

HW 5 Solutions

Rocker (Note depending on how you define where the spring and damper act when the rocker moves you may end up with sec and tan in your NL equation – which as it is drawn is more correct)

$$ml^2\ddot{\theta} + bl^2 \cos^2(\theta)\dot{\theta} + kl^2 \sin(\theta)\cos(\theta) = mgl \cos(\theta)$$
$$ml^2\ddot{\theta} + bl^2\dot{\theta} + kl^2\theta = mgl$$

6.5)

$$(RC)\dot{V}_1 + V_1 = Ri_s$$

6.6)

$$(R_1 + R_2)C\dot{V}_o + V_o = V_s$$

6.7)

$$(R_1C)\dot{V}_o + V_o = V_s$$

$$V_o(t) = V_o(0)e^{-t/R_1C} + V_s(1 - e^{-t/R_1C})$$

6.8)

$$(R_1R_2C)\dot{V}_o + (R_1 + R_2)V_o = R_2V_s$$

$$V_o(t) = \frac{R_2}{R_1 + R_2} V_s \left(1 - e^{-\frac{R_1R_2C}{R_1+R_2}t} \right)$$

6.9)

$$L\dot{I} + RI = V_s$$

$$I(t) = \frac{V_s}{R} \left(1 - e^{-\frac{R}{L}t} \right)$$

6.10)

$$L\dot{I} + RI = V_s$$

$$I(t) = 0.72e^{-2t}$$

6.11)

$$L\dot{V}_o + RV_o = LV_s$$

6.12)

$$(R_1 + R_2)C\dot{V}_o + V_o = R_2C\dot{V}_s + V_s$$

6.13)

$$LC\ddot{I} + RC\dot{I} + I = CV_s$$

6.14)

$$LC\ddot{V}_o + RC\dot{V}_o + V_o = Ri_s$$

6.15)

6.16)

6.17)

$$L\dot{I}_1 + RI_1 = V_1$$

$$CV_1 = I_s - I_1$$

6.18)

$$L\dot{I}_1 = V_1 - V_2 - V_c$$

$$C\dot{V}_c + (1/R)V_c = I_1 - (1/R)V_2$$

6.19)

$$L_1\dot{I}_1 + RI_1 = V_1 - V_3$$

$$L_2\dot{I}_2 = V_3 - V_2$$

$$C\dot{V}_3 = I_1 - I_2$$

NOTE: Electrical Motors material will not be on Exam #1 (they can potentially be on Exam #2).

6.33)

$$\frac{\Omega(s)}{T_L(s)} = \frac{-1}{Is + c}$$

$$\frac{\Theta(s)}{V_f(s)} = \frac{K_T}{s[L_f Is^2 + (L_f c + R_f I)s + R_f c]}$$

$$\frac{\Theta(s)}{V_f(s)} = \frac{-1}{s(Is + c)}$$

6.34)

$$\frac{\Omega_f(s)}{V_f(s)} = \frac{K_T / N}{L_f I_e s^2 + (L_f c_e + R_f I_e) s + R_f c_e}$$

$$\frac{\Omega_f(s)}{T_L(s)} = \frac{1/N^2}{I_e s + c_e}$$

6.35)

$$L\dot{I} + RI = V_s$$

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6.36)

$$L_f \dot{I}_f + R_f I_f = V_f$$

$$I\dot{\omega} + c\omega = K_T I_a - T_L$$

$$L_a \dot{I}_a + R_a I_a = K_f I_f - K_b \omega$$

$$\begin{bmatrix} \dot{I}_f \\ \dot{\omega} \\ \dot{I}_a \end{bmatrix} = \begin{bmatrix} \frac{-R_f}{L_f} & 0 & 0 \\ 0 & \frac{-c}{I} & \frac{K_T}{I} \\ \frac{K_f}{L_a} & \frac{-K_b}{L_a} & \frac{-R_a}{L_a} \end{bmatrix} \begin{bmatrix} I_f \\ \omega \\ I_a \end{bmatrix} + \begin{bmatrix} \frac{1}{L_f} & 0 \\ 0 & \frac{-1}{I} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_f \\ T_L \end{bmatrix}$$

6.38)

$$L_a I s^2 + (L_a c + R_a I) s + (R_a c + K_b K_T) = 0$$

$$s = -219, -47.5 \quad (c = 0)$$

$$s = -196 \pm 73.5i \quad (c = 0.01)$$

$$s = -1240, -277 \quad (c = 0.1)$$