Selected HW #8 Answers

Palm 9.13 Using the "rule of thumb" approximations:

$$\omega_r \approx 1.4\, rad/s \qquad M_r(\zeta = 0.1) \approx \frac{5}{20} \qquad M_r(\zeta = 0.3) \approx \frac{1.67}{20}$$

Note the actual answers for ζ=0.3 is a bit different:

$$\omega_r = 1.28\, rad/s \qquad M_r(\zeta = 0.3) = \frac{1.74}{20}$$

Palm 9.14 c>2.6

Palm 9.15 k=208 $\frac{10}{8}\sin\left(4t - \frac{\pi}{2}\right)$

Palm 9.18 $\omega_r \approx \sqrt{600}\, rad/s$ $\quad G(\omega_n) = \frac{1}{c\sqrt{600}}$

1. $\omega_n \approx 13\ Hz$

2. a) $G(\omega) = \frac{1}{\sqrt{(20-2\omega^2)^2+(22\omega)^2}}$ $\quad G_{DC} = \frac{1}{20}$ $\quad \emptyset(\omega) = -tan^{-1}\left(\frac{22\omega}{20-2\omega^2}\right)$

   b) $G(\omega) = \frac{1}{\sqrt{(27-3\omega^2)^2+(4\omega)^2}}$ $\quad G_{DC} = \frac{1}{27}$ $\quad \emptyset(\omega) = -tan^{-1}\left(\frac{4\omega}{27-3\omega^2}\right)$

   c) $G(\omega) = \frac{1}{|100-4\omega^2|}$ $\quad G_{DC} = \frac{1}{100}$ $\quad \emptyset(\omega) = -tan^{-1}\left(\frac{0}{100-4\omega^2}\right)$

   Note: $\emptyset(\omega < 5) = 0$
   $\emptyset(\omega > 5) = -180$

   d) $G(\omega) = \frac{1}{\sqrt{(4\omega^2)^2+(100\omega)^2}}$ $\quad G_{DC} = \infty$ $\quad \emptyset(\omega) = -tan^{-1}\left(\frac{100}{-4\omega}\right)$

   Note: $\emptyset(0) = -90\ deg$

3.

4. $x(t) = A_1 e^{-2t}\cos\left(\sqrt{12}t + A_2\right) + \frac{3}{\sqrt{481}}\sin\left(5t - tan^{-1}\left(\frac{20}{-9}\right)\right)$

5. a) $K_I = 0.2531$

   b) $\tau = 10e + 8\int e\, dt$

6. a) $F = [J(18)^2 - kl^2]e + [9.2J - bl^2]\dot{e}$

Palm 10.5

For small k_p, the dominant eigenvalue is approximately $s \cong -\frac{k_p+1}{5}$.

For mid k_p, the second eigenvalue has a significant impact on the dominant eigenvalue (i.e. no longer equal to value above.

For large k_p, eigenvalues are complex

Palm 10.39

$$k_p \geq \sqrt{10}$$
$$k_d \geq 40\sqrt{5} - 10$$