HW 4 Solutions

1.9)

$$|x| \le 0.5 \text{ rad}$$

1.10)

$$f(\theta)|_{\theta_o=\frac{\pi}{4}} \approx \sin(\tfrac{\pi}{4}) + \left(\theta - \tfrac{\pi}{4}\right)\cos(\tfrac{\pi}{4})$$

$$f(\theta)|_{\theta_o=\frac{3\pi}{4}} \approx \sin(\tfrac{3\pi}{4}) + \left(\theta - \tfrac{3\pi}{4}\right)\cos(\tfrac{3\pi}{4})$$

1.11)

$$f(\theta)|_{\theta_o=\frac{\pi}{3}} \approx \cos(\tfrac{\pi}{3}) - \left(\theta - \tfrac{\pi}{3}\right)\sin(\tfrac{\pi}{3})$$

$$f(\theta)|_{\theta_o=\frac{2\pi}{3}} \approx \cos(\tfrac{3\pi}{3}) + \left(\theta - \tfrac{2\pi}{3}\right)\sin(\tfrac{2\pi}{3})$$

1.12)

$$f(h)|_{h_o=25} \approx \sqrt{25} + \frac{1}{2\sqrt{25}(h-25)}$$

1.13)

$$f(r)|_{r_o=5} \approx 25 + 10(r-5)$$

$$f(r)|_{r_o=10} \approx 100 + 20(r-10)$$

1.14)

$$f(h)|_{h_o=16} \approx 4 + \tfrac{1}{8}(h-16)$$

1.15)

$$f(p) = \frac{0.06}{900}p$$

2)

$$\left(m+\frac{J}{R^2}\right)\ddot{x}+b\dot{x}+kx^3 = \frac{\tau}{R}$$

$$M_{eff}\,\ddot{x}+b\dot{x}+(3kx_o^2)x=+2kx_o^2$$

$$x_o = x_{ss} = \sqrt[3]{\frac{mg\cos(\theta)}{k}}$$

$$M_{eff} = \left(m+\frac{J}{R^2}\right)$$

3)

$$J_1\ddot{\theta}_1 + k(\theta_1-\theta_2)^3 = \tau$$
$$J_2\ddot{\theta}_2 + b\dot{\theta}_2^2 + k(\theta_2-\theta_1)^3 = 0$$

4)

$$ml^2\ddot{\theta}+bl^2\dot{\theta}\cos^2(\theta)+kl^2\sin(\theta)\cos(\theta)=mgl\cos(\theta)$$

Linearized about θ=0:

$$ml^2\ddot{\theta}+bl^2\dot{\theta}+kl^2\theta=mgl$$

Gears

$$\left(J_2 + \frac{R_2^2}{R_1^2} J_1\right)\ddot{\theta}_2 + \left(b_2 + \frac{R_2^2}{R_1^2} b_1\right)\dot{\theta}_2 = \frac{R_2}{R_1}\tau_{in} - \tau_{out}$$

Ball on the Beam

$$\left(J + mx^2\right)\ddot{\theta} = \tau + mgx\cos(\theta)$$

$$m\ddot{x} = mg\sin(\theta)$$

Two Masses (*y_{out}* is the position of the bottom mass – positive to the right)

$$m_2\ddot{x}_{out} + b\dot{x}_{out} + k_2 x_{out} = b\dot{y}_{out}$$
$$m_1\ddot{y}_{out} + b\dot{y}_{out} + k_1 y_{out} = b\dot{x}_{out} + k_1 x_{in}$$