The next topic in SSSF CV analysis will deal with devices that have multiple inlets and/or outlets: Sec. 4 in chapter 5 of the text. Mass conservation for such devices states that the net mass flow rate into the CV will equal the net mass flow out. Say the CV has \( N \) inlets and \( M \) outlets, then

\[
\sum_{\text{N inlets}} \dot{m}_{i,\text{in}} = \sum_{\text{M outlets}} \dot{m}_{i,\text{out}}
\]

Neglecting KE and PE, the first law for this device will appear as

\[
\dot{Q} - \dot{W} = \sum_{\text{M outlets}} \dot{m}_{i,\text{out}} h_{i,\text{out}} - \sum_{\text{N inlets}} \dot{m}_{i,\text{in}} h_{i,\text{in}}
\]

The common devices in this category are mixing chambers and two fluid heat exchangers.

A mixing chamber typically takes two streams and mixes them together. The device, as a whole, can be considered adiabatic. There is no CV work, and kinetic energies can be neglected. The entire device is also typically taken to operate at a constant pressure, so that \( P_1 = P_2 = P_3 \). Mass conservation has

\[
\dot{m}_3 = \dot{m}_1 + \dot{m}_2
\]

and the first law is

\[
\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2
\]

A two fluid heat exchanger transfers heat from a "hot" to a "cold" fluid. The device as a whole is adiabatic: all the heat transfer occurs within the CV and no heat transfer crosses the CV boundary. There is no CV work, and KE is usually neglected. The hot and cold fluids can be different substances and they do not mix. Usually there is negligible pressure change across each fluid, i.e., \( P_{1h} = P_{2h}, \; P_{1c} = P_{2c} \). The first law will appear as

\[
\dot{m}_h (h_{2h} - h_{1h}) + \dot{m}_c (h_{2c} - h_{1c}) = 0
\]

Note that the terms for the hot and cold streams each represent the net heat transfer to/from the particular stream, so that

\[
\dot{Q}_h + \dot{Q}_c = 0
\]

That is, the heat transfer from the hot stream is equal to the heat transfer to the cold stream.

**Exercises**

1. A mass flow of 10 kg/s of compressed liquid water at 2 MPa, 40°C enters a mixing chamber, and is mixed with a flow of superheated steam at 2 MPa, 300°C. The mixed flow exits as a saturated liquid at 2 MPa. Calculate the mass flow rate of the superheated steam flow.

Let states 1 and 2 denote in compressed liquid and SH steam inlets. From the tables,

\[
h_1 = 169 \text{ kJ/kg}, \quad h_2 = 3024 \text{ kJ/kg}, \quad h_3 = 909 \text{ kJ/kg}
\]

and using the first law for the mixing chamber,

\[
\dot{m}_2 = \dot{m}_1 \frac{h_3 - h_1}{h_2 - h_3} = 3.50 \text{ kg/s}
\]

2. A volumetric flow of 0.4 m³/s of air at 400°C is adiabatically mixed with 1.2 m³/s at 10°C. Find the volumetric flow rate and temperature of the mixed stream. All streams are at 1 atm pressure.

\[
\dot{V}_1 = \frac{P \dot{V}_1}{RT_1} = 0.210 \text{ kg/s}, \quad \dot{m}_2 = 1.50 \text{ kg/s}
\]

and \( \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 1.71 \text{ kg/s} \). The outlet enthalpy is

\[
h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_1 + \dot{m}_2}
\]

Assume constant specific heat, so

\[
h(T) - h(T_{ref}) = C_p(T - T_{ref})
\]

where \( T_{ref} \) is some reference temperature. It does not matter what this temperature is, because it will cancel out in the end result (you should prove this). The outlet temperature is

\[
T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} = 331 \text{ K}
\]

and

\[
\dot{V}_3 = \frac{\dot{m}_3 RT_3}{P} = 1.6000 \text{ m³/s}
\]

The result suggests that volumetric flow rates are additive. It turns out that for the ideal gas with constant specific heats, the volumetric flow rates in a constant—\( P \) mixing chamber will be additive, i.e., the net volumetric flow out is the net flow in. To show this, rearrange the formula for the outlet temperature, and multiply by \( R/P \):

\[
\frac{\dot{m}_3 RT_3}{P} = \frac{\dot{m}_1 RT_1}{P} + \frac{\dot{m}_2 RT_2}{P} \rightarrow \dot{V}_3 = \dot{V}_1 + \dot{V}_2
\]

Remember that – unlike mass conservation – this equation does not hold for all situations.

3. Air at 1500 K is to be used to vaporize a flow of water in a two-fluid heat exchanger. The water enters as a saturated liquid at 2 MPa, and leaves as a saturated vapor at the same pressure. The air exits at a temperature 50°C above the water inlet temperature. Calculate the required air-to-water mass flow ratio \( \dot{m}_a/\dot{m}_w \).

Water properties are

\[
h_{1c} = 909 \text{ kJ/kg}, \quad T_{1c} = 212°C, \quad h_{2c} = 2800 \text{ kJ/kg}
\]

The air exit temperature is 262°C = 535 K. Enthalpy values are

\[
h_{1h} = 1637 \text{ kJ/kg}, \quad h_{2h} = 541 \text{ kJ/kg}
\]

and, from the first law,

\[
\frac{\dot{m}_a}{\dot{m}_w} = \frac{h_{2c} - h_{1c}}{h_{1h} - h_{2h}} = 1.73
\]
4. River water at 20°C is used to cool the condenser in a steam power plant. The steam enters the condenser as a saturated vapor at 50 kPa and exits as a saturated liquid at the same pressure. The mass flow rate of the steam is 20 kg/s. Calculate the mass flow of river water needed to condense the steam, given the constraint that the maximum temperature rise of the river water is 5°C. You can use a specific heat relation to calculate the change in enthalpy of the river water: \( \Delta h_c = C \Delta T_c \), with \( C = 4.19 \text{ kJ/kg K} \).

Same basic approach as the previous problem.

5. A flow of 2 kg/s of saturated liquid R-134a at 200 kPa is mixed with 3 kg/s of R-134a at 200 kPa and 10°C. The liquid and vapor phases of the mixture are separated, and each exit separately at 200 kPa. Find the mass flow rates of the exiting liquid and vapor streams. Note that this problem involves two exits and two inlets.

Here is the general approach: All the states are fixed: 1 and 2 are given, and the two exits (3 and 4) correspond to saturated liquid and vapor at 200 kPa, respectively. Let \( \dot{m}_T = \dot{m}_1 + \dot{m}_2 \) be the total mass flow rate into the CV, then

\[
\dot{m}_4 = \dot{m}_T - \dot{m}_3
\]

and from the first law,

\[
\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_4 h_4
\]

Combine the previous two equations and solve for \( \dot{m}_3 \).