1) Derive the equation for the longitudinal motion for the car:

\[ M_{\text{eff}} \ddot{x} + B_{\text{eff}} \dot{x} + D \dot{x}^2 = \frac{\tau_c N_i N_d}{R_w} - F_{\text{loss}} \]

At each shaft, place a viscous drag (loss proportional to rotational velocity) and a coulomb drag (constant “friction” loss). Solve for \( M_{\text{eff}}, B_{\text{eff}}, F_{\text{loss}} \)

2) Find data on a car you wish to simulate running the quarter mile. You will need to find information such as max engine torque, max horsepower, ¼ mile time and speed, vehicle mass, and transmission and rear-end gearing. Some magazines will provide a plot of the quarter mile run (velocity vs. time) which would be good for comparing with your simulations. Assume the following for the car (unless you can find information on these values as well):

- \( R_{\text{tire}} = 0.35 \text{ m} \)
- \( f_{\text{rr}} = 0.03 \)
- \( D = 0.3 \text{ Ns}^2/\text{m}^2 \)
- \( I_{\text{e}} = 0.4 \text{ kgm}^2 \)

We will neglect the inertia of the transmission, rear end, axles, and wheels (we can just assume these are small relative to the mass of the car and inertia of the engine, or lump them in with the engine!) Neglect traction effect – i.e. assume perfect traction (no slip). We will also assume “perfect” gearshifts. Think about how you could include shift delay and impact losses during shifting. Write down all of the information for your car (or turn in a copy of the data sheet for the vehicle).

a) Using your engine torque, engine horsepower data develop and equation for the engine torque as a function of engine speed. You will need to add points such as max torque and stall torque (This is fairly easy to do in excel – make sure that the trend line equation from excel has at least 3 decimal places. Plot the torque speed graph for the engine model in Matlab (including the your data points).

b) Assuming a constant engine torque (\( T_e \)) and assuming the air drag force is proportional to velocity, solve for the position of the car as a function of time. Why must we make these assumptions? How does the time constant change for each gear (in terms of the variables). You will probably need to pull out your Differential Equations Book/Notes.

c) Simulate the car running the quarter mile (with air drag proportional to velocity squared and your engine model) in each gear. What is the time constant for gears 1-3 (time it take the system to reach 62% of its final/steady-state value – the term “time constant” applies only to linear systems, however we can still use it as a measure to speed, time response, for this non-linear problem). How do the results compare to the theoretical prediction in part b. What is the quarter mile time for each gear? You should be able to show answers to each of these questions on one piece of paper with 6 plots!
d) Allowing shifting, minimize the quarter mile time? Where are the shift points?
   Again one sheet of paper with 3 plots should answer this part!

e) Scale the engine torque (by adding an engine efficiency constant as discussed in
class) and adjust values of $I_e, f_{rr}$, and/or $D$ to match time and speed from that of the
vehicle specs. Record your final parameters.

f) How sticky of tire (value of $\mu$) do you need to realize this quarter mile time
(neglect transverse weight transfer)?

g) Can you increase performance by changing the gearing? (This is how simulations
are used for design!)

Try and get in the habit of writing very clean and flexible code. Think about how to
“design” your code before you just starting typing! You should be able to write a fairly
piece of simulation code to do all parts of this problem and with a comment (“%”) or an
“if” statement vary which part of the problem you are doing.