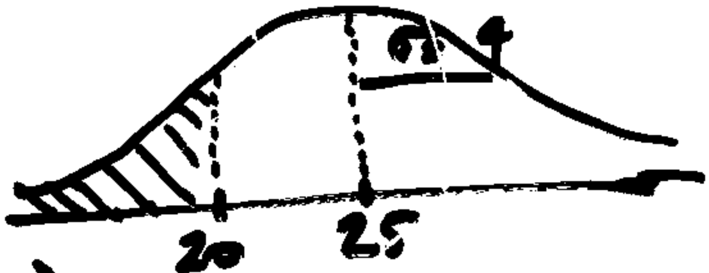
HW 5

$T \sim (25, 16)$

1)



a) $P(T \leq 20) = P\left(Z \leq \frac{20-\mu}{\sigma}\right)$

$= P(Z \leq -5/4) = \phi(-1.25)$

$= .1056$

b) $P\left(Z \leq \frac{30-\mu}{4}\right)$

$P(Z \leq 5/4) = \phi(1.25) = .8944$

c) Find the 99^{th} p-tile.

Step 1 Fin $Z_{0.01} = 2.33$

99^{th} p-tile $= \mu + Z_{.01} \cdot \sigma$

$= 25 + (2.33)(4)$

$= 34.32$ sec.

P.2

$.500 \pm 0.004$

$(.5 - 0.004) <$ Bearing Diameter $< (.5 + 0.004)$

$N(.499, (0.02)^2)$

$P(\text{acceptance})$

$$= P\left(\frac{0.496 - \mu}{\sigma} < z < \frac{0.504 - \mu}{\sigma} \right)$$

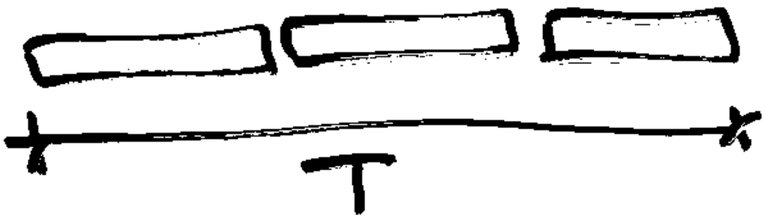
$$\frac{0.496 - 0.499}{0.02} < z < \frac{0.504 - 0.499}{0.02}$$

$$-1.5 < z < 2.5$$

$$P(\text{acceptance}) = \Phi(2.5) - \Phi(-1.5)$$
$$= .9938 - .0668$$
$$= .927$$

$$P(\text{reject}) = 1 - 0.927 = 0.073$$

3) 3 pipes $N(15.00\,;\,0.0064)$

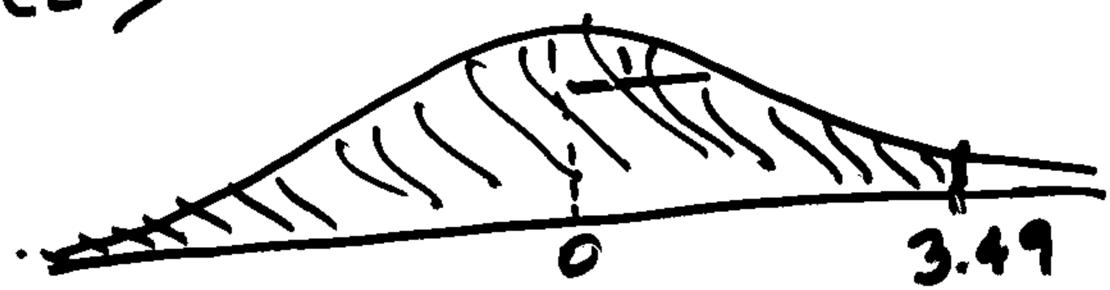


$$T \sim N\Big(\underbrace{15\times 3}_{45.00}\,,\,(3)(0.0064)\Big)$$

$$T \sim N(45.00\,,\,0.0192)$$

$$P(T > 48.25) = P\left(z > \frac{48.25 - 45.00}{\sqrt{0.0192}}\right)$$

$$P(z > 23.45)$$



Standard Normal

$$\phi(3.49) = 0.9998$$

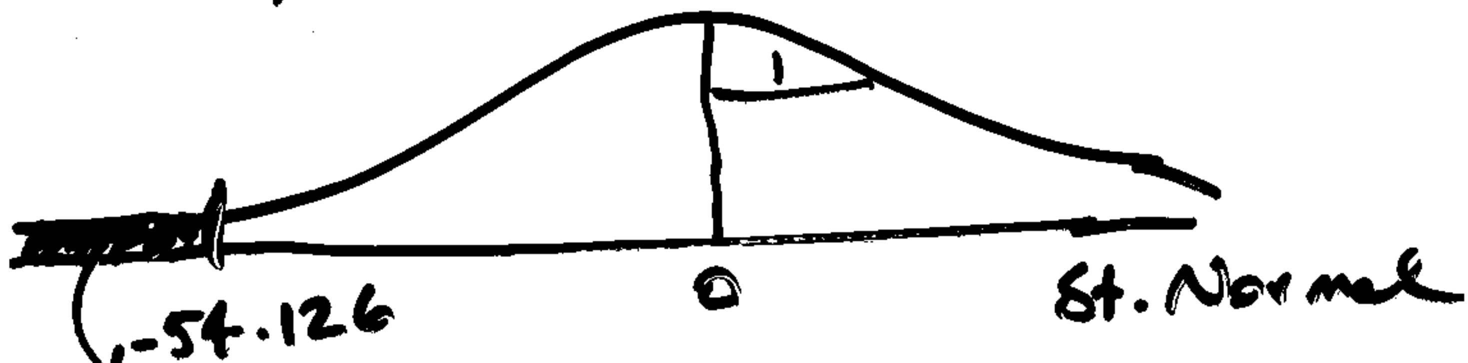
$$\phi(x) > 0.9998 \text{ if } x > 3.49$$

$$1 - \phi(23.45) < 0.0002$$

3) b) $\bar{X} \sim N\left(15.00, \frac{0.0064}{3}\right)$

$P(\bar{X} > 12.50) = P\left(Z > \frac{12.50 - 15.00}{\sqrt{0.00213}}\right)$

$P(Z > -54.126)$



$\phi(-54.126) < 0.0002$

$1 - \phi(-54.126) > 0.9998$

4) $E(Life) = (\alpha)(\beta) = (4)(20) = 80$ weeks

$V(Life) = (\alpha)(\beta)^2 = (4)(400) = 1600$ week^2

b) $P(Life > 100) = 1 - \underbrace{P(Life \le 100)}_{cdf}$

$= 1 - F\left(\frac{100}{\beta}; \alpha\right) = 1 - F(5;4)$

$= 1 - 0.735 = 0.265$

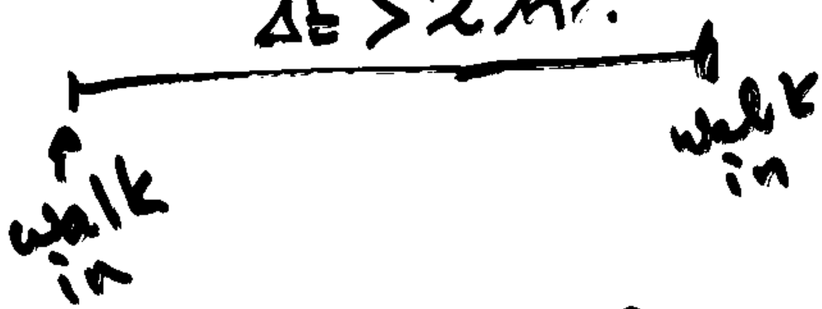
c) $P(Life < 50) = F\left(\frac{50}{20}; 4\right) = F(2.5; 4) = (0.143 + 0.353)/2$

$= 0.248$

5) $\lambda = 5$ cust/hr

Mean interarrival time $= 1/\lambda = 1/5 = 0.2$ hr/cust

a)



$$\text{Poisson}(0;\ \underset{\substack{\downarrow \\ 5}}{\lambda}\underset{\substack{\downarrow \\ 2}}{t}) = e^{-\lambda t} = e^{-10}$$
$$= 4.5 \times 10^{5}$$

$\lambda t = 10$

$$F_{\text{exponential}}(x) = P(X < x)$$

$$P(X > 2) = 1 - F(2)$$
$$= \cancel{1} - [\cancel{1} - e^{-\lambda t}]$$
$$= e^{-\lambda t}$$

b) $$P(X < 3) = 1 - e^{-\lambda t}$$
$$= 1 - e^{-15}$$

6) min life = 35,000 miles
Char. life = 80,000 miles
α = slope = 4

a) $R(60{,}000 \text{ miles}) = e^{-\left(\frac{60{,}000 - 35{,}000}{80{,}000 - 35{,}000}\right)^4}$

$\rightarrow R(60{,}000) = 0.909137$

$\text{Fails}(60{,}000) = 1 - 0.909137 = \underline{0.090863}$

b) Sell 1,000,000
The expected # of failures
$(1{,}000{,}000)(0.090863) = 90{,}863$

Total Cost = $(300)(90{,}863) = \$27{,}258{,}900$
$< \$45M$

X\Y	1	2	3	4	5
1		.04	.08		
2		.05	α		
3		.05	.02		
4		.02	.07		
			0.03		
$P_y(y)$		.16	.20		

for Y=2

$$\underbrace{P(X|Y=2)}_{\frac{p(x,y)}{P_y(y)_{=2}}} =$$

X	P(X|Y=2)
1	.04/.16
2	.05/.16
3	.05/.16
4	.02/.16
	1.0

$$E(X|Y=2)$$

$$=(1)\left(\frac{.04}{.16}\right)+(2)\left(\frac{.05}{.16}\right)+(3)\left(\frac{.05}{.16}\right)+(4)\left(\frac{.02}{.16}\right)$$

Percentile of a Gamma dist.

$\alpha = 4$ $\qquad$ 99^{th} p-tile.

$\beta = 20$

$(.990)$ is at 10^{th} row, 4^{th} column.

$$F(10;4) = F\left(\frac{x_{.99}}{\beta};4\right)$$

$$x_{.99} = 10 \cdot \beta = 200.$$

$$f(x,y) = c\cdot(10 - x - y)$$

$$-2 \leq x \leq 4$$

$$0 \leq y \leq 3$$

$$f_X(x) = \int_{R_y} f(x,y)\,dy$$

$$f_X(x) = c\int_0^3 (10 - x - y)\,dy$$

$$f_Y(y) = c\int_2^4 (10 - x - y)\,dx$$

$E(X|Y) \Rightarrow$ 1st find the conditional pr.

$$\underbrace{f(X|Y)} = \frac{f(x,y)}{f_Y(y)} \Rightarrow \text{plug-in } y=2$$

$$E(X|Y=2) = \int_2^4 x \cdot f(x|y=2)\,dx$$

$$f_y(y) = c\int_2^4 (10-x-y)\,dx$$

$$= c\left(10x - \frac{x^2}{2} - xy\right)\Big|_2^4$$

$$= c\left[\left(40-8-4y\right) - \left(20-2-2y\right)\right]$$

$$1/33 = c\left[14-2y\right] = f_y(y)$$

$$f(x|y) = \frac{f(x,y)}{f_y(y)} = \frac{\cancel{c}\cdot(10-x-y)}{\cancel{c}\left[14-2y\right]}$$

$$E(x|y)\underset{y=2}{=} \int_2^4 x\cdot\frac{(10-x-y)}{(14-2y)}$$

$$= \int_2^4 x\cdot\frac{(8-x)}{(10)} = 1/10\int_2^4 8x - x^2$$

10

$$= 1/10\left(4x^2 - \frac{x^3}{3}\right)_2^4 \quad \frac{1}{10}\left[\left(64-\frac{64}{3}\right) - \left(16-\frac{8}{3}\right)\right]$$

$$\frac{1}{10}\left[\left(64-\frac{64}{3}\right)-\left(16-\frac{8}{3}\right)\right]$$

$$1/10\left[\left(\frac{2\times 64}{3}\right)-\left(\frac{5\times 8}{3}\right)\right]$$

$$\boxed{\frac{88}{30}}$$