Ride Models

Full Car: Bounce, Pitch & Roll

- 14th order
- 7 DOF
  - 4 wheel ($Z_w$)
  - Roll ($\phi$)
  - Pitch ($\theta$)
  - Bounce ($Z_s$)
Ride Models

$\frac{1}{2} \text{ Car : Bounce & Pitch}$

- 8th order
- 4 DOF
  - 2 wheel ($Z_w$)
  - Pitch ($\theta$)
  - Bounce ($Z_s$)
Ride Models

\[ \frac{1}{4} \text{ Car} \quad \text{:} \quad \text{Bounce} \]

- 4th Order
- 2 DOF
  - wheel \((Z_u)\)
  - Car \((Z_s)\)
Ride Model

• Assuming SEP:

\[
\begin{bmatrix}
\ddot{z}_s \\
\dot{z}_s \\
\ddot{z}_u \\
\dot{z}_u
\end{bmatrix} = \begin{bmatrix}
\frac{-b_s}{m_s} & -\frac{k_s}{m_s} & \frac{b_s}{m_s} & \frac{k_s}{m_s} \\
1 & 0 & 0 & 0 \\
\frac{b_s}{m_u} & \frac{k_s}{m_u} & -(b_s + b_t) & -(k_s + k_t) \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\dot{z}_s \\
z_s \\
\dot{z}_u \\
z_u
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & b_s \\
0 & 0 & k_t & 0 \\
0 & 0 & m_u & m_u \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dot{z}_r \\
z_r \\
\dot{z}_r \\
z_r
\end{bmatrix}
\]

\[m_u \approx 0.1m_{total} \quad m_s \approx 0.9m_{total} \quad k_t \approx 10k_s\]

• Neglecting the tire damping:

\[
\frac{\ddot{z}_s}{z_s} = \frac{z_s}{z_s} = \frac{\frac{k_t b_s s + k_s k_t}{m_s m_u s^4 + b_s (m_s + m_u) s^3 + \{k_s m_u + (k_s + k_t) m_s\} s^2 + b_s k_t s + k_s k_t}}{m_s m_u s^4 + b_s (m_s + m_u) s^3 + \{k_s m_u + (k_s + k_t) m_s\} s^2 + b_s k_t s + k_s k_t}
\]
Ride Model

• Note:

\[ \omega_{n1} \neq \sqrt{\frac{k_s}{m_s}} \quad \text{and} \quad \omega_{n2} \neq \sqrt{\frac{k_t}{m_u}} \]

• However due to the values for a typical car:

\[ \omega_{n1} \approx \sqrt{\frac{k_s}{m_s}} \approx 6.2 \frac{\text{rad}}{s} \]

\[ \omega_{n2} \approx \sqrt{\frac{k_t}{m_u}} \approx 62 \frac{\text{rad}}{s} \]
Bode Plot of $z_S$

- **Small Suspension Damping**
- **Realistic Suspension Damping**

- Note $G_{DC} = 1$ (0 dB)
Simplified Ride Model

- Neglecting the sprung mass:

\[ \ddot{z}_s \quad z_s \quad = \quad \frac{k_t b_s s + k_s k_t}{b_s m_s s^3 + (k_s + k_t)m_s s^2 + b_s k_t s + k_s k_t} \]

- Defining the effective stiffness (also called the ride rate) using springs in series:

\[ RR = k_{eff} = \frac{k_s k_t}{k_s + k_t} \]

- Writing the above transfer function w/ the RR:

\[ \ddot{z}_s \quad z_s \quad = \quad \frac{k_{eff}}{k_s} b_s s + k_{eff} \]

\[ \ddot{z}_r \quad z_r \quad = \quad \frac{k_{eff}}{k_s k_t} b_s m_s s^3 + m_s s^2 + \frac{k_{eff}}{k_s} b_s s + k_{eff} \]
Simplified Ride Model

• Then for large $k_t$:

\[
\frac{\ddot{z}_s}{\ddot{z}_r} = \frac{z_s}{z_r} = \frac{k_{\text{eff}}}{k_s} \frac{b_s s + k_{\text{eff}}}{m_s s^2 + \frac{k_{\text{eff}}}{k_s} b_s s + k_{\text{eff}}}
\]

• This results in the dominate natural frequency (i.e. the suspension natural frequency) of:

\[
\omega_n = \sqrt{\frac{k_{\text{eff}}}{m_s}}
\]
Bode Plot of $z_S$

- Comparison the 2\textsuperscript{nd} and 4\textsuperscript{th} order suspension models:
Pitch Dynamics

\[
\begin{align*}
\text{Recall:} & \quad F_{zF} &= \frac{mg b}{L} - \frac{mh_{CG}}{L} \ddot{x} \\
F_{zR} &= \frac{mg a}{L} + \frac{mh_{CG}}{L} \ddot{x}
\end{align*}
\]
Given $K_t$ and $K_s$, what will $\Delta z_{\text{tire}} + \Delta z_{\text{susp}}$ equal??

$\Delta z_{\text{tire}} = \frac{m h c c}{L K_{tire}}$  

(additional deflection)

$\Delta z_{\text{susp}} = \frac{m h c c}{L K_{susp}}$

• All loading goes through tire

• Not all loading goes through suspension spring

$\Rightarrow$ Some goes through a-arms
Equivalent Trailing Arm

upper control arm

Lower Control arm

"Equivalent" Control Trailing Arm
\( F_{z\text{TA}} \Rightarrow \text{Vertical load through control Arms} \)

\( F_{z\text{susp}} \Rightarrow \text{Vertical load through suspension spring} \)
\[ F_{xf} = \frac{1}{2} ma_x = \frac{1}{2} m \ddot{x} \]

\[ F_{xr} = (1-\frac{1}{2}) ma_x = (1-\frac{1}{2}) m \ddot{x} \]

\[ \frac{1}{2} \Rightarrow \text{Defines } \% \text{ of Force on Front vs. Rear} \]

\[ \Theta_{pitch} = \frac{1}{L} M \dddot{x} \left[ \frac{h_{cc}}{K_{RL}} - \frac{(1-\frac{1}{2})}{K_R} \left( \frac{e_R - R_w}{d_R} \right) + \frac{h_{cc}}{K_{FL}} + \frac{\frac{1}{2}}{K_F} \left( \frac{e_F - R_w}{d_F} \right) \right] \]
Solving for front and rear suspension deflection

\[ S_R = \frac{1}{K_R} \left[ \frac{mh}{L} \ddot{x} - F_{xR} \left( \frac{e_R - R_w}{d_R} \right) \right] \]

\[ S_F = \frac{1}{K_F} \left[ -\frac{mh}{L} \ddot{x} - F_{xF} \left( \frac{e_f - R_w}{d_f} \right) \right] \]

\[ \Theta_{\text{pitch}} \approx \frac{S_R - S_F}{L} \]

Can design the Control Arms to set how much the vehicle pitches under deceleration!!
Anti-Dive Geometry

\[
\frac{e_f}{d_f} = \tan(\beta_f) = \frac{-h}{\xi L}
\]

\[
\frac{e_r}{d_r} = \tan(\beta_r) = \frac{h}{(1-\delta)L}
\]

(\delta \Rightarrow \% Braking of front)