The last topic covered in Ch. 5 deals with unsteady state and unsteady flow (USUF) problems, such as those encountered when filling/emptying a tank or when the CV properties change with time. The basic principles for such processes are conservation of mass:

\[ m_2 - m_1 = m_{\text{in}} - m_{\text{out}} \]  

and conservation of energy (1st law):

\[ m_2 \dot{u}_2 - m_1 \dot{u}_1 = Q_{1-2} - W_{1-2} \]

\[ + \int_1^2 (\dot{m}_{\text{in}} h_{\text{in}} - \dot{m}_{\text{out}} h_{\text{out}}) \, dt \]  

Note that these formulas are applied on a process basis (state 1 to 2) as opposed to the rate basis used in SSSF problems. Subscripts 1 and 2 refer to properties of the CV at states 1 and 2, and \( m_{\text{in}} \) and \( m_{\text{out}} \) refer to the properties of the mass entering and leaving the CV during the process.

Equation (2) can be evaluated only when the \( \dot{m}_{\text{in}} h \) quantities are some known function of the process path from 1 to 2. In general, the mass flow rates and the properties of the fluids entering and leaving the CV will change with time. In certain situations, however, \( h_{\text{in}} \) and/or \( h_{\text{out}} \) remain constant during the process. For such cases, energy conservation becomes

\[ m_2 \dot{u}_2 - m_1 \dot{u}_1 = Q_{1-2} - W_{1-2} \]

\[ + m_{\text{in}} h_{\text{in}} - m_{\text{out}} h_{\text{out}} \]  

All of the assigned problems correspond to this special situation.

**Exercises**

1. An insulated rigid tank, of volume \( V = 0.1 \) m\(^3\), is initially empty. The tank is connected to a supply line of air at 2 MPa and 300 K by a valve. The tank is allowed to fill, and the process ends when the pressure in the tank reaches the supply line pressure. Calculate the final temperature of the air in the tank and the mass of air that entered.

   The initial mass is zero, and \( m_2 = m_{\text{in}} \). The first law has

   \[ u_2 = h_{\text{in}} \]

   From the tables, \( h_{\text{in}} = 300.2 \) kJ/kg = \( u_2(T_2) \). Interpolation gives \( T_2 = 419 \) K. An approximation is to assume constant specific heats, so

   \[ T_2 = \frac{C_p}{C_v} T_{\text{in}} = 1.4 \cdot 300 = 420 \) K

2. Repeat problem 1, except now assume that the tank initially contains air at 300 K and 1 atm pressure.

   There is no work and the system is adiabatic, so

   \[ m_2 \dot{u}_2 - m_1 \dot{u}_1 = (m_2 - m_1) h_{\text{in}} \]

   Using

   \[ m_1 = \frac{P_1 V}{RT_1}, \quad m_2 = \frac{P_2 V}{RT_2} \]

   then

   \[ \frac{P_2 u_2}{T_2} - \frac{P_1 u_1}{T_1} = \left( \frac{P_2}{T_2} \frac{P_1}{T_1} \right) h_{\text{in}} \]

   or

   \[ u_2 = h_{\text{in}} - \frac{P_1 T_2}{P_2 T_1} (h_{\text{in}} - u_1) \]

   The procedure for solving this equation using air table data is to 1) guess a \( T_2 \); 2) use the equation to calculate \( u_2 \); 3) use the \( u_2 \) in the tables to get a new \( T_2 \); and 4) go to 2) and repeat until the solution converges. I started with \( T_2 = 400 \) K and got \( T_2 = 411 \) K after a couple of iterations. An approximation is to use the relations

   \[ h = C_p T, \quad u = C_v T \]

   with \( T \) in K; this results in

   \[ T_2 = \frac{\gamma P_2 T_1 T_{\text{in}}}{P_2 T_1 + P_1 (\gamma T_{\text{in}} - T_1)} = 412 \] K

   in which \( \gamma = C_p/C_v = 1.4 \)

3. Consider the case in problem 2, except now the tank does not have insulation, and is immersed in a water bath which maintains the system at 300 K. Calculate the total heat transfer from the tank during the process and the total mass in the tank at the end of the process.

   The process has \( T_1 = T_2 = T_{\text{in}} = 300 \) K. The masses are

   \[ m_1 = \frac{P_1 V}{RT_1} = 0.118 \) kg, \( m_2 = \frac{P_2 V}{RT_2} = 2.323 \) kg

   and

   \[ Q_{1-2} = m_2 u_2 - m_1 u_1 - (m_2 - m_1) h_{\text{in}} \]

   and since \( u_2 = u_1 = u_{\text{in}} = h_{\text{in}} - RT_{\text{in}} \) for this problem,

   \[ Q_{1-2} = -(m_2 - m_1) RT_{\text{in}} = -190 \) kJ

   Note that the heat transfer for this isothermal, constant volume, ideal gas case is

   \[ Q_{1-2} = -(P_2 - P_1) V \]

   which is equivalent to the flow work required to push the gas into the tank.

4. A pressure cooker consists of a pot with a tight–fitting lid. The function of the device is to boil food at an elevated yet constant pressure; the increased pressure results in a higher boiling temperature (and hence faster cooking), yet also maintains the constant–temperature advantages of boiling and thus reduces the chance of burning the food. The lid is equipped with a pressure–regulating valve, which bleeds off steam to maintain the constant pressure. Consider a pressure cooker of volume \( V = 4 \) L, which initially contains 2 L of water at 25°C and 1 atm. The lid is sealed and the pot heated. At a pressure of
5. An insulated rigid tank, of volume 0.1 m³, initially contains 1 kg of water at 20°C. The tank is connected to a supply line containing superheated steam at 10 MPa. The tank is filled, and when the pressure in the tank reaches the line pressure the water in the tank becomes a single phase. Find the temperature of the supply line steam.

The initial state of the water has

\[ u_1 = 105 \text{ kJ/kg}, \quad v_1 = 0.0010 \text{ m}^3/\text{kg} \]

so

\[ m_1 = \frac{2 \times 10^{-3} \text{m}^3}{v_1} = 2 \text{ kg} \]

and at 300 kPa,

\[ u_f = 561.2 \text{ kJ/kg}, \quad u_g = 2543 \text{ kJ/kg} \]

\[ h_g = h_{out} = 2725 \text{ kJ/kg} \]

\[ v_f = 0.0011 \text{ m}^3/\text{kg}, \quad v_g = 0.606 \text{ m}^3/\text{kg} \]

The total volume at state 2 is 4 L (the volume of the pot), so \( v_2 = V/m_2 = 0.004 \text{ m}^3/\text{kg} \), and

\[ x_2 = \frac{v_2 - v_f}{v_g} = 0.0048 \]

\[ u_2 = u_f + x_2 u_g = 570.7 \text{ kJ/kg} \]

The heat transfer is

\[ Q_{1-2} = m_2 u_2 - m_1 u_1 - (m_2 - m_1) h_{out} = 1996 \text{ kJ} \]

6. A balloon, initially empty, is attached to a supply line containing air at 200 kPa and 300 K. The balloon is surrounded by a vacuum. The valve is opened, and the balloon inflates. During the process the pressure in the balloon is proportional to the diameter of the balloon. When the pressure in the balloon reaches 200 kPa the diameter of the balloon is 0.2 m.

(a) Assuming the process is adiabatic, calculate the total work done by the system, and the final temperature and mass of air in the balloon.

(b) The balloon is now allowed to cool to 300 K. What is the final diameter of the balloon?

This process is adiabatic yet there is work present. The air in the balloon undergoes a polytropic process with exponent \(-1/3\), because

\[ P \sim D \sim V^{1/3} \]

and using the polytropic process formula for work gives

\[ W_{1\rightarrow 2} = \frac{3}{4} (P_2 V_2 - P_1 V_1) = \frac{3}{4} m_2 R T_2 \]

Note that \( m_1 = V_1 = 0 \) for this problem. The first law reduces to:

\[ m_2 u_2 = -W_{1\rightarrow 2} + m_2 h_{in} \]

Using the formula for work and the constant specific heat assumption;

\[ \left( C_v + \frac{3R}{4} \right) T_2 = C_P T_{in} \]

or, using \( R = C_P - C_v \) and \( C_P = \gamma C_v \);

\[ T_2 = \frac{4\gamma}{1+3\gamma} T_{in} = 323.1 \text{ K} \]

As the gas cools from state 2 to 3 the system – which is now closed – undergoes the polytropic process. For an ideal gas, polytropic process with exponent \( n \);

\[ \frac{P_3}{P_2} = \left( \frac{T_3}{T_2} \right)^{n/(n-1)} = \left( \frac{V_2}{V_3} \right)^n \]

or

\[ P_3 = 200 \cdot \left( \frac{300}{323.1} \right)^{1/4} = 196.3 \text{ kPa} \]

and

\[ D_3 = D_2 \frac{P_3}{P_2} = 0.196 \text{ m} \]