MECH 4420 Lecture: Vehicle Dynamics Control

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Various Yaw Dynamic Models

**Bicycle Model**

\[
\frac{R(s)}{\delta(s)} = \frac{a C_{\alpha f} s + \frac{a C_{\alpha f} - c_1 C_{\alpha f}}{mV}}{I_z s^2 + \frac{c_0 I_z + mc_2}{mV} s + \left(\frac{c_0 c_2 - c_1 mV^2 - c_1^2}{mV^2}\right)}
\]

DC Gain:

\[
R_{ss} = \frac{V_x}{L + K_{US} V_x^2} \delta
\]

**Neutral Steer Model** \((K_{US} = 0)\)

\[
\frac{R(s)}{\delta(s)} = \frac{a C_{\alpha f}}{I_z s + \frac{c_2}{V}}
\]

**Kinematic Model**

\[
R = \frac{V_x}{L} \delta
\]
Classical (3140) Yaw Dynamic Control

• Placing the bicycle model transfer function into a classic block diagram format results in:

\[
\begin{align*}
K(s) &= \frac{a C_{\alpha f} - c_1 C_{\alpha f}}{mV} \\
\delta &= \frac{a C_{\alpha f} + \frac{a C_{\alpha f} - c_1 C_{\alpha f}}{mV}}{I_Z s^2 + \frac{c_0 I_Z + mc_2}{mV}s + \left(\frac{c_0 c_2 - c_1 mV^2 - c_1^2}{mV^2}\right)}
\end{align*}
\]
Classical (3140) Yaw Dynamic Control

• Where does desired yaw rate come from?
  – From knowledge of trajectory (and speed)
  – From an “ideal” bicycle model
  • Use measured steer angle and speed to determine desired yaw rate

• Design Methods
  – Classical Design (need good model)
    • Have to design in Bode or Root Locus
      – Why? Because of zero in the bicycle model TF
      – Prevents from being able to do coefficient matching
  – Hand-Tuning
    • Need access to the vehicle
Classical Control Options for $K(s)$

- **P**
  - Takes advantage of “free” zero (zero in the TF)
  - Closed loop $G_{DC}$ will not be one!
    - Need to add feedforward or pre-reference scaling
    - Don’t want zero steer angle when error is zero

- **PI (or Lag)**
  - Drive steady state error to zero (even with disturbances)

- **PD (or Lead)**
  - If desire better (faster, higher closed-loop bandwidth)
    - Note: Very likely actuator limited in achieving this through steering

- **PID (or Lead+Lag)**
  - If desire both of the PI and PD.
Root Locus with Proportional Control

- P controller using the G35 Specifications
  - Note that (limited) improvement in settle time and bandwidth does exist with P control

\[ K(s) = K \]
Root Locus with PI Control

• PI controller using the G35 specifications
  – Note that the time to steady state is decreased due to the additional integrator pole which will be dominant
  • But guarantees zero steady state error

\[ K(s) = K \frac{s + 10}{s} = K + \frac{10K}{s} \]

\[ K_p = K \]
\[ K_i = 10K \]
What about controlling Position

• Various Strategies:
  – Waypoint Following
    • Easier to implement and tune (by hand)
    • Requires fewer model parameters
    • One less integrator (i.e. lower order)
  – Line Tracking
    • Need good model
    • Requires more parameters/states
    • Harder to define error
    • Provides improved tracking
Waypoint (Heading Control)

- Ignore the position and drive to “waypoints”
  - Reduce the order of the dynamics by one
    - One integrator

- Error is defined to be the angle pointing to the waypoint
  - Difference in the angle between the vehicle and the waypoint
Waypoint (Heading) Control

• One way of doing vehicle position/later control is to drive to “waypoints.”
  – Feedback three measurements
    • Heading error (modified to scale gain with Vx)
    • Yaw rate
    • Steer angle
  – Some gains set to zero because of lack of reliable vehicle model
  – If gain is too large the vehicle becomes unstable
    • Models predict this result
  – Use a “look ahead” distance (blue semi-circle) to select which point to drive to
    • Increasing the look ahead distance adds damping and increases stability
    • Increasing the look ahead distance leads to more error as the vehicle will cut corners in tight turns

\[ x = [\psi \hspace{1em} r \hspace{1em} \delta]^T \]
• **Heading Error**

\[ \psi_{error} = \psi - \tan^{-1}\left(\frac{E_{des} - E}{N_{des} - N}\right) \]

• **Look Ahead Radius**

\[ R_{min} = 1 \text{ m} + \left(1 \frac{\text{m}}{\text{m/s}}\right) V \]
Waypoint (Heading) with P Control

- Proportional heading controller using the G35 Specifications
  - Stable with P only

\[ K(s) = K \]
Waypoint with PD Control

- PD heading controller using the G35 specifications
  - Note this is exactly the same root locus as PI on yaw dynamic control

\[ K(s) = K(s + 10) = Ks + 10K \]

- \( K_p = 10K \)
- \( K_D = K \)
Simplified Waypoint Control

• Waypoint control using a simplified vehicle model (1st order yaw dynamics) and ignoring steering dynamics, \( S(s) \)

Vehicle Model:
\[
\frac{r(s)}{\delta(s)} = \frac{G_{DC}}{\tau_r s + 1} \quad G_{DC} = \frac{V}{L + K_{US} V^2}
\]

PD Controller:
\[
\delta = K_\psi \psi_{\text{err}} + K_r \dot{\psi}_{\text{err}} = K_\psi \psi_{\text{err}} + K_r r
\]

Desired Closed Loop C.E.:
\[
\frac{\psi}{\psi_{\text{des}}} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Resulting Gains:
\[
K_\psi = \frac{\omega_n^2 \tau_r}{G_{DC}} \quad K_r = \frac{2\zeta \omega_n \tau_r - 1}{G_{DC}}
\]
Waypoint Following Results

- Test performed on ATV at Auburn
- Waypoint controller works o.k. with correct parameters
- The incorrect model results in instability
Further Improvements

• Driving to waypoints is not best control method
• Results in decreased tracking accuracy
  – Vehicle oscillates more
• In limited scenarios this caused instability on RASCAL

• Two errors are important
  • Heading
  • Lateral position
• Driving both errors to zero requires line-tracking controller
• Requires a more accurate model
Line Tracking

• Heading Error

\[
\psi_{error} = \psi - \tan^{-1}\left( \frac{E_{des} - E}{N_{des} - N} \right)
\]

• Lateral Error
  – Neglecting \( V_y \)

\[
\dot{y} = V \frac{X}{\sin(\psi)} \approx V \frac{X}{\psi}
\]
Lateral Position Dynamics

\[ V = \text{velocity} \]
\[ r = \text{yaw rate} \]
\[ \psi = \text{heading (or yaw)} \]
\[ \nu = \text{vehicle course} \]
\[ \delta = \text{steer angle} \]
\[ \beta = \text{body sideslip angle} \]
\[ \alpha = \text{tire sideslip angle} \]

- Lateral velocity is defined as:
  \[ \dot{y} = V \sin(\psi + \beta) = V_x \sin(\psi) + V_y \cos(\psi) \]

- Assuming small angles:
  \[ \dot{y} \approx V_x \psi + V_y \]
Full Yaw Vehicle Dynamics

• **Steering**
  – 1\(^{\text{st}}\) order
    \[ \frac{\delta}{u} = \frac{K_\delta}{\tau_\delta s + 1} \]
    • Neglecting motor dynamics \((\dot{\delta} = u)\)
  – 0\(^{\text{th}}\) order if neglect steering dynamics \((\delta = u)\)

• **Vehicle**
  – 2\(^{\text{nd}}\) Order
    \[ \frac{r}{\delta} = \frac{k_v(s + n_v)}{s^2 + 2\zeta_v\omega_vs + \omega_v^2} \]

• **Error**
  – Heading (1\(^{\text{st}}\) order) \(\dot{\psi} = r\)
  – Lateral Position (1\(^{\text{st}}\) Order) \(\dot{y} \approx V_x\psi + V_y\)
• Note that to put the lateral position into a transfer function, $V_y$ has to be neglected (or treated as a disturbance)
  – Recall TF only allow one input and one output
• Not the case for state space

$$y(s)s = V_x \psi(s) + V_y(s)$$
Control Strategies

• Classical (i.e. 3140)
  – Single Controller
    • This must be done in Root Locus or Bode
  – Cascaded
    • Break up the system into more manageable systems
      – Yaw dynamic loop still must be done with root locus or bode due to the zero
      – Inner and outer loops could be designed with coefficient matching, root locus, or bode
      – Hand-tune each loop

• State Feedback (we will get into this later)
  – Need Good Model for Design
  – Need Estimator (also requires good model)
Lateral Control Dynamics

- Single Controller Design
- Neglecting Lateral Velocity, the system can be represented as follows (\(V_y\) is treated as a disturbance):

\[
\begin{align*}
S(s) &= \frac{1}{s} \\
\psi &= \frac{V_x}{s} \\
y &= \frac{1}{s} \\
r &= \frac{1}{s} \\
\delta &= \frac{a C_{af} s + a C_{af} - c_I C_{af}}{m V} \\
P &= \frac{a C_{af} s + a C_{af} - c_I C_{af}}{m V} \\
\end{align*}
\]
Lateral Control Dynamics

• Can use any type of controller for $K(s)$
• Note the closed loop $G_{DC}=1$ for any $K(s)$
  • This is because the plant already had integrators (2 to be exact)
  • Therefore if $y_{ss}=y_{des}$ (if $y_{des}$ = constant) so $e_{ss}=0$
  • However might still need integrator for:
    – Changing $y_{des}$ (i.e. tracking curves)
    – Disturbances (banks, cross-wind, etc.)
Lateral Proportional Controller

- Lateral P controller using the G35 specifications and ignoring steering dynamics ($S(s)=1$)
  - Unstable with P only (for any value of $K$)
  - Therefore also unstable for PI
Lateral PD Control

- PD lateral controller using the G35 specifications and ignoring steering dynamics ($S(s)=1$)
  - Stable, but not that great of performance (settle time of 2 seconds)

\[ K(s) = K(s + 2) = Ks + 2K \]

\[ K_p = 2K \]
\[ K_D = K \]
Cascaded Approach

• Cascaded into 3 consecutive controllers
  – Inner loop: Steering Control
  – Middle Loop: Yaw Dynamics
  – Outer Loop: Position Control (waypoint or line)

• Each consecutive controller should be designed “slower” than the previous
  – Therefore previous “inner” dynamics can be neglected
  – Can use P, PI, and/or PID for any of the loops
Line Tracking of a Tractor with GPS

Mean = 5 mm  \( 1\sigma = 3 \text{ cm} \)