Multi-Input Ground Vehicle Control Using Quadratic Programming Based Control Allocation

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Outline

- Motivation and Contributions
- The Problem and Objectives
- Vehicle Models
- Controller Design
- Control Allocation
- Background
- Simulation Results
- Conclusions
- Future Work

Motivation

- Multiple inputs are becoming more economically feasible
  - rear steer, differential braking, active suspension, active stabilizer, individual torque control of each wheel.
- Highway vehicles, Military applications
- Vehicle Stability Control systems – obstacle avoidance, rollover prevention

Recent Publications

B. Chen and H. Peng. *Differential braking based rollover prevention for SUVs w/ Human-in-the-loop evaluations.* Vehicle System Dynamics, 2001.

C. Carlson and J. C. Gerdes. *Optimal rollover prevention w/ steer-by-wire and differential braking.* Proceedings of IMECE, 2003.

A. Savkoor and C. T. Chou. *Application of Aerodynamic Actuators to Improve Vehicle Handling.* Vehicle System Dynamics, 1999.

Contributions

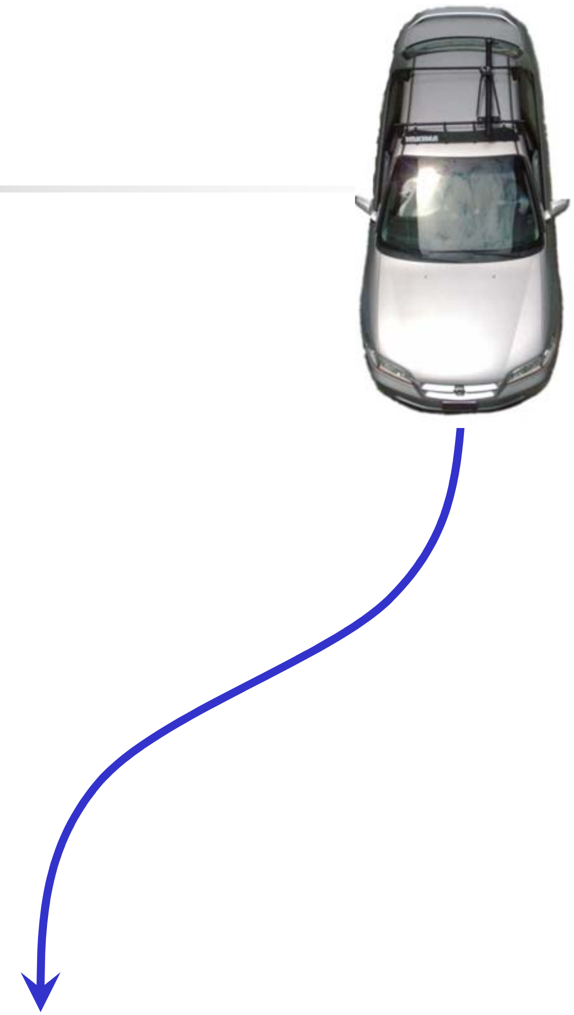
- A control strategy to provide intelligent combinations of input commands to a ground vehicle in order to accomplish multiple objectives.
- A method for the application of Control Allocation techniques to a ground vehicle with coupled dynamics
- Demonstrate ability to maintain successful tracking in the event of a failure.

The Problem

We want to track a yaw rate trajectory with a 4 wheeled ground vehicle while minimizing the sideslip angle.

Question: But if there are multiple inputs available, How do you choose which to use and how much?

Answer: Control Allocation!!



Control Allocation (CA)

- CA involves generating a set of effector commands that produce a desired control effect while minimizing the control effort.

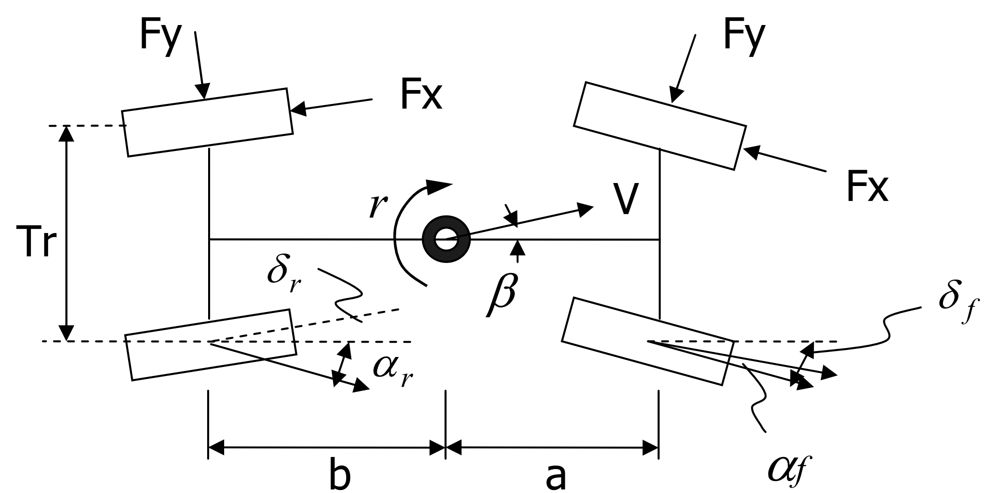
- A CA approach is generally used for a redundant set of effectors.

- CA allows for reconfiguration in the event of an effector failure.

Vehicle Model (nonlinear)

Assumptions made:

- The vehicle runs at constant velocity
- No longitudinal weight transfer (pitch is neglected)
- Constant roll center.
- A Pacejka tire model was used to represent nonlinear tire behavior.



Vehicle Model (linear)

The vehicle model in state space form:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + Bu$$

Linearized about constant velocity

Vehicle Model (linear)

3-Input vehicle model:

$$Bu = \begin{bmatrix} \dfrac{C_{\alpha f}}{mV} & 0 & 0 \\ a\dfrac{C_{\alpha f}}{I_z} & \dfrac{T_f}{2I_z} & \dfrac{T_r}{2I_z} \end{bmatrix} \begin{bmatrix} \delta \\ \Delta F_{xf} \\ \Delta F_{xr} \end{bmatrix}$$

δ = steering angle

ΔF_{xf} = differential braking force on front axle

ΔF_{xr} = differential braking force on rear axle

Vehicle Model (linear)

4-Input vehicle model:

$$Bu = \begin{bmatrix} \dfrac{C_{\alpha f}}{mV} & \dfrac{C_{\alpha r}}{mV} & 0 & 0 \\ a\dfrac{C_{\alpha f}}{I_z} & -b\dfrac{C_{\alpha r}}{I_z} & \dfrac{T_f}{2I_z} & \dfrac{T_r}{2I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ \Delta F_{xf} \\ \Delta F_{xr} \end{bmatrix}$$

δ_f = front steering angle
δ_r = rear steering angle
ΔF_{xf} = differential braking force on front axle
ΔF_{xr} = differential braking force on rear axle

Vehicle Model (linear)

6-Input vehicle model:

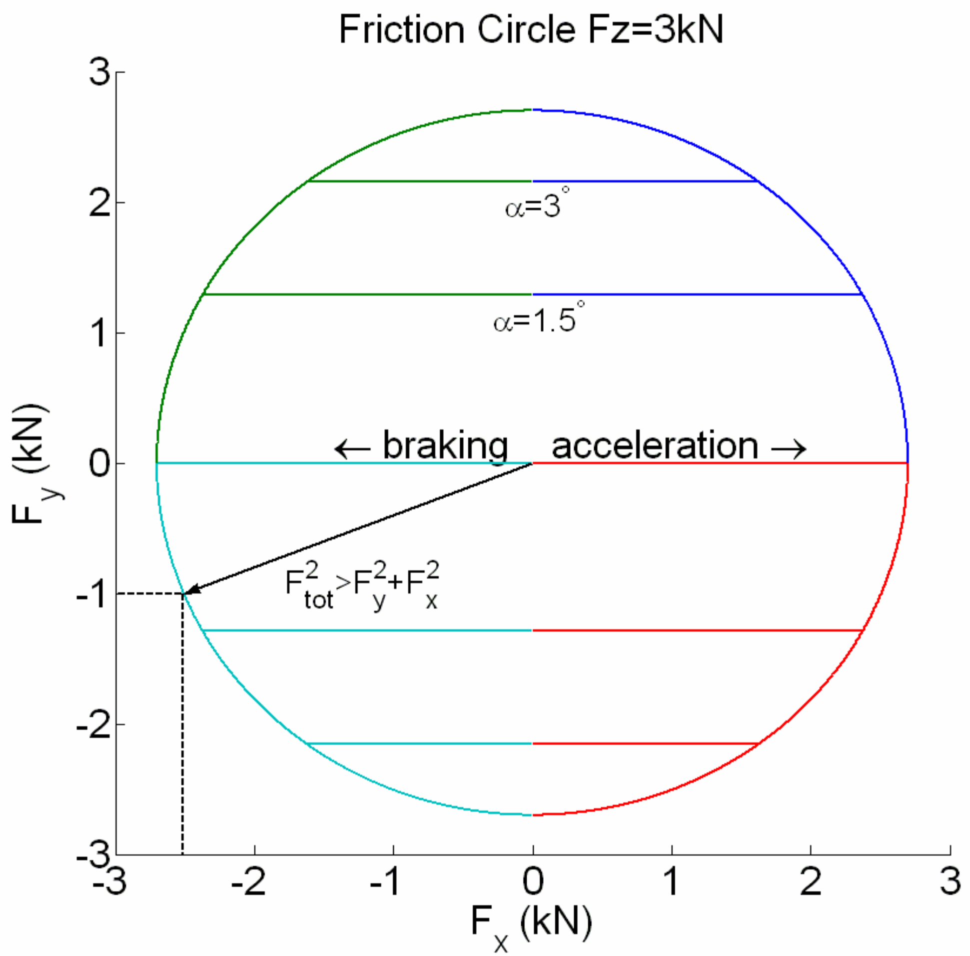
$$Bu = \begin{bmatrix} \frac{C_{\alpha f}}{mV} & \frac{C_{\alpha r}}{mV} & 0 & 0 & 0 & 0 \\ a\frac{C_{\alpha f}}{I_z} & -b\frac{C_{\alpha r}}{I_z} & -\frac{T_f}{2I_z} & \frac{T_f}{2I_z} & -\frac{T_r}{2I_z} & \frac{T_r}{2I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ F_{xfR} \\ F_{xfL} \\ F_{xrR} \\ F_{xrL} \end{bmatrix}$$

δ_f = front steering angle

δ_r = rear steering angle

F_x = force at each wheel

Vehicle Model
Effector limitations



Front and rear steering angle: $\pm 0.5\, rad$

Longitudinal wheel force:

$$F_{tot} = \mu F_z$$

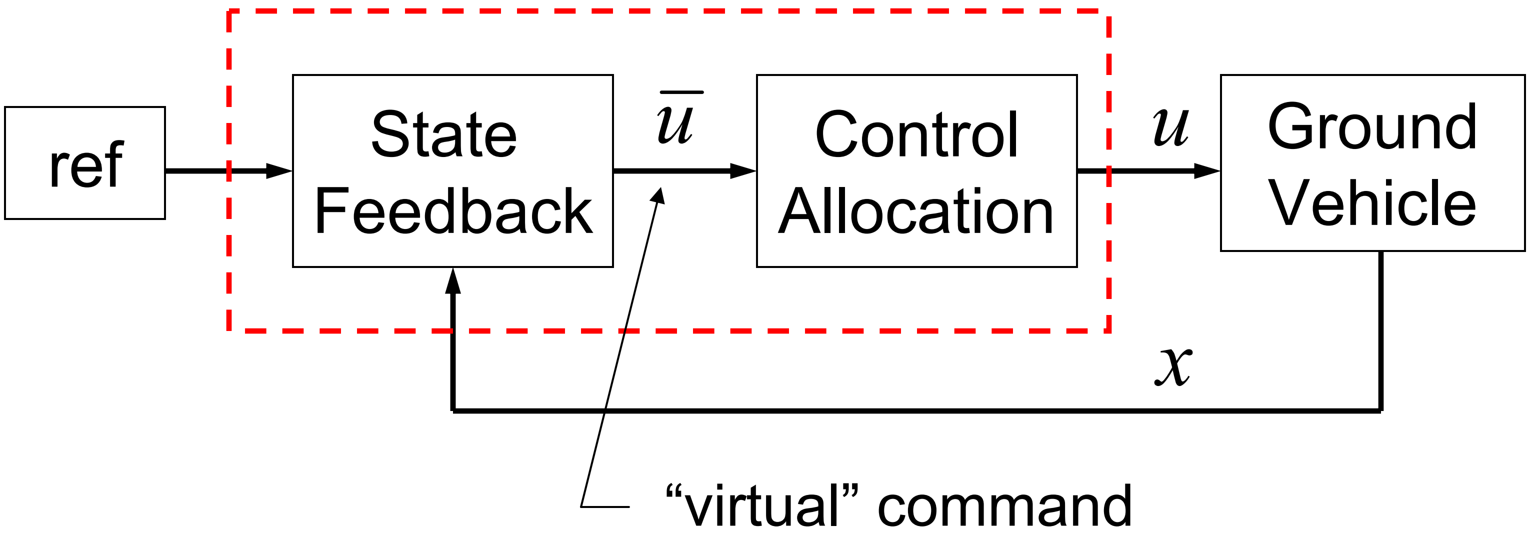
$$F_x^{\pm} = \sqrt{F_{tot}^{\;2} - F_y^{\;2}}$$

where $\mu = 0.8$

Controller Design Approach

Controller has 2 main tasks:

1. Generate Control Effort ⟶ State Feedback
2. Generate Effector Commands ⟶ Control Allocation



Control Effort

LQR gains are designed for the following modified system:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 & 0 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \overline{u}_\beta \\ \overline{u}_r \end{bmatrix}$$

This model is used solely for generating the virtual command effort.

The gain matrix K is applied to the error vector to produce the overall desired effect.

$$\begin{bmatrix} \overline{u}_\beta \\ \overline{u}_r \end{bmatrix} = -K_{2x3} \begin{bmatrix} \beta_{error} \\ r_{error} \\ \psi_{error} \end{bmatrix}$$

Command Generation

Physical effector commands are generated by the Control Allocation routine.

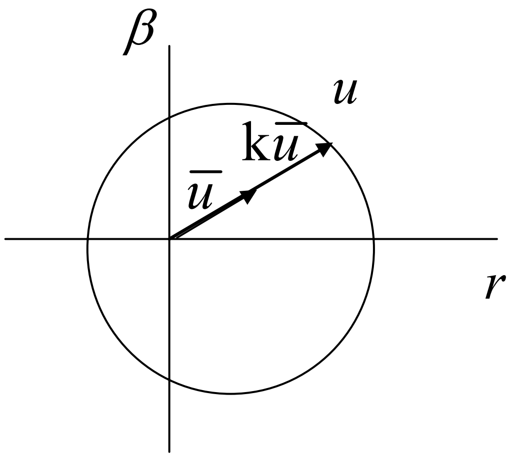
Given the virtual command $\bar{u}$, solve for the effector commands u.

$$Bu = \bar{u}$$

$$u^- \leq u \leq u^+$$

Control Allocation background
Traditional Approaches

- Direct CA involves the calculation of the Attainable Set of virtual commands
  1. Determine AS
  2. Scale Desired Effect Vector to boundary of AS
  3. Solve for AS and virtual command vector intersection
  4. Scale by inverse of Step 2 scaling (u, effector commands)



Output $= {}^{1}/_{k}u = u_c$

- Generalized Inverse
  - weighted pseudo inverse $u = Q^{-1}B^T\left[BQ^{-1}B^T\right]^{-1}\bar{u}$

Control Allocation background
Optimization Techniques

Least Squares

$$\min_{u} \quad \frac{1}{2} u^T Q u + c^T u$$
$$s.t. \quad Bu = \overline{u}$$
$$-\infty \leq u \leq +\infty$$

Q and c are weighting matrices

No inequality constraints

Linear Programming

$$\min_{u} \quad c^T u$$
$$s.t. \quad Bu = \overline{u}$$
$$-u \leq u \leq +u$$

Linear cost function

Quadratic Programming

$$\min_{u} \quad \frac{1}{2} u^T Q u + c^T u$$
$$s.t. \quad Bu = \overline{u}$$
$$-u \leq u \leq +u$$

Nonlinear cost function and inequality constraints

Command Generation: slack variable

Problem: The coupled dynamics make it impossible to perfectly satisfy both parts of the virtual command .

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{\alpha f}}{mV} & \frac{C_{\alpha r}}{mV} & 0 & 0 \\ a\frac{C_{\alpha f}}{I_z} & -b\frac{C_{\alpha r}}{I_z} & \frac{T_f}{2I_z} & \frac{T_r}{2I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ \Delta F_{xf} \\ \Delta F_{xr} \end{bmatrix}$$

QP is forced to make a compromise between $\bar{u}_\beta$ and $\bar{u}_r$. Poor tracking will result.

Solution: Add a pseudo-input ν.

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} B & \vdots & \begin{matrix} \kappa_\beta \\ \kappa_r \end{matrix} \end{bmatrix} \begin{bmatrix} u \\ \nu \end{bmatrix}$$

Reasoning: The CA law could use ν to satisfy the infeasible portion of $\bar{u}_\beta$ while leaving the yaw rate tracking up to the real effectors.

Command Generation: slack variable

Plan: Choose the effectiveness values κ_β and κ_r so that the DC gain from v to r is zero.

Step 1: Calculate the Transfer Function.

Step 2: Set the numerator equal to zero.

Step 3: Choose κ_β=1 and solve for κ_r.

$$\frac{R(s)}{N(s)} = \frac{\kappa_r m I_z V^2 s + \kappa_r I_z V C_0 - \kappa_\beta C_1 m V^2}{m I_z V^2 s^2 + I_z C_0 m s V - C_1 m V^2 - C_1^2}$$

$$0 = \kappa_r m I_z V^2 s + \kappa_r I_z V C_0 - \kappa_\beta C_1 m V^2$$

$$\kappa_\beta = 1$$

$$\kappa_r = \frac{C_1 m V}{C_0 I_z}$$

Command Generation: slack variable

The QP problem is set up as follows:

min $u,\ v$: $\frac{1}{2}\begin{bmatrix} u \\ v \end{bmatrix}\begin{bmatrix} Q & 0 \\ 0 & Q_v \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} + c^T \begin{bmatrix} u \\ v \end{bmatrix}$

A large penalty is placed on v to reduce its use.

subject to: $\begin{bmatrix} B & \vdots & \dfrac{1}{\dfrac{C_1 mV}{C_0 I_z}} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \overline{u}_\beta \\ \overline{u}_r \end{bmatrix}$

$$\begin{bmatrix} u^- \\ \infty^- \end{bmatrix} \le \begin{bmatrix} u \\ v \end{bmatrix} \le \begin{bmatrix} u^+ \\ \infty^+ \end{bmatrix}$$

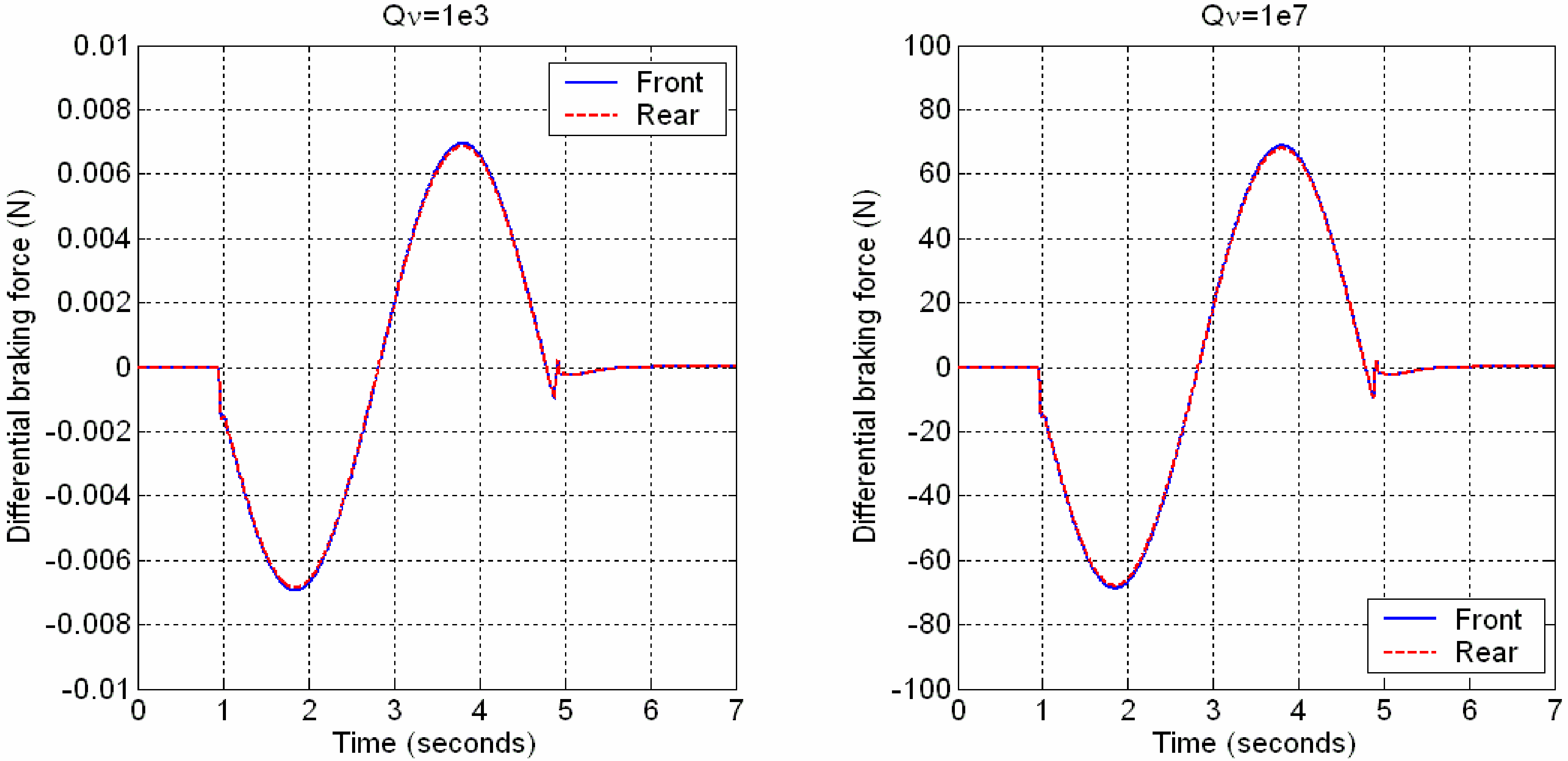
Nonrestrictive inequality constraints

Simulation

- The vehicles are simulated traveling at constant velocities of 45, 55, and 65mph.
- Scenarios examined:
  1) Nominal – no failures experienced.
  2) Failure – steering angle jams at 2.25s.
- Penalty on pseudo effector: $Q_v = 10^3, 10^7$.
- The desired yaw rate trajectory simulates a double lane change maneuver.

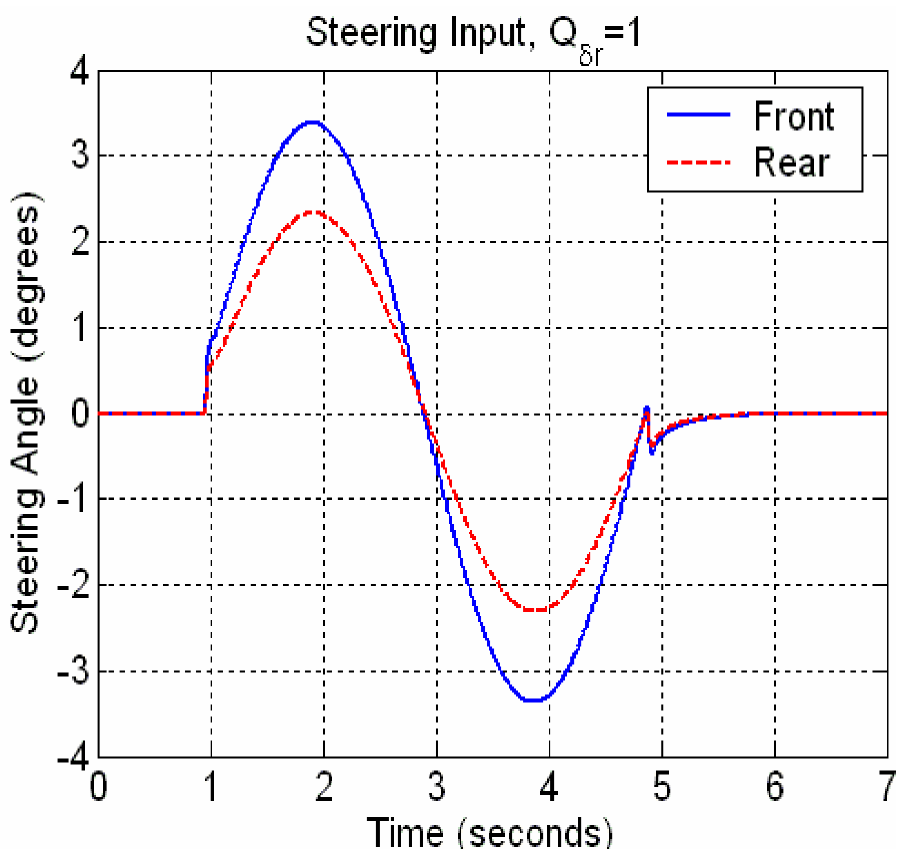
Weighting the pseudo effector

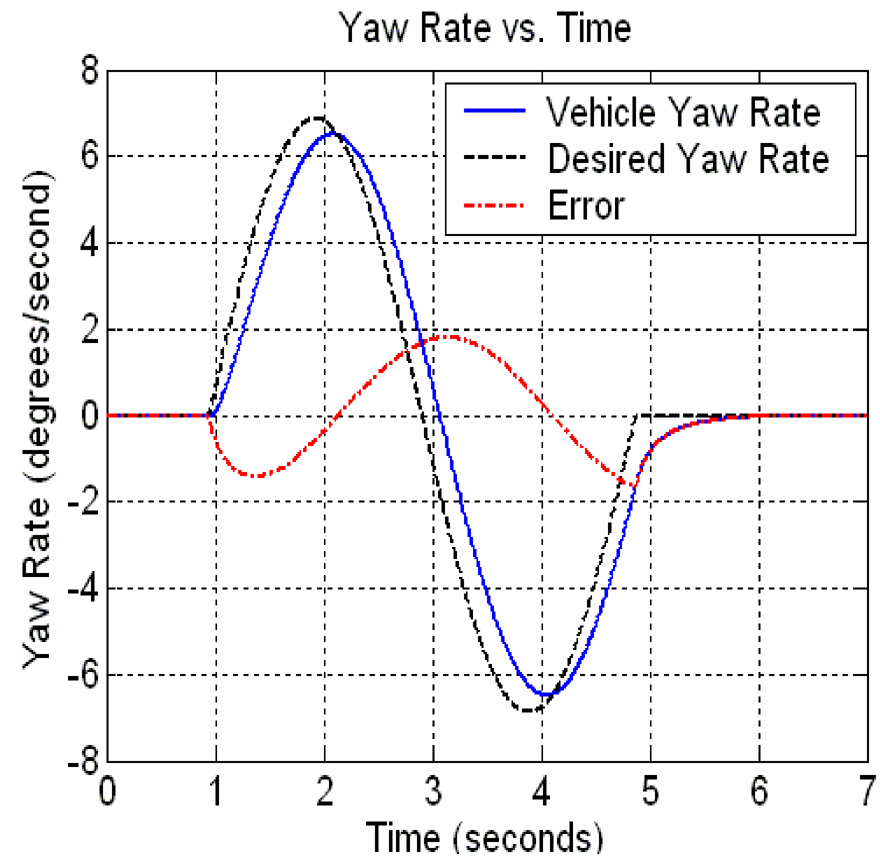
3-Input, Nominal , 55mph



Rear Steering Issue

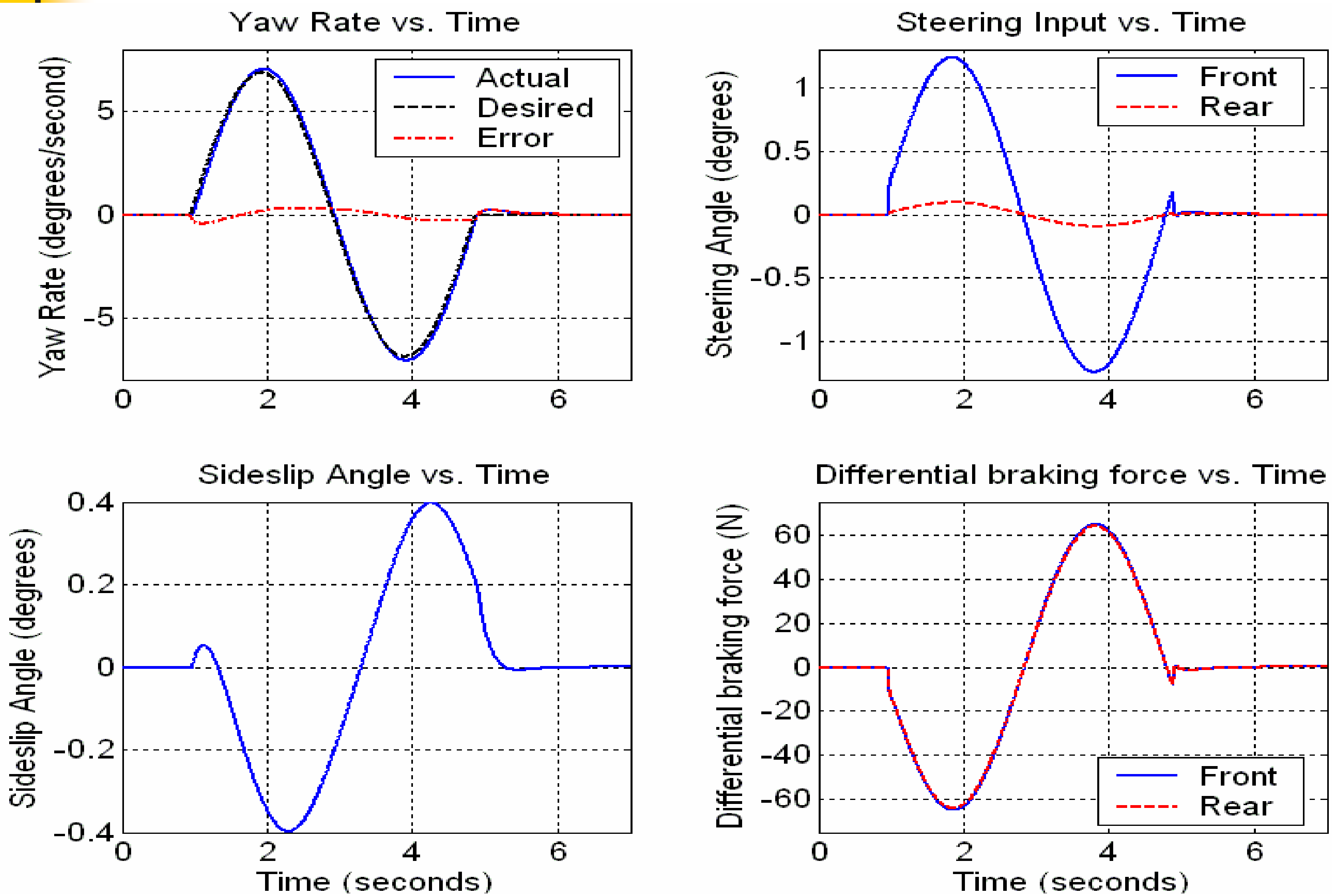
4-input, Nominal, 55mph



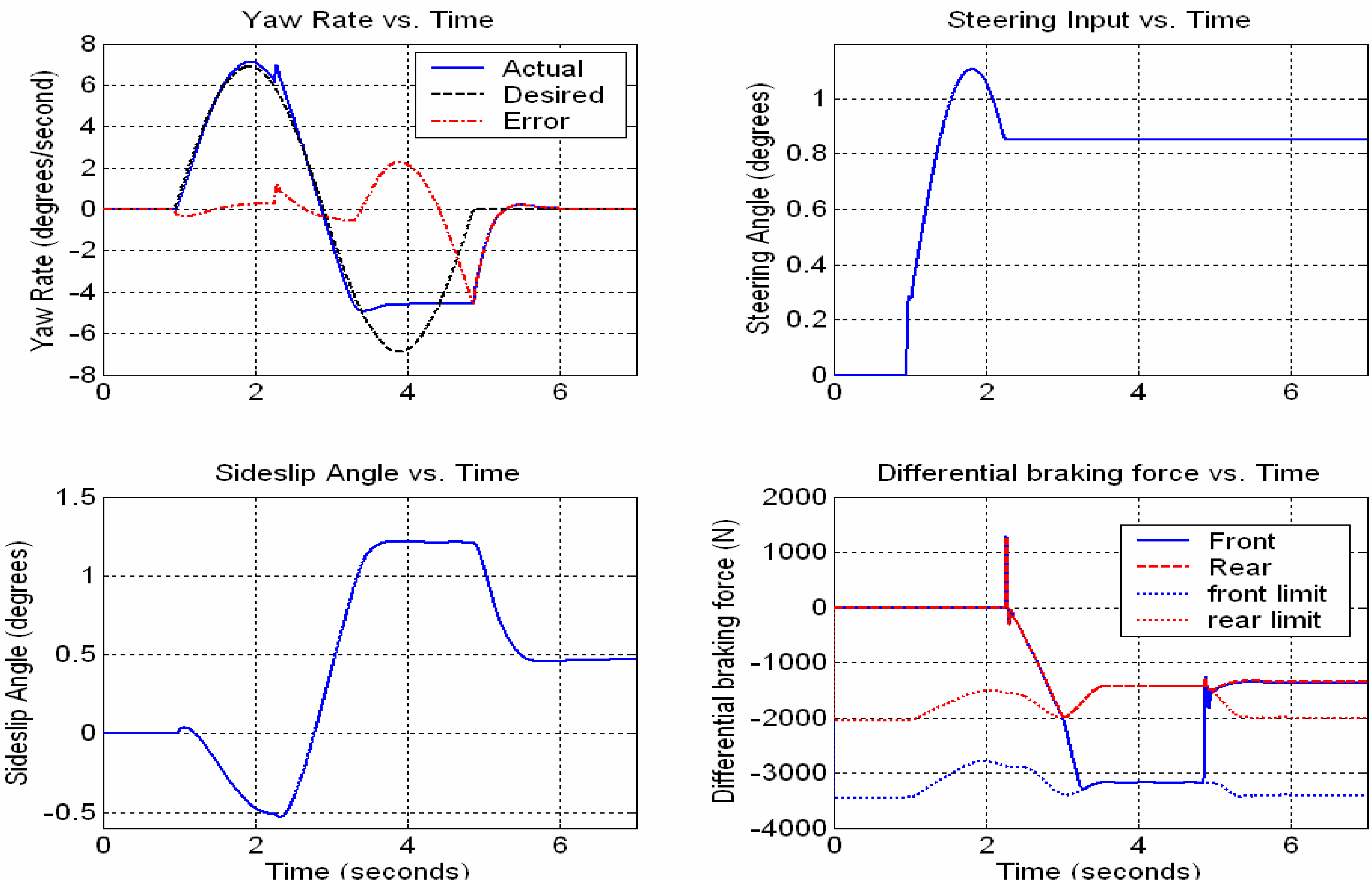


Nominal Case Performance

4-input 55mph QP

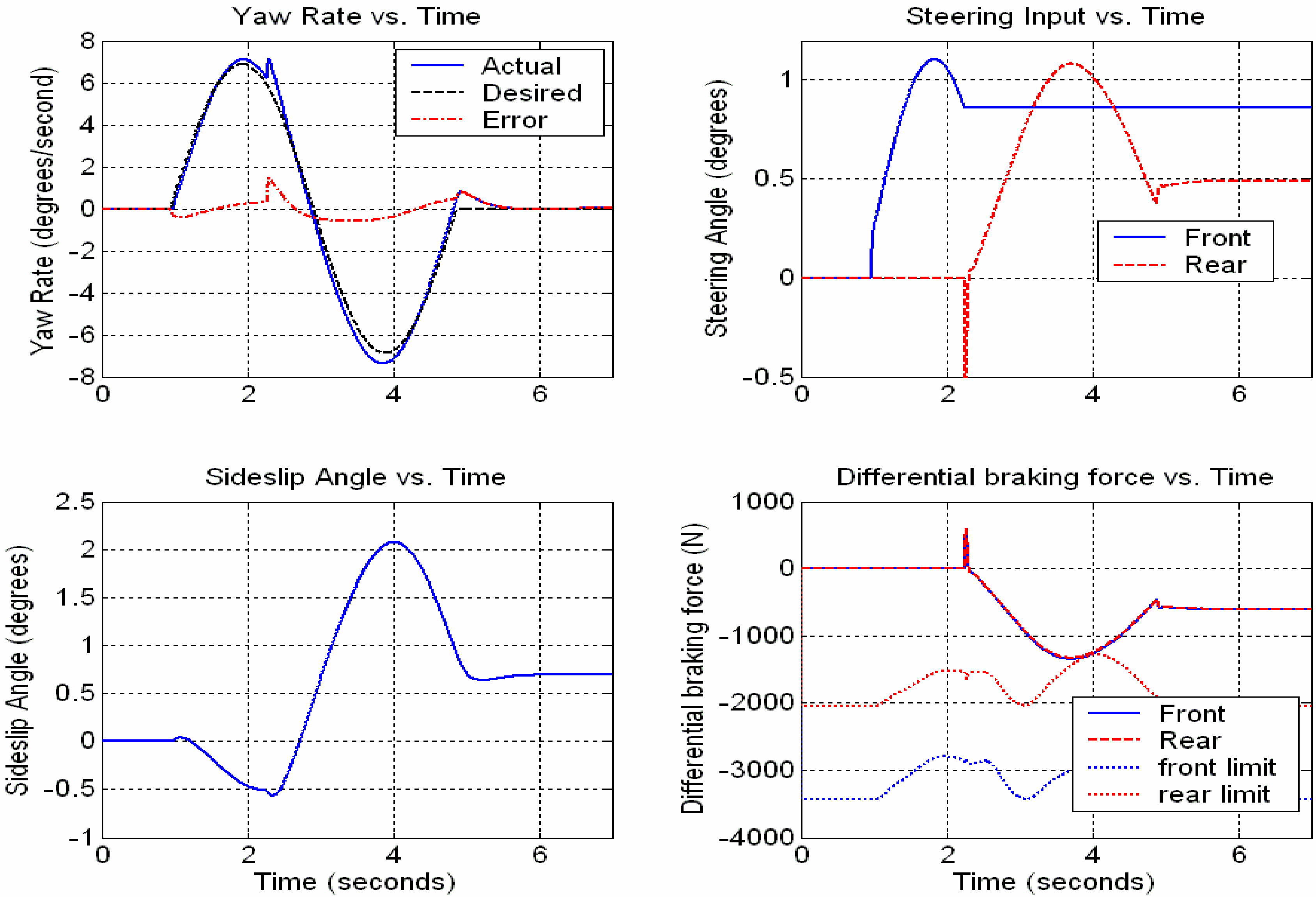


Failure Case Performance
3-input, 55mph



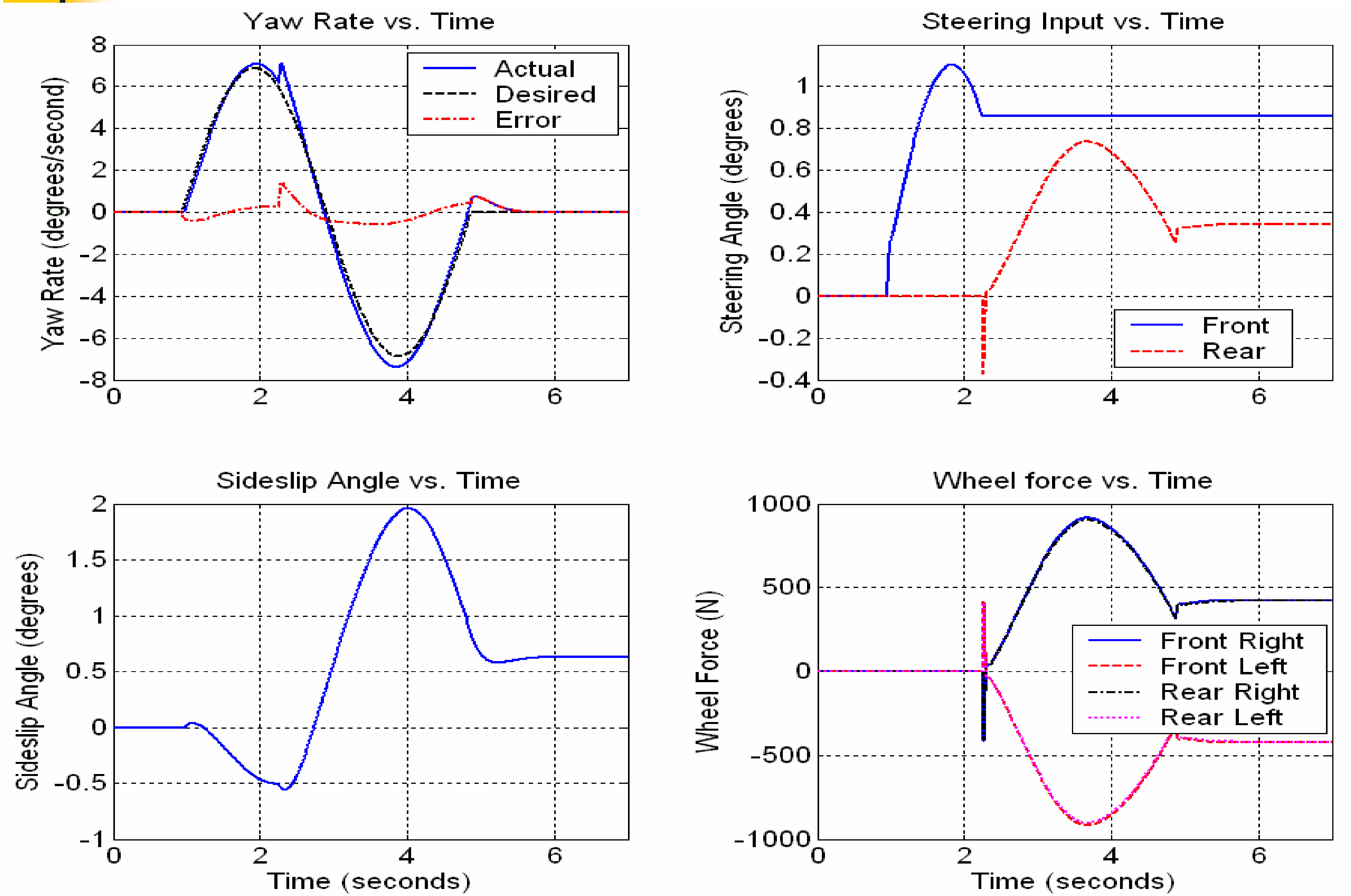
Failure Case Performance

4-input, 55mph



Failure Case Performance

6-input, 55mph



Conclusions

- The addition of the pseudo effector successfully allows the sideslip angle to vary so that opposing commands would be minimized in the presence of conflicting objectives.
- Large Q_v produced better sideslip and yaw rate tracking in the nominal case at high speeds.
- Small Q_v gave the best tracking of both objectives for the failure case.
- The proposed QP based CA strategy provides an intelligent reconfiguration of control effort in the event of a failure while respecting the effector limits.

Future Work

- Study controller performance for maneuvers other than a lane change.
- Investigate ways to alter the frequency content of the QP solution in the presence of noisy state estimates.
- Simulate driver in the loop. Evaluate performance.
- Development of an interior point algorithm for finding a fast solution to small scale QP problems.
- Actual implementation

Questions

???

Vehicle Dynamics Lab Reminder: I am Dr Bevly's 1st student to graduate. The difficulty of your questions will be setting the standard for your own defense.

Tire Force Estimation

- Laura R. Ray. *Nonlinear tire force estimation and road friction identification: Simulation and Experiments*. Automatica, 33(10): 1819-1833, 1997.

  - Can give online estimates of μ and Fy using Extended Kalman filtering and Bayesian Hypothesis Selection

- Steffen Muller, et al. *Estimation of the Maximum Tire-Road Friction Coefficient*. Journal of Dynamic Systems, Measurement, and Control, 125: 607-617, 2003

  - Presents a method for estimating the maximum tire-road friction during braking. Also gives a method for estimating Fz as an alternative to direct measurement.

Fault Detection

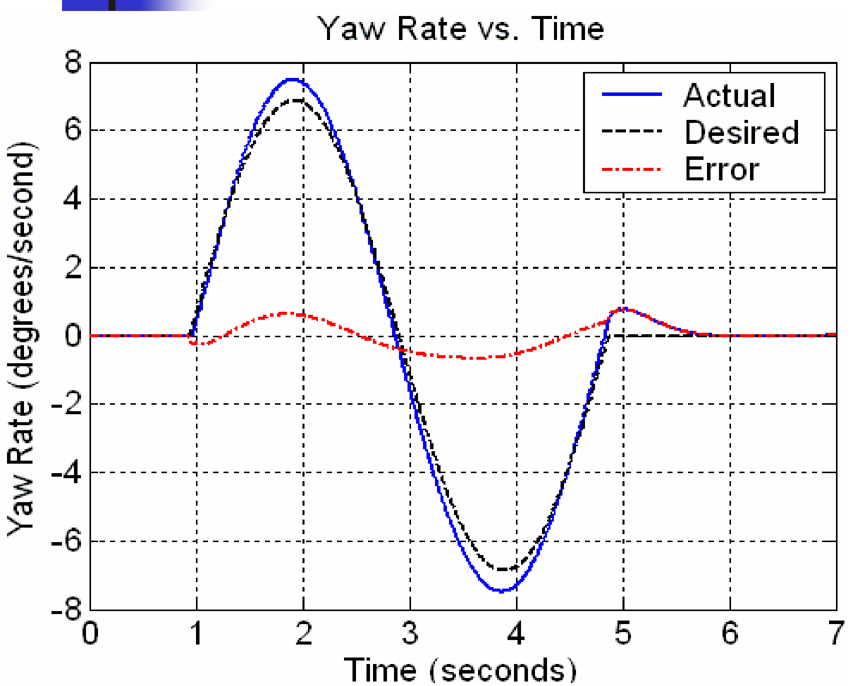
- Rajamani, et al. *A Complete Fault Diagnostic System for Automated Vehicles Operating in a Platoon*. IEEE Transactions on Control Systems Technology. vol. 9, no. 4: 553-564, July 2001.

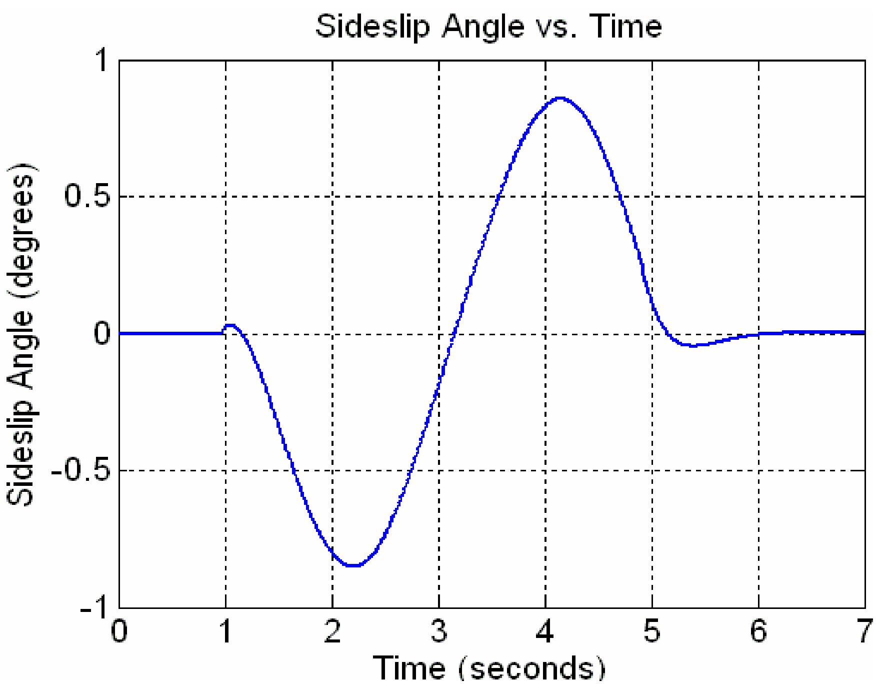
  - Uses nonlinear observer design techniques to construct a complete fault detection system for vehicle sensors including steering angle sensor, and brake pressure sensor.

- Swaroop, et al. *Fault Tolerant Control of Automatically Controlled Vehicles in Response to Brake System Failures.* IEEE International Conference on Control Applications. 705-710, October 1997.

  - Brake pressure sensor and braking actuator failure detection.

Performance with incorrect parameters, 4-input



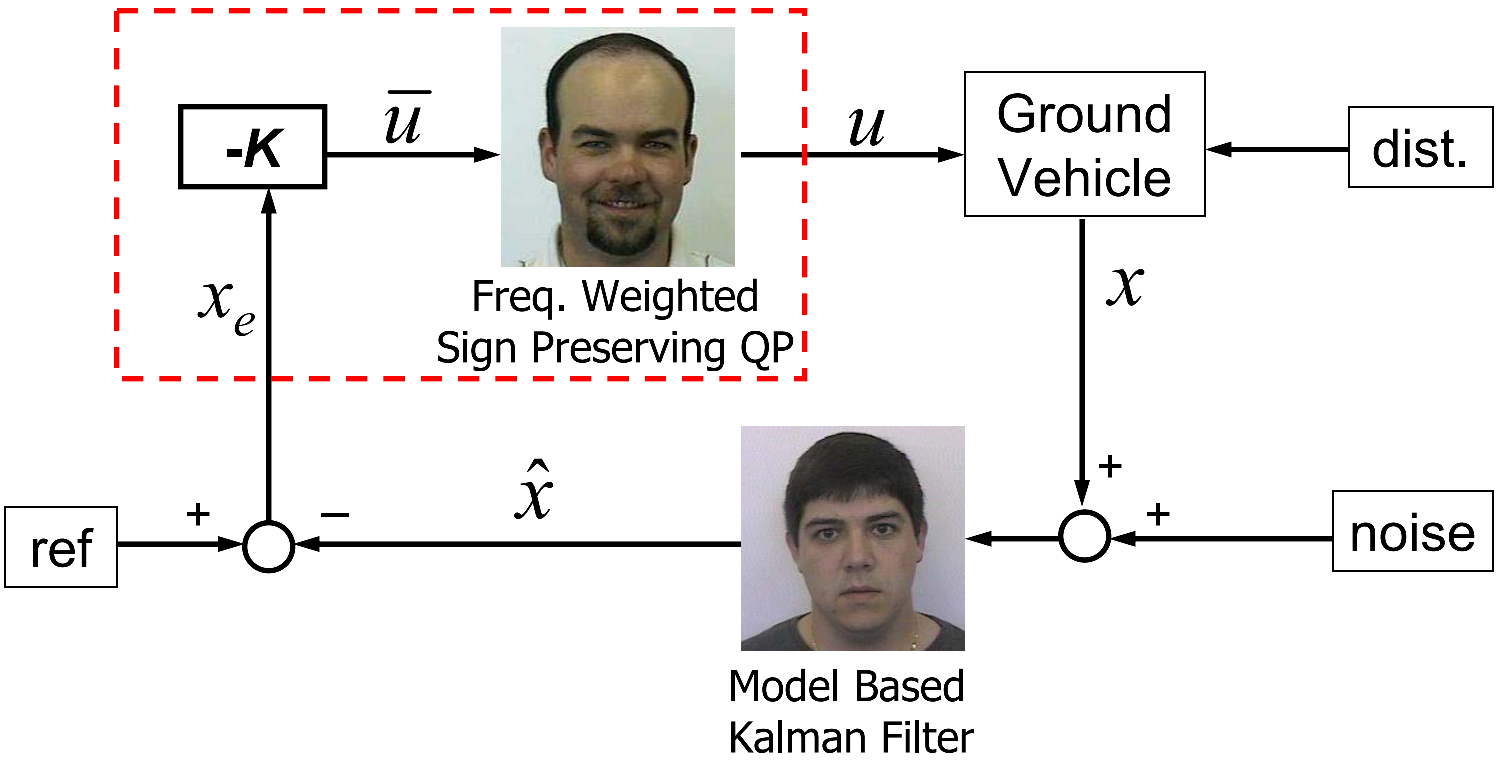


- Controller designed with 2000 Honda Accord parameters
- 2001 Chevy Blazer parameters were used for simulation
- Sideslip error is significantly larger

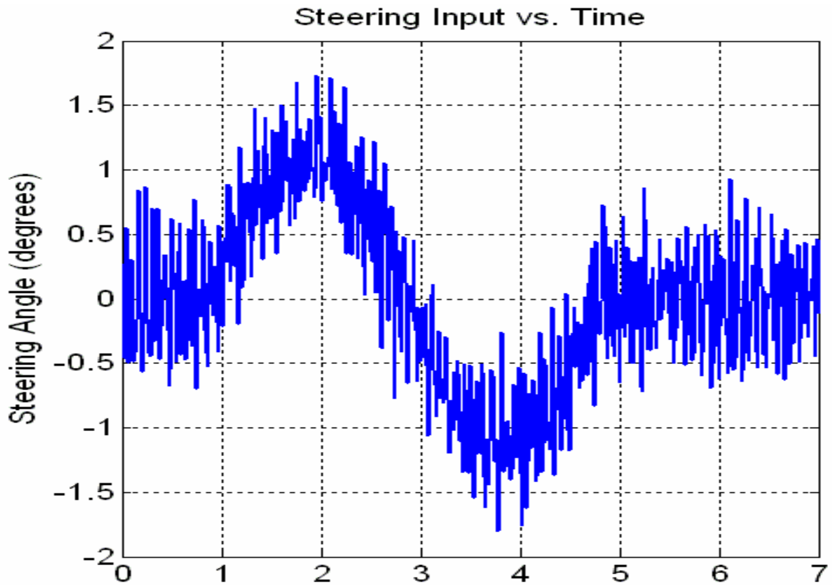
Actual Implementation

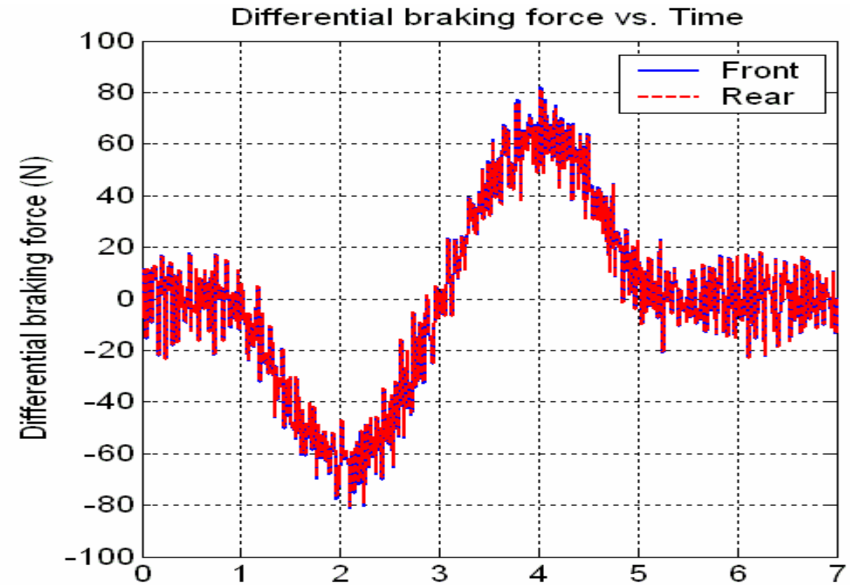
In reality, exact state estimates are not available.

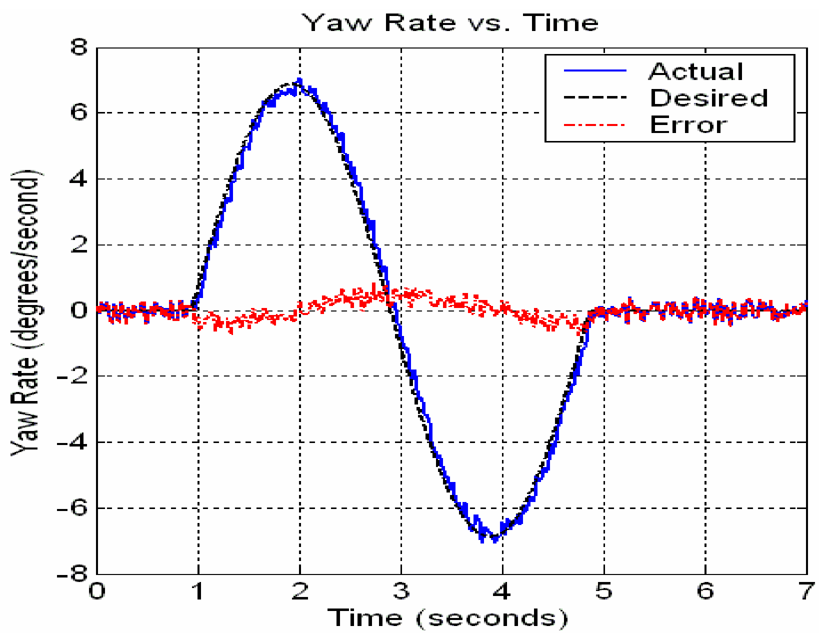
1. Use Kalman filter to get rid of some noise
2. Set up QP problem to minimize high frequency commands.

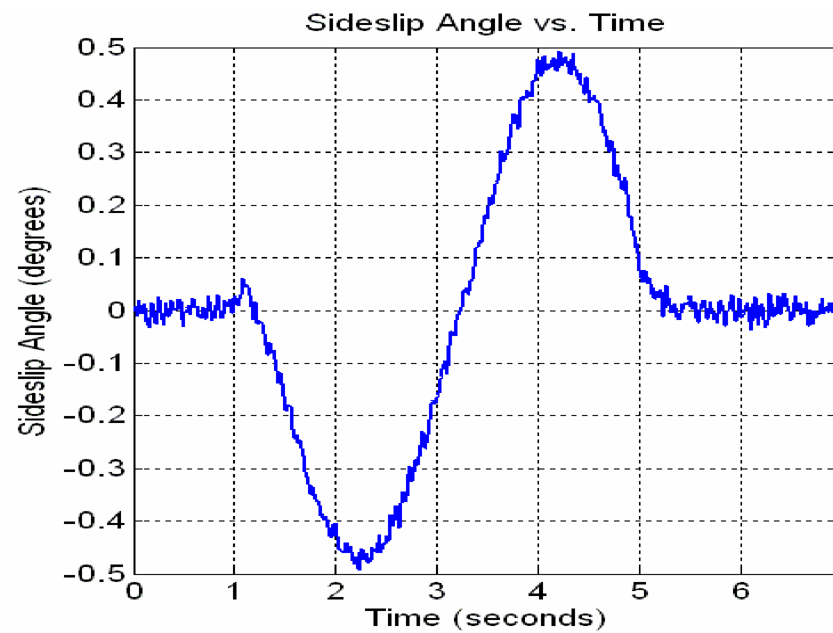


Commands w/ noisy state estimates

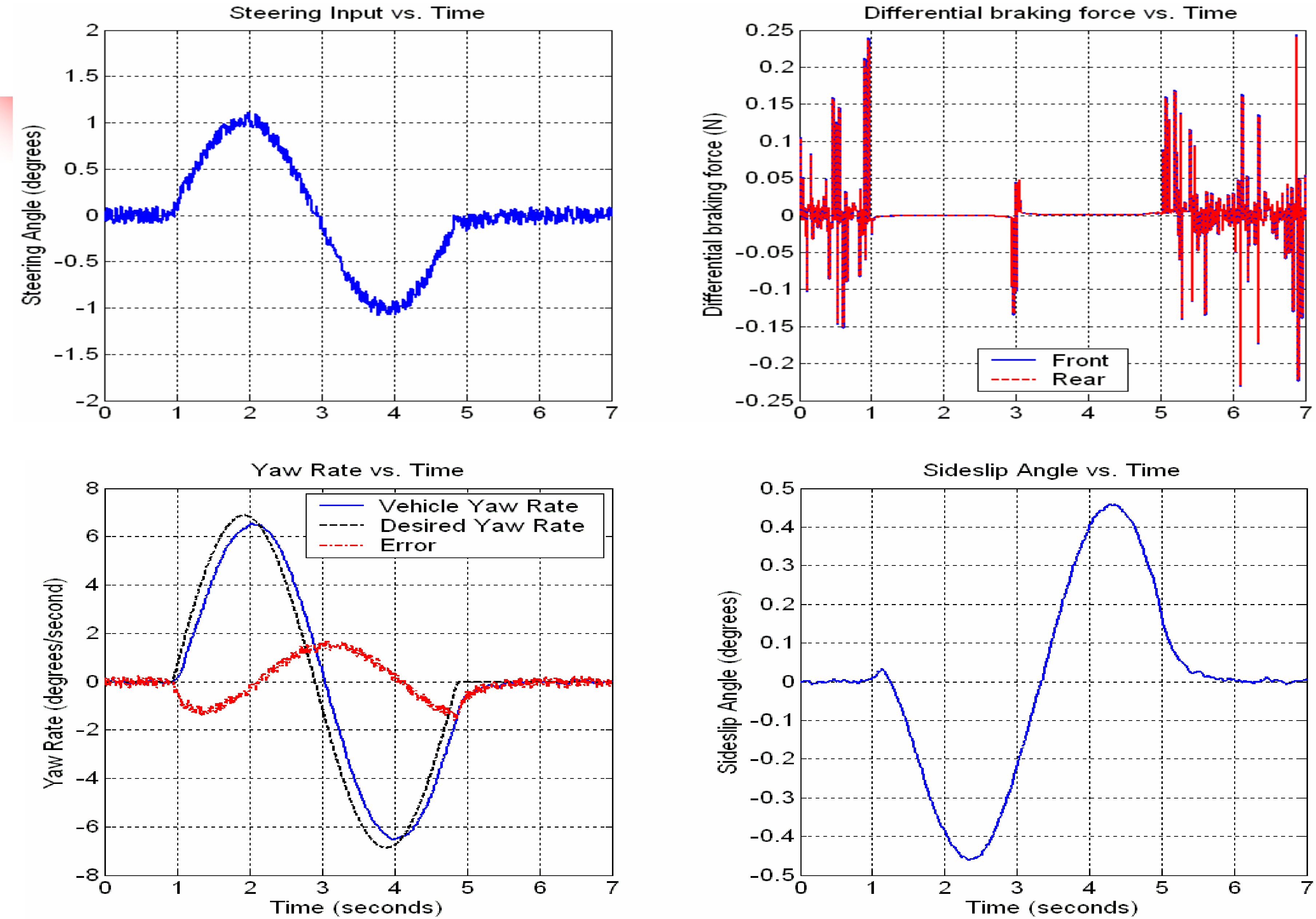




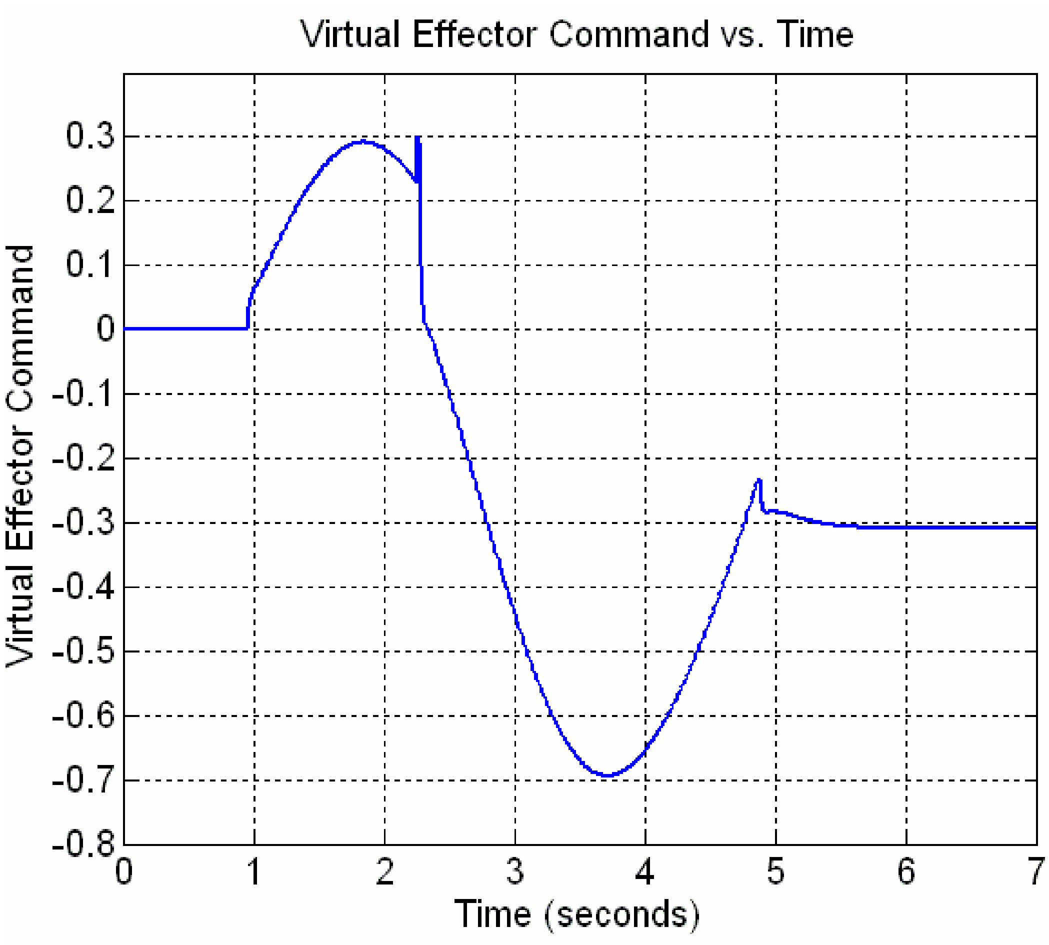




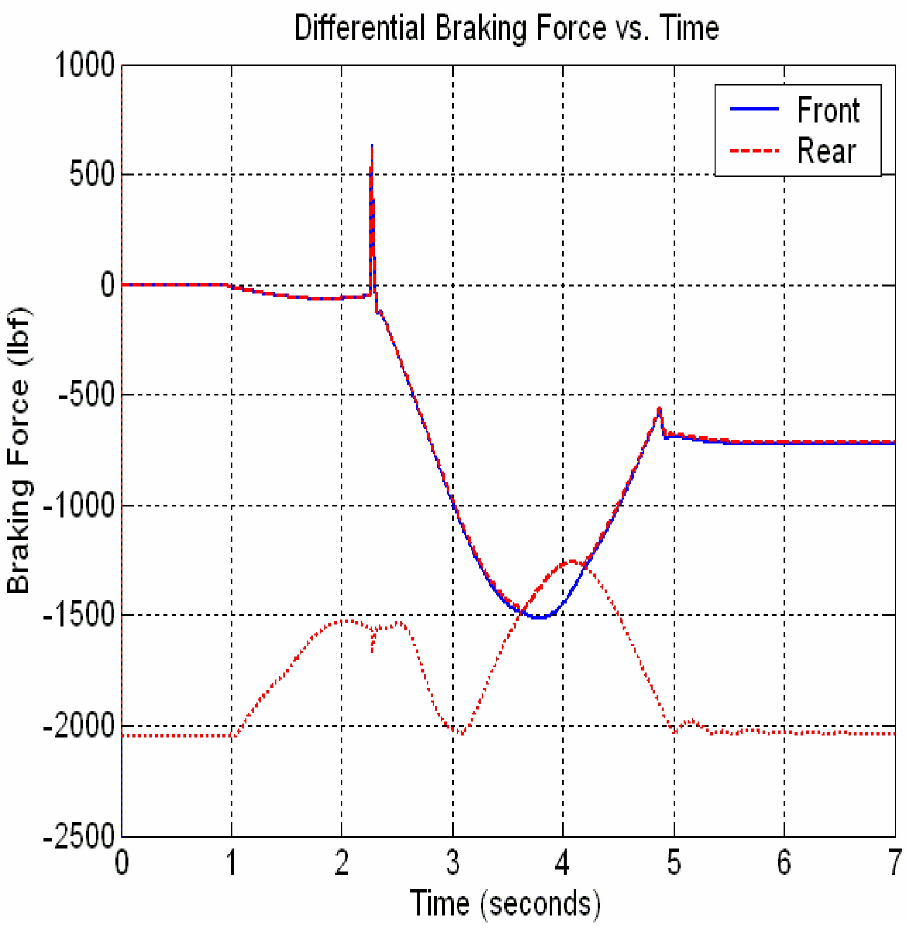
Frequency Weighted Sign Preserving QP



Pseudo effector command 4-input, failure, Qv=1e3



Friction circle
4-input, 55mph, failure



Rear Left Tire Friction circle