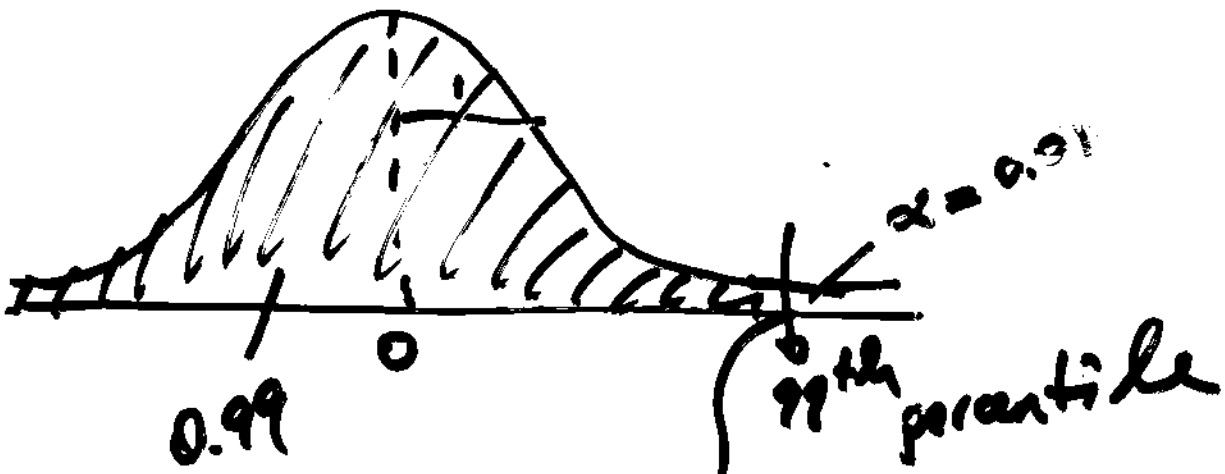
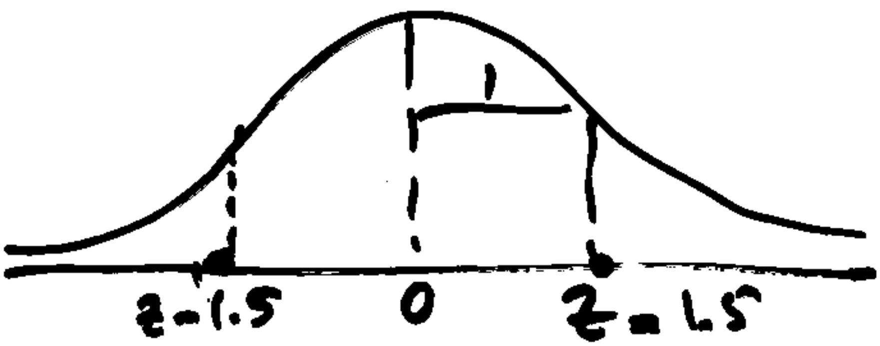


$Z_{.01} = Z_\alpha$

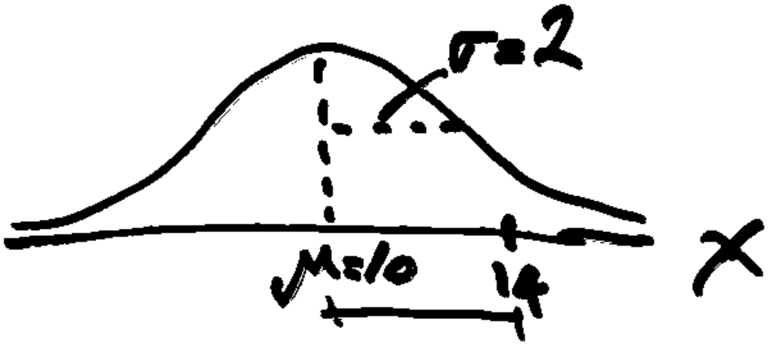
99^{th} percentile $= Z_{.01}$



distance from the mean in terms of the standard deviation.

mean $= \mu$ stdev $= \sigma$

For any Normal r.v. $\dfrac{X-\mu}{\sigma}$

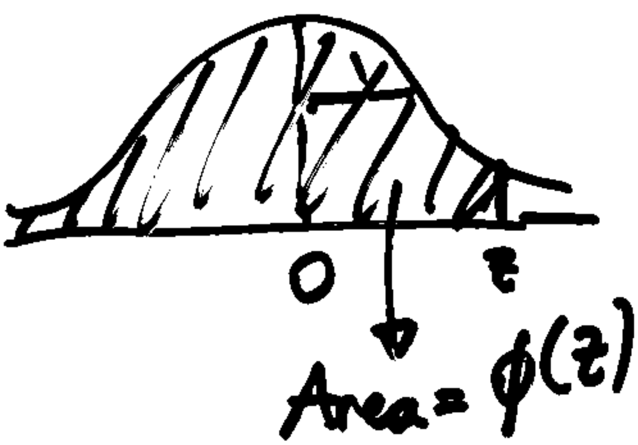


$P(x \le 14)$

$$\underbrace{\boxed{\frac{14-10}{\sigma_{(2)}}}}_{z} = 2 =$$

$$P(x \le 14) = P\left(\underbrace{\frac{x-10}{\sigma}}_{z} \le \frac{14-10}{\sigma}\right)$$

$$= P(z \le 2)$$



Ex. 4.15 on page 166

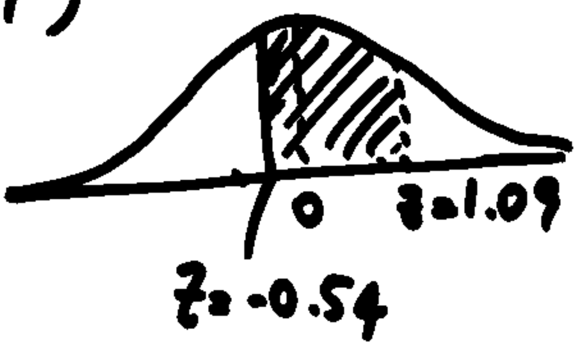
$$X \sim N(\underbrace{1.25}_{\mu}\text{ sec}, \underbrace{0.46^2}_{\sigma^2})$$

$$P(1.00 \leq X \leq 1.75) = ?$$

$$P\left(\frac{1.00-\mu}{\sigma} \leq \underbrace{\frac{X-\mu}{\sigma}}_{z} \leq \frac{1.75-\mu}{\sigma}\right)$$

$$P\left(\frac{1.00-1.25}{.46} \leq z \leq \frac{1.75-1.25}{.46}\right)$$

$$P(-0.54 \leq z \leq 1.09)$$



$$= P(z \leq 1.09) - P(z \leq -0.54)$$

$$= \phi(1.09) - \phi(-0.54)$$

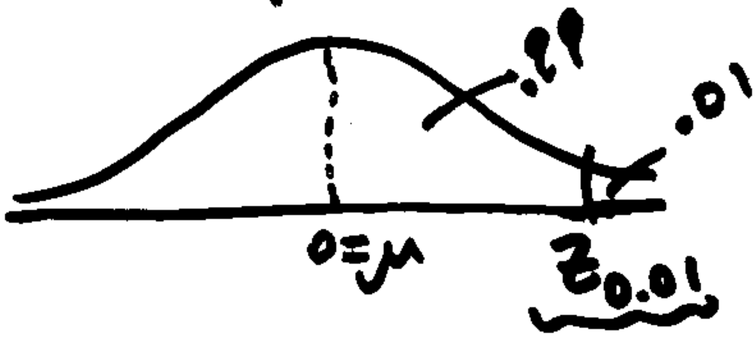
$$= 0.8621 - 0.2946 = 0.5675$$

Finding the percentiles for any Normal Variable.

Lets take Ex. 4.15 page 166

$X \sim N(1.25, 0.46^2)$

99^{th} percentile, and 75^{th} percentile.

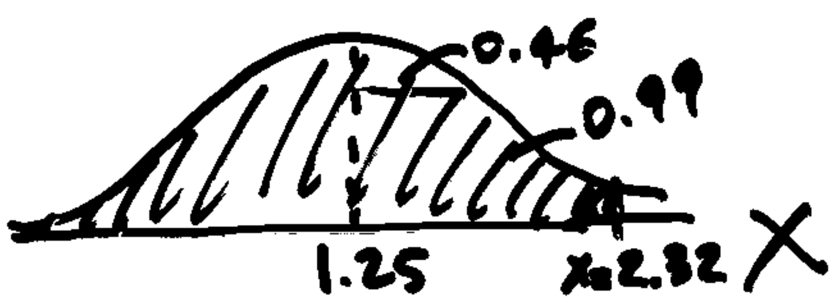


$z_{.01} \cong 2.33$

$(99^{th}\text{ percentile})$ of $X = \mu_x + z_{.01} \times \sigma_x$

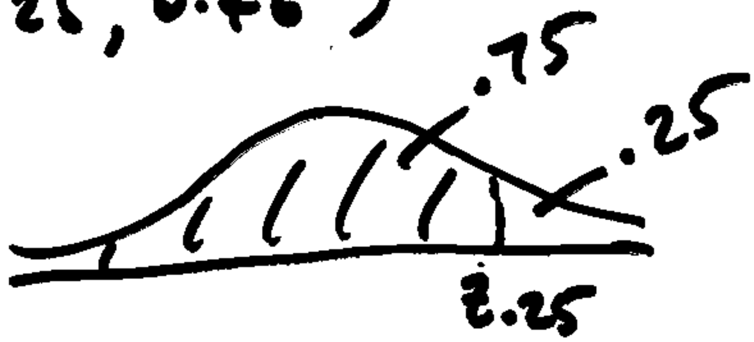
$= 1.25 + (2.33)(0.46)$

$X_{99\%} = 2.32$ sec.



$X \sim N(1.25, 0.46^2)$

$X_{75\%} \Rightarrow$



$z_{.25} =$

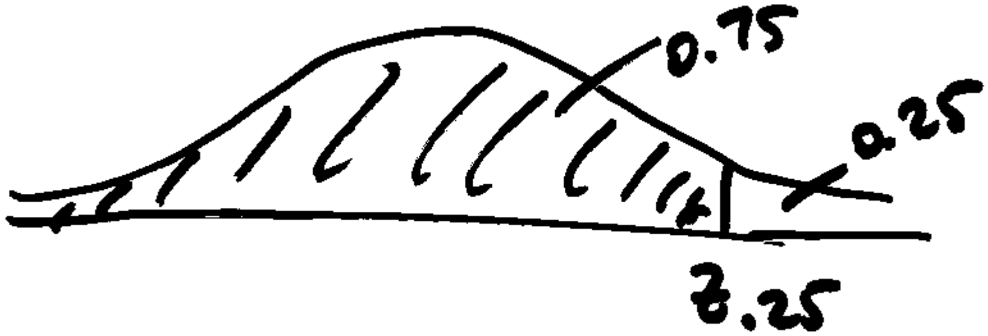
$\phi(0.67) = 0.7486$
$\phi(0.68) = 0.7517$ > 0.7500

$$z_{.25} = \frac{0.67 + 0.68}{2} = \underline{\underline{0.675}}$$

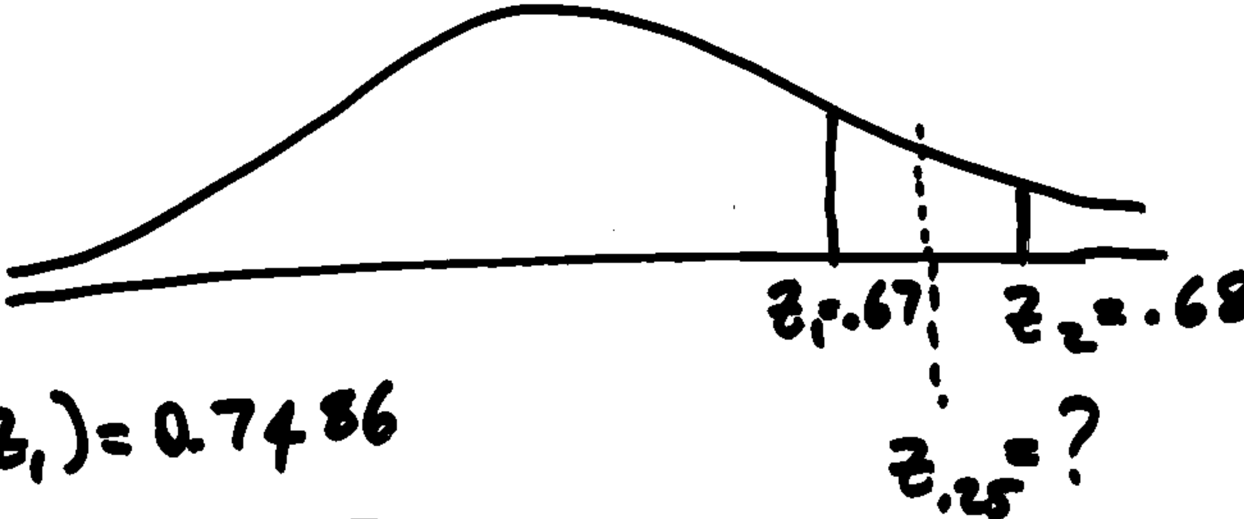
$$X_{(75\%)} = \mu_x + Z_{.25} * (\sigma_x)$$
$$= 1.25 + (0.675)(0.46)$$
$$= 1.56 \text{ sec.}$$

$P(X \leq 1.56) = 0.75$

$z_{.25}$



$\phi(z_{.25}) = 0.75$



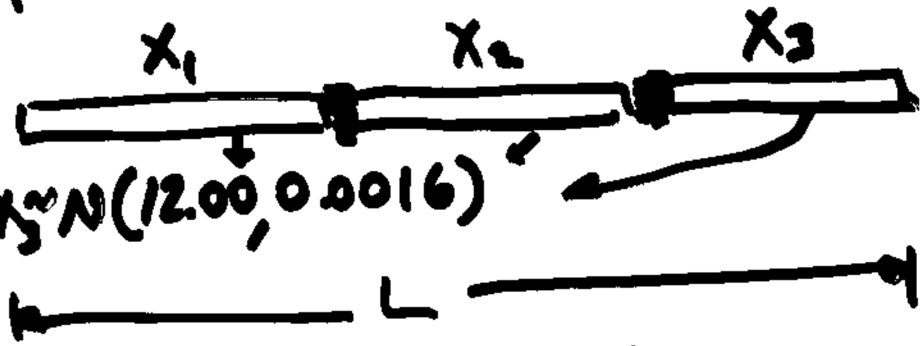
$\phi(z_1) = 0.7486$

$\phi(z_2) = 0.7517$

$\phi(z_{.25}) = 0.7500$

$z_{.25} \cong \dfrac{.67 + .68}{2}$

Reproductive property of Normal Dist



$X_1, X_2, X_3 \sim N(12.00, 0.0016)$

$Y = L = X_1 + X_2 + X_3$

$$E(Y) = \sum_{i=1}^{3} c_i \mu_{x_i}$$

$$E(Y) = \mu_{x_1} + \mu_{x_2} + \mu_{x_3} = 12 + 12 + 12 = 36.00$$

$$V(Y) = \sum_{i=1}^{3} (c_i)^2 \underbrace{(\sigma_{x_i})^2}$$

$(\sigma_{x_i})^2 = 0.0016$

$\sigma_{x_i} = 0.04$

$$= (0.0016)(3) = 0.0048$$

$$Y \sim N(36.00,\ 0.0048)$$

$$P(Y > 36.12'') = ?$$

$$P\left(Z > \frac{36.12 - \mu_y}{\sigma_y}\right) = P\left(Z > \frac{36.12 - 36.00}{\sqrt{0.0048}}\right)$$

$$P(Z > 1.732) = 1 - P(Z \leq 1.732)$$

$$= 1 - .9582 = 0.0418$$

Take 4 pipes

$E(Y) = (12)(4) = 48$

$V(Y) = (0.0016)(4) = 0.0064$

$\sigma_Y = 0.08$

$Y \sim N(48.00,\ 0.0064)$

a)

$P(Y \leq 47.90'') = ?$

$$P\left(Z \leq \frac{47.90 - 48.00}{0.08}\right) = P(Z \leq -1.25)$$

$P(Z \leq -1.25) = \phi(-1.25) = 0.1056$

b) 

$Y =$ the average length of $X_1, X_2, X_3 X_4$

$$Y = \frac{X_1 + X_2 + X_3 + X_4}{4} = (1/4)X_1 + (1/4)X_2 + (1/4)X_3 + (1/4)X_4$$

$c_i = 1/4 \ \forall i \in 1..4$

$E(Y) = 1/4\,\mu_{X_1} + 1/4\,\mu_{X_2} + 1/4\,\mu_{X_3} + 1/4\,\mu_{X_4} = 12.$

$V(Y) = (1/4)^2 V(X_1) + (1/4)^2 V(X_2) + (1/4)^2 V(X_3) + (1/4)^2 V(X_4)$

$V(Y) = 0.0004$

$$Y \sim N(12.00, 0.0004)$$

$$\sigma_y = 0.02$$

$$P(Y > 12.04) = P\left(z > \frac{12.04 - 12}{.02}\right)$$

$$= 1 - P\left(z \leq \frac{12.04 - 12.00}{0.02}\right)$$

$$= 1 - P(z \leq 2.0)$$

$$= 1 - \Phi(2.0) = 1 - 0.9772 = 0.0228$$