MECH 7710 February 16, 2011
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MECH 7710 Homework Assignment #3

1. Repeat problem #4 on HW#2. Convince yourself that the Least Squares solution you developed is the same as the solution given from arx.m. (What are the ARX inputs to recover the least squares solution you developed in HW#2?)
   a) Now crank up the sensor noise to σ=1.0. Try using higher order ARX fits? Can you identify the model?
   b) What about with another model form? Which model form worked best? How good is the fit (provide plots for proof) What was the order of the fit?
   c) Now color the white noise (σ=1.0) with the following shaping filter:

$$H(z) = \frac{z^2 - 1.6z + 0.75}{z^2 - 1.86z + 0.985}$$

   Now add the colored noise to the measurements and repeat part (b). How well can you I.D. the shaping filter?

2. Download the data ***hw3_data*** from the website. The data is from excitation of a cart and dual pendulum system and is in the form ***[time input ouput]***. (collected at 50 Hz). The input is a voltage to drive the cart and the output is the encoder of the first pendulum (attached to the cart).
   a) Compute the output and input periodograms $\Phi_y(\omega)$ and $\Phi_u(\omega)$ (plot smoothed version of the periodograms). Was a good input used?
   b) Compute the periodogram $\hat{G}(\omega) = \frac{\Phi_y}{\Phi_u}$ of the transfer function.

   Show a sufficient number of plots to show how you picked the final smoothing value for the transfer function. What are the approximate locations of the poles and zeros?
   c) Using the SISO techniques (ARX, ARMAX, OE, BJ) – produce a model of the data. Compare the model TF’s to the smoothed TF’s (from part b) by plotting the two on the same graph.
   d) Use the validation tools discussed in class and discuss your confidence in the model (provide plot(s) to backup your arguments).

3. Download the data ***hw3_3_mod*** from the website. The data is in the form [***t*** ***h***]. Fit the data using least-squares assuming *h(t)* is a polynomial of the form:

$$h(t) = a_0 + a_1 t + \cdots + a_n t^n$$

Determine the order ***n*** of the best fit. Use ***hw3_3_val*** (validation data) to justify your results. (DO NOT TRY AND FIT THE VALIDATION DATA – IT IS FOR VALIDATION ONLY!) Plot your model against both data sets. Also plot the rms of the residuals for each fit from ***n***=0 to ***n***=5 on both the model data and the validation data (explain why the curves look like they do). What order did you use (why) and what were the resulting coefficients in the polynomial? You can assume ***n***≤5. This simple example is used to show that you should always use at least 2 sets of data – one to provide the model and one for validation of that model.

4 .Basic Optimal Estimation. Consider estimating a static, scalar variable *x*. We take two noisy measurements of the scalar *x* of the from:

$$z_i = h_i x + \theta_i$$

where ***i*** ranges from 1 to 2. Also h_1=1 and h_2=2. Assume that the noise terms θ_i are uncorrelated and have statistics $E[\theta_i]=0$ and $E[\theta_i^2]=\sigma_i^2$. Also assume that $E[x\theta_i]=0$, but little else is known about ***x***.

a) Assume an optimal estimate of the form:

$$\hat{x} = l_1 z_1 + l_2 z_2$$

where l_i are constants that are statistically independent of ***x***. Compute the values of l_i that

i. Ensure the estimate $\hat{x}$ is unbiased

ii. Minimizes the mean square estimation error $E[(x-\hat{x})^2]$.

Use these gains to compute the minimum value of the mean square estimation error. Hint: the minimization can be performed with a simple optimization with respect to the estimator gains.

b) Discuss how the optimal estimator uses the two measurements for the following cases:

i. $\sigma_2 >> \sigma_1$

ii. $\sigma_2 = \sigma_1$

iii. $\sigma_2 << \sigma_1$

c) Determine the weighted least-squares estimate for this problem. Make sure that you pick the weighting matrix appropriately. Compare the gains of the WLSE with those from part (a) and (b).