma_{\max})_c = \frac{My}{I} = \frac{75(0.0175)}{0.3633(10^{-6})} = 3.61\ \text{MPa}$$ **Ans.**

Ans:
$(\sigma_{\max})_t = 6.71$ MPa, $(\sigma_{\max})_c = 3.61$ MPa

6–61. The beam is subjected to a moment of 15 kip · ft. Determine the percentage of this moment that is resisted by the web D of the beam.

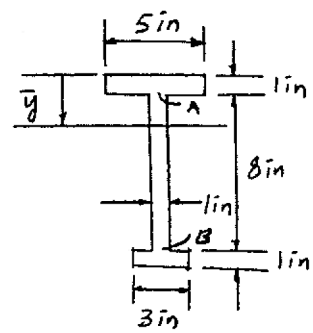


$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(1)(5) + 5(8)(1) + 9.5(3)(1)}{1(5) + 8(1) + 3(1)} = 4.4375 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(4.4375 - 0.5)^2 + \frac{1}{12}(1)(8^3) + 8(1)(5 - 4.4375)^2$$

$$+ \frac{1}{12}(3)(1^3) + 3(1)(9.5 - 4.4375)^2$$

$$= 200.27 \text{ in}^4$$



Using flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_A = \frac{15(12)(4.4375 - 1)}{200.27} = 3.0896 \text{ ksi}$$

$$\sigma_B = \frac{15(12)(9 - 4.4375)}{200.27} = 4.1007 \text{ ksi}$$

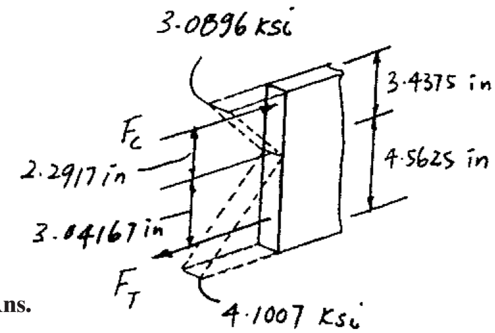
$$F_C = \frac{1}{2}(3.0896)(3.4375)(1) = 5.3102 \text{ kip}$$

$$F_T = \frac{1}{2}(4.1007)(4.5625)(1) = 9.3547 \text{ kip}$$

$$M = 5.3102(2.2917) + 9.3547(3.0417)$$

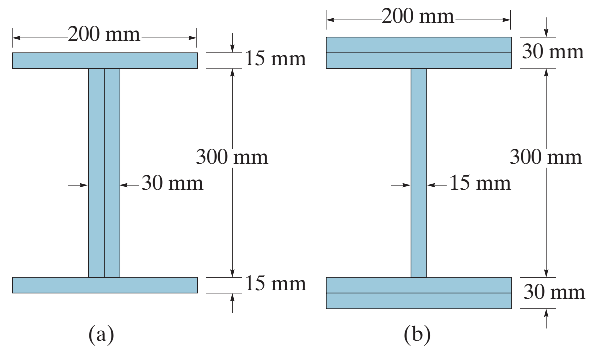
$$= 40.623 \text{ kip} \cdot \text{in.} = 3.3852 \text{ kip} \cdot \text{ft}$$

$$\% \text{ of moment carried by web} = \frac{3.3852}{15} \times 100 = 22.6\ \%$$ **Ans.**



Ans:
% of moment carried by web = 22.6 %

6–69. Two designs for a beam are to be considered. Determine which one will support a moment of $M = 150\text{ kN}\cdot\text{m}$ with the least amount of bending stress. What is that stress?



Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3)^3 = 0.21645(10^{-3})\text{ m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3})\text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\text{max}} = \dfrac{Mc}{I}$

For section (a)

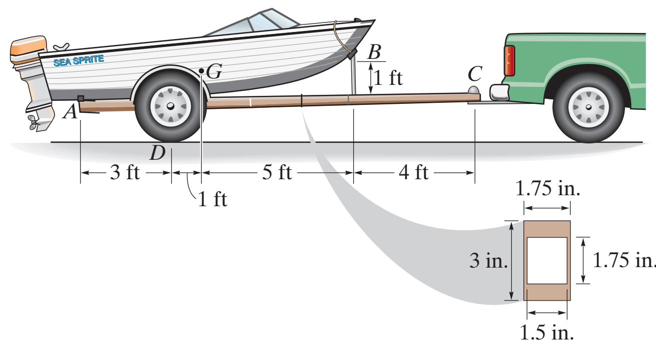
$$\sigma_{\text{max}} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3\text{ MPa}$$

For section (b)

$$\sigma_{\text{min}} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72\text{ MPa} = 74.7\text{ MPa}$$ **Ans.**

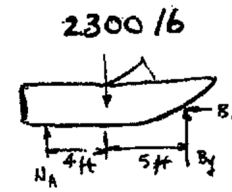
Ans:
$\sigma_{\text{min}} = 74.7\text{ MPa}$

6–71. The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.



Boat:

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$



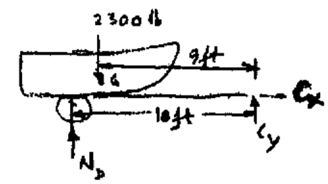
$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A(9) + 2300(5) = 0$

$N_A = 1277.78 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad 1277.78 - 2300 + B_y = 0$

$B_y = 1022.22 \text{ lb}$

Assembly:



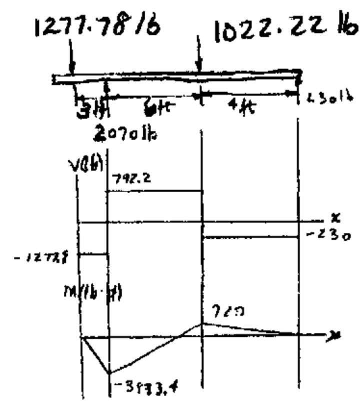
$\circlearrowleft + \Sigma M_C = 0; \qquad -N_D(10) + 2300(9) = 0$

$N_D = 2070 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad C_y + 2070 - 2300 = 0$

$C_y = 230 \text{ lb}$

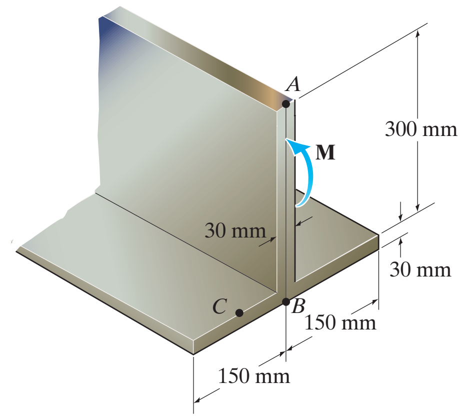
$$I = \frac{1}{12}(1.75)(3)^3 - \frac{1}{12}(1.5)(1.75)^3 = 3.2676 \text{ in}^4$$



$$\sigma_{max} = \frac{Mc}{I} = \frac{3833.3(12)(1.5)}{3.2676} = 21.1 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_{max} = 21.1$ ksi

6–81. If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.03)(0.3) + 0.18(0.3)(0.03)}{0.03(0.3) + 0.3(0.03)} = 0.0975 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.03^3) + 0.3(0.03)(0.0975 - 0.015)^2 + \frac{1}{12}(0.03)(0.3^3)$$

$$+ 0.03(0.3)(0.18 - 0.0975)^2$$

$$= 0.1907(10^{-3})\,\text{m}^4$$

Allowable Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section. For the top-most fiber,

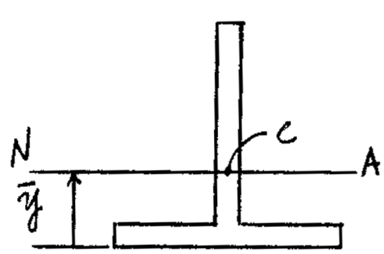
$$(\sigma_{\text{allow}})_c = \frac{Mc}{I} \qquad 150(10^6) = \frac{M(0.33 - 0.0975)}{0.1907(10^{-3})}$$

$$M = 123024.19 \text{ N}\cdot\text{m} = 123 \text{ kN}\cdot\text{m (controls)}$$ **Ans.**

For the bottom-most fiber,

$$(\sigma_{\text{allow}})_t = \frac{My}{I} \qquad 125(10^6) = \frac{M(0.0975)}{0.1907(10^{-3})}$$

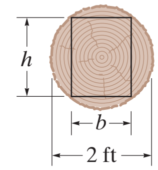
$$M = 244\,471.15 \text{ N}\cdot\text{m} = 244 \text{ kN}\cdot\text{m}$$

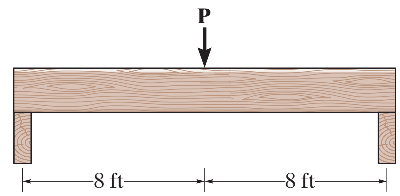


(a)

Ans:
$M = 123$ kN·m

6–97. A log that is 2 ft in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 8$ ksi, determine the largest load P that can be supported if the width of the beam is $b = 8$ in.





$$24^2 = h^2 + 8^2$$

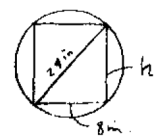
$$h = 22.63 \text{ in.}$$

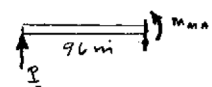
$$M_{max} = \frac{P}{2}(96) = 48\,P$$

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$8(10^3) = \frac{48P\left(\frac{22.63}{2}\right)}{\frac{1}{12}(8)(22.63)^3}$$

$$P = 114 \text{ kip}$$ **Ans.**





Ans:
$P = 114$ kip