MECH 7710 January 26, 2011
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Homework Assignment #1

1. Use the MATLAB convolve function to produce discrete probability functions (PDF's) for throws of six dice as follows (note: this is effectively the sum of 6 random variables)
   a) 6 numbered 1,2,3,4,5,6
   b) 6 numbered 4,5,6,7,8,9
   c) 6 numbered 1,1,3,3,3,5
   d) 3 numbered 1,2,3,4,5,6 and 3 numbered 1,1,3,3,3,5

   Check that the ΣPDF = 1.0
   Plot each PDF with a normal distribution plot of same average and sigma.

   Note that even peculiar random distributions, taken in aggregate, tend to produce "normal" error distributions

2. What is the joint PDF for 2 fair dice (x_1,x_2) (make this a 6x6 matrix with the indices equal to the values of the random variables). Note each row should add to the probability of the index for x_1 and each column to the probability of the index for x_2

   a) What are $E(X_1)$ (*called the mean*), $E(X_1-E(X_1))$, $E(X_1^2)$ (*called mean squared value*), $E((X_1-E(X_1))^2)$ (*called the variance*) and $E(((X_1-E(X_1))*(X_2-E(X_2)))$ (*called the covariance*)

   b) Form the covariance matrix for x_1 and x_2 (a 2x2 matrix- we will frequently use this, it is called P. It's transformation with time, and through the reactions of dynamics is the heart of this course).

   Note: this is a manual pain, but by defining vectors with the values of the random variables and using the "dot" multiply and "sum" functions it is trivial. Something like: sum(PDF*(V_2.*V_1)).

   c) Now find the PDF matrix for the variables v_1=x_1 and v_2=x_1+x_2.
      Note: Use [1:6, 1:12], with the first column zeros.
   d) Now what is the mean, $E(v_1-E(v_1))$, rms, and variance of v_1
   e) What is the mean, $E(v_2-E(v_2))$, rms and variance of v_2
   f) What is the new covariance matrix P.

3. Two random vectors $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are called uncorrelated if

$$P = E\left\{(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)^T\right\} = 0$$

   Show that: a) Independent random vectors are uncorrelated
   b) Uncorrelated Gaussian random vectors are independent

4. Consider a sequence created by throwing a pair of dice and summing the numbers which are {-2.5, -1.5, -0.5, 0.5, 1.5, 2.5}. Call this $V_o(k)$. What is the PDF? (*you can set this up as a pair of vectors in MATLAB*).

   a) What are the mean and variance of this sequence?

   If we generate a new random sequence as:

   $V_N(k+1) = (1-r)*V_N(k)+r*V_o(k)$, $V_N(k)$ is an example of serially-correlated (not white) noise.

   b) In steady state, what are the mean and variance of this new sequence (V_N)?

   c) What is the covariance function: $R_V(k) = E\{V_N(k)*V_N(k-L)\}$ (Hint: $V_N(k)$ and $V_o(k)$ are uncorrelated).

   d) Are there any practical constraints on ***r***?

5. A random variable ***x*** has a PDF given by:

$$PDF(x) = \begin{Bmatrix} 0 & x < 0 \\ x/2 & 0 \le x < 2 \\ 0 & x \ge 2 \end{Bmatrix}$$

   a) what is the mean of ***x***?

   b) what is the variance of ***x***?

6. Consider a ***normally*** distributed two-dimensional vector ***x***, with mean value zero and

$$P_X = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

   a) Find the eigenvalues of P_x

   b) The likelihood ellipses are given by an equation of the form: $x^T P_X^{-1} x = c^2$

      What are the principle axes in this case?

   c) Plot the likelihood ellipses for c=0.25, 1, 1.5

   d) What is the probability of finding ***x*** inside each of these ellipses?

7. If $x \sim N(0,\sigma_x^2)$ and $y=2x^2$, then:

   a) Find the appropriate output PDF

   b) Draw the input output PDF on the same plot for $\sigma_x=2.0$

   c) How has the density function changed by this transformation

   d) Is ***y*** a normal random variable?