MECH 7710
David M. Bevly                                                                                    Due: 4/28/15

# MECH 7710 Homework Assignment #5

*(LQG Regulation)*

Please limit the number of plots. I ask a lot of questions but you can simply state the answer. Show the "important" plots to demonstrate your estimator or controller is working. I would simulate systems for about 5-10 time constants (which will depend on you controller). This will allow you to see the transient response of your controller as well has it behavior at steady state with the disturbances acting on it.

1. Lets consider a simple $1/s^2$ plant (i.e. a frictionless cart with m=10 kg). We are going to create an LQG (linear quadratic guassian) compensator which is simply a Kalman filter combined with a LQR controller. Lets say we can measure position ($\sigma$=0.1 m). The process disturbance on the system is a Force with statistics ~ N(0,1.0]).
   a) Simulate the system with an LQR controller (select $R_{xx}=C^TC$ – i.e. place the roots on the symmetric root locus) and a Kalman Filter. Start the System from rest 10 m away from the regulated position (0 m).
   b) Where are the poles of your estimator? Where are the poles of your controller? Calculate the equivalent combined compensator and comment on what it looks like. What is the effect of changing $R_{uu}$ (look at control effort, position error, time response, etc.)
   c) Change $R_{uu}$ to move the poles outside of the Kalman Filter poles. Repeat parts (a) and (b). What affect does this have on your response. How much controller authority do you use?
   d) Now let $R_{xx}$=I . Repeat parts (a) and (b) What is the affect of increasing diagonal term of $R_{xx}$ associated with the second state (i.e. $R_{xx}[2,2]$ - look at the poles and control response). Does this make intuitive sense?
   e) Lets say the process disturbance has the characteristics N~(5.0, 1.0). Redo part (a). What is the steady state error?
   f) Now lets augment the State equations to incorporate the bias. Redo part (e). What is the effect of augmenting the state equations (vs. part e). Note the effect of adding the bias state on the compensator pole locations. What is the steady state error? What is this very equivalent to doing in classical designs.

2. Lets consider the same system as in problem #1, but with position and velocity measurements. The velocity sensor noise is given as $\sigma$=0.03 m/sec and the position measurement remains $\sigma$=0.1. The process disturbance on the system is again a Force with statistics ~ N(0,1.0). Notice that we now use the velocity measurement as a "sensor" as opposed to an "input" as we did in problem #3 on HW #6. This is because for full state feedback we must estimate velocity – therefore velocity is now a state.

a) Simulate the system with an LQR controller and a Kalman Filter. Start the System from rest 10 m away from the regulated position (0 m). How has the velocity sensor affected our compensator and estimator. Do we get better performance.
b) Calculate the equivalent combined compensator and comment on what it looks like.
c) Now add a bias with statistics ~ $N[1.0,0.01)^2]$ on the position sensor. Redo part (a). How well do you estimate the position bias? Why?
d) Now lets place the bias in part (c) on the velocity sensor (no bias on the position sensor). Redo part (a) – but don't estimate the velocity bias (i.e. we don't know the sensor is biased). What is the mean position error?
e) Now lets estimate the velocity sensor bias and Redo part (d). Does the mean position error go to zero now? Note similarity between the effect of the sensor bias and sensor bias estimate with the force disturbance and force disturbance estimate in problem #1 parts (e) and (f).
f) Now lets assume the bias on the velocity sensor is a 1st order Markov process with a time constant of 10 seconds driven by white noise ($\sigma$=0.2). Make sure to properly augment the state equations. Redo part (c). Show how well you can estimate/track the bias.

3. (Reference Scaling). Consider the inverted pendulum control problem where the pendulum is driven by a motor. The equation of motion, linearized about inversion, is:

$$ml^2\ddot{\theta} + b\dot{\theta} - mgl\theta = T_m$$

The mass is 5 kg, the length is 1 meter and the damping is 3 N-m-s. Assuming full state feedback (don't design and estimator). Design a LQR controller for the system (use the symmetric root locus to show where your design will lie)

a) Start the pendulum at some initial offset angle and show the controller performs as expected
b) Now start the pendulum inverted and command a reference of 20 degrees. What is the steady state error
c) Now design a reference matrix for the system. Repeat part (b)

# Possible Project Idea:

Cart and Inverted Pendulum. Our measurements for this system are cart position ($\sigma$=0.005 m) and pendulum angle ($\sigma$=0.01 rad). The equations of a cart and inverted pendulum are defined by (see FPE p.32):

$$\left(I + m_p l^2\right)\ddot{\theta} - m_p g l \sin(\theta) = m_p l \ddot{x} \cos(\theta)$$
$$\left(m_c + m_p\right)\ddot{x} + b\dot{x} - m_p l \ddot{\theta}\cos(\theta) + m_p l \dot{\theta}^2 \sin(\theta) = u$$

Linearizing the equations about $\theta$=0:

$$\left(I + m_p l^2\right)\ddot{\theta} - m_p g l \theta = m_p l \ddot{x}$$
$$\left(m_c + m_p\right)\ddot{x} + b\dot{x} - m_p l \ddot{\theta} = u$$

Accounting for the actuator (motor) dynamics and utilizing the parameters for the pendulum in the lab we get the following state space model (A, B, and C):

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.4 & -9.1 & 0 \\ 0 & 40.1 & 29.8 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2.03 \\ -6.66 \end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

where $x$ is in meters, $\theta$ is in radians, and $V$ is the voltage from simulink to the motors. There are several ways to size $R_{xx}$. One is $R_{xx}=C^T C$ and the other is Bryson's Rule. For Bryson's rule we will choose $R_{xx}$ by:

$$R_{xx} = C^T \begin{bmatrix} \frac{1}{\left(y_{\max}^1\right)^2} & & \\ & \ddots & \\ & & \frac{1}{\left(y_{\max}^m\right)^2} \end{bmatrix} C$$

or:

$$R_{xx} = \begin{bmatrix} \frac{1}{\left(x_{\max}^1\right)^2} & & \\ & \ddots & \\ & & \frac{1}{\left(x_{\max}^l\right)^2} \end{bmatrix}$$

and $R_{uu}$ by:

$$R_{uu} = \rho \begin{bmatrix} \frac{1}{\left(u_{\max}^1\right)^2} & & \\ & \ddots & \\ & & \frac{1}{\left(u_{\max}^n\right)^2} \end{bmatrix}$$

where ρ is a "tweaking" knob, $y_{max}$ or $x_{max}$ are the maximum desired error in each output or state, and $u_{max}$ is the maximum control effort. Again note this is very similar to how we "hand tuned" our $Q_d$ in the Kalman filter. However with the Kalman filter the assumption is we are tuning to try and match the input disturbance characteristics (that we have no control over) and with the LQR controller we are tuning parameters to give us the output response we desire. For example on the cart and pendulum we may say we want to control pendulum to 0.02 radians and the cart to 3 cm with a maximum control authority of 7 volts. This gives:

$$R_{xx} = \begin{bmatrix} \frac{1}{(0.03)^2} & & & \\ & \frac{1}{(0.02)^2} & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \text{ and } R_{uu} = \rho\left[\tfrac{1}{49}\right]$$

This gives a possible starting point. However we have seen in problem #1 how we can modify the response by hand tweaking $R_{xx}$.

d) Design an LQR compensator and Kalman Filter for this system.

e) First simulate with no actual process disturbance or sensor noise on the system. Look at the actual states – does this look realistic why or why not. Start the system at $X_0$=0.1 meters and $\theta_o$=0.1 radians.

f) Now add the sensor noise and some "realistic" process noise. There should be two separate disturbances - one to the cart and one to the pendulum. To get an approximate idea for the sizes of these you may want to simulate varying sizes of disturbances on the cart with the pendulum hanging downward. Pick values that seem to disturb the system in a reasonable amount. To simulate the cart with the pendulum hanging downward, simply change A to:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.4 & -9.1 & 0 \\ 0 & -40.1 & 29.8 & 0 \end{bmatrix}$$

Design your compensator and estimator for the system with the disturbance parameters you have chosen. Be sure and check actuator saturation (limit V input to $\pm$10 volts).

g) How Robust is your controller to "stronger" disturbances?

h) Show how you would augment to the state equations to track a disturbance acting on the pendulum if the disturbance had a correlation time of 5 seconds and a nominal value of 0.1 Newtons (1σ). Be sure and calculate the augment components completely.

i) Show how you might augment the State Equations to include neglected input lag:

$$\frac{V}{V_{commanded}} = \frac{\alpha}{s+\alpha}$$