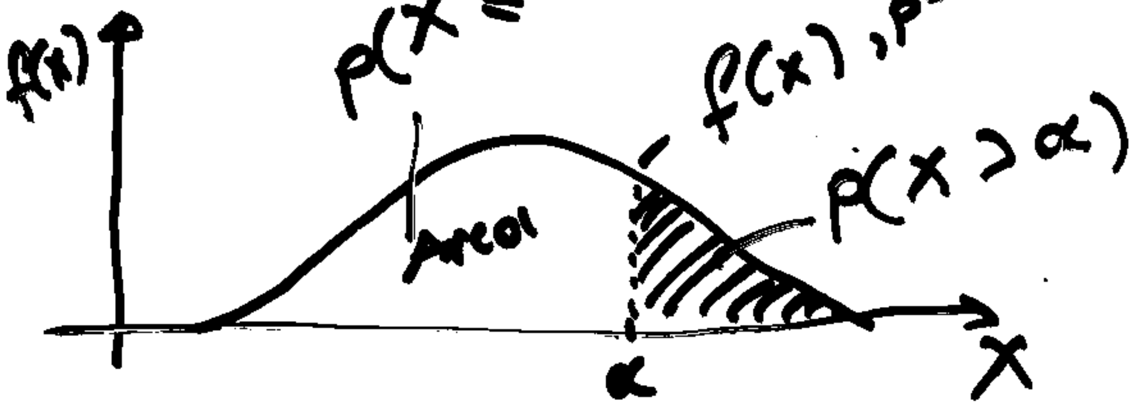
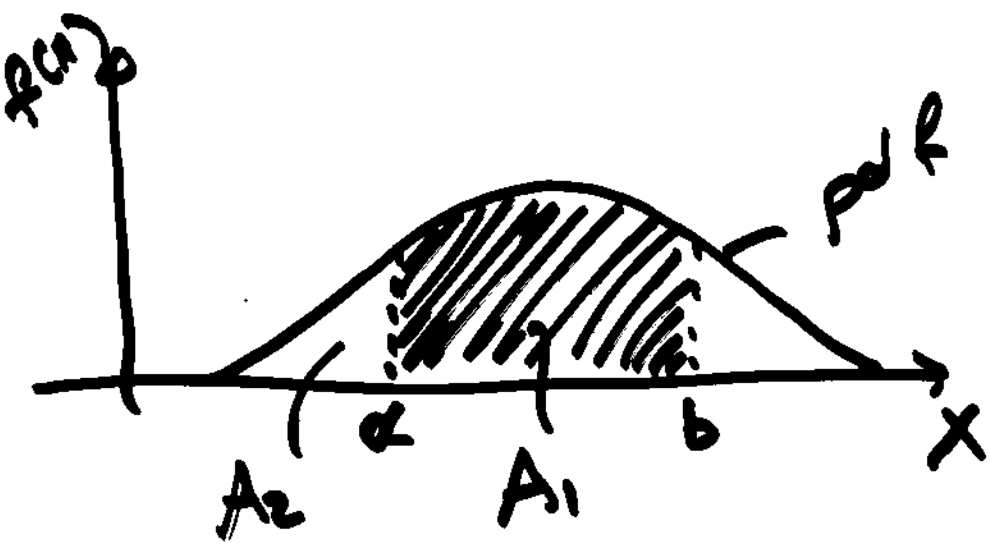
$P(X>\alpha)$

$p(X \le \alpha) = F(\alpha)$



Total area under the curve = 1

$$p(X>\alpha) = 1 - P(X \le \alpha)$$
$$= 1 - F(\alpha)$$



$p(\alpha \le X \le b) = A_1$

$p(\alpha \le X \le b) \Rightarrow \quad F(b) = A_1 + A_2$

$F(\alpha) = A_2$

$A_1 = F(b) - F(\alpha)$

$$\boxed{P(\alpha \le X \le b) = F(b) - F(\alpha)}$$

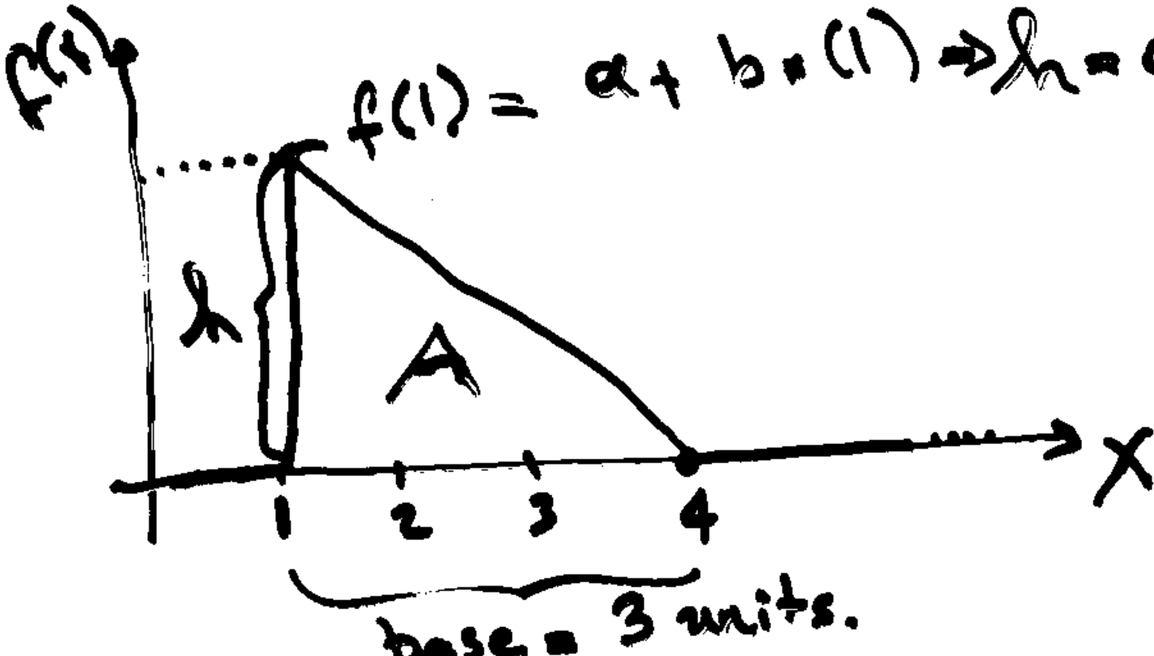
Ex

$$pdf = f(x) = a + bx$$

$$1 \leq X \leq 4 \quad \underline{f(x) = 0 \text{ elsewhere}}$$

$$f(4) = 0$$

$$f(1) = a + b \cdot (1) \Rightarrow h = a + b$$



For $f(x)$ to be a pdf, $A = 1.0$

$$\frac{h \cdot 3}{2} = 1 \Rightarrow h = 2/3$$

$$f(x) = a + bx$$

$$f(4) = 0$$

$$f(1) = 2/3$$

$$f(4) = a + 4b = 0 \quad (1)$$

$$f(1) = a + b = 2/3 \quad (2)$$

$a = -4b \rightarrow$ from eq (1)

$$-4b + b = 2/3 \qquad \boxed{b = -2/9}$$

$$-3b = 2/3$$

eq (2)

$$\alpha - 2/9 = 2/3$$

$$\alpha = 2/3 + 2/9 = 8/9$$

$$f(x) = 8/9 - 2/9\,x \qquad 1 \leq x \leq 4$$

if $f(x)$ is a pdf, the total area under $f(x)$ curve is 1.0

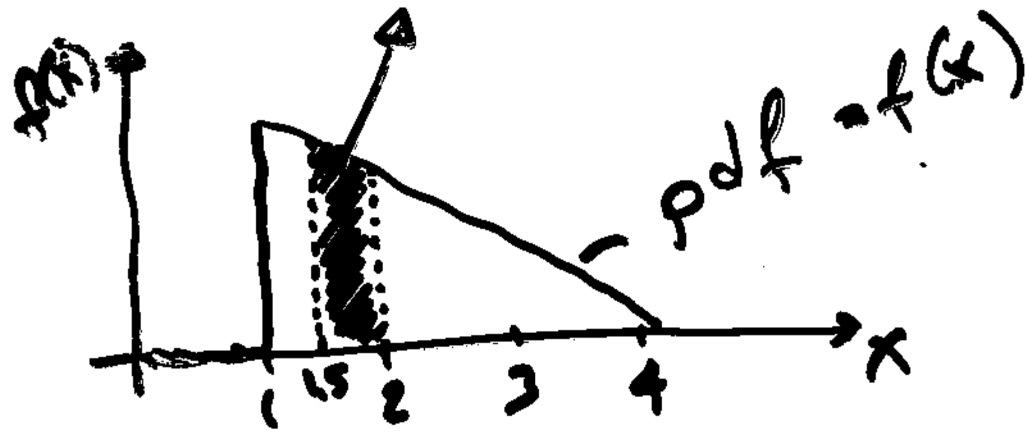
$$\text{Total Area} = \int_1^4 f(x)\,dx = \int_1^4 \left(\frac{-2x+8}{9}\right)dx$$

$$= \frac{1}{9}\int(-2x+8)\,dx = \frac{1}{9}\Big[-x^2 + 8x\Big]_1^4$$

$$= 1/9\Big[\{-(4)^2 + 32\} - \{-(1)^2 + 8\}\Big]$$

$$= 1/9\,[16 - 7] = 1/9 \cdot 9 = \underline{\underline{1.0}}$$

$$P(1.5 \leqslant X \leqslant 2) = ?$$



$$P(1.5 \leq X \leq 2) = \int_{1.5}^{2} f(x)\,dx$$

$$= \frac{1}{9}\left[-x^2 + 8x\right]_{1.5}^{2} = \frac{1}{9}\left[\{-(2)^2 + 16\} - \{-(1.5)^2 + 12\}\right]$$

$$= \frac{1}{9}\left[\{12\} - \{9.75\}\right] = \frac{2.25}{9} = \underline{\underline{0.25\%}}$$

$\underline{OR}$

$$P(1.5 \leqslant X \leqslant 2) = F(2) - F(1.5)$$

$$F(2) = \int_{1}^{2} f(x)\,dx = \frac{1}{9}\left[\{-(2)^2 + 16\} - \{-(1)^2 + 8\}\right]$$

$$F(1.5) = \int_{1}^{1.5} f(x)\,dx = \frac{1}{9}\left[\{-(1.5)^2 + 12\} - \{-(1)^2 + 8\}\right]$$

$$F(2) = 5/9 \quad F(1.5) = \frac{1}{9}\left[9.75 - 7\right] = \frac{2.75}{9}$$

$$F(2) - F(1.5) = \frac{2.25}{9} = \underline{\underline{0.25}}$$

4

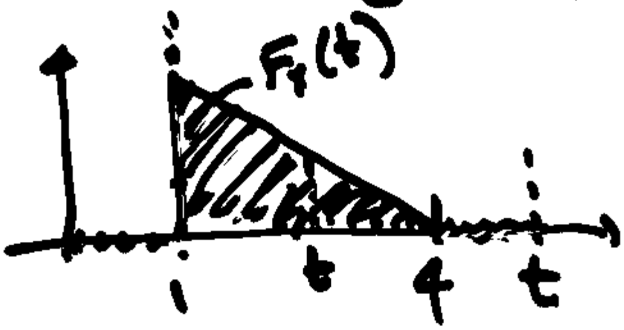
c) $F(X) = ?$

$$F_x(t) = \int_1^t f(x)\,dx = 1/9\left[-(x)^2 + 8x\right]_1^t$$

$$= 1/9\left[\{-(t)^2 + 8t\} - \{-(1)^2 + 8\}\right]$$

$$= 1/9\left[-t^2 + 8t - 7\right]$$

$$F_x(t) = \begin{cases} 0, & \text{if} \\ 1/9(-t^2 + 8t - 7) & 1 \leq t \leq 4 \\ 1.0, & \text{if } t > 4 \end{cases}$$



$$F_x(t) = 1/9\,(-t^2 + 8t - 7)$$

for $t = 1$ $F_x(t) = 0.0$

for $t = 4$ $F_x(t) = 1.0$

$F_X(t)$

$P(1.5 \leq X \leq 2)$

$= F_X(2) - F_X(1.5)$

$= \underbrace{\frac{1}{9}\left[-2^2 + 8 \times 2 - 7\right]}_{F_X(2)} - \underbrace{\frac{1}{9}\left[-(1.5)^2 + 8 \times 1.5 - 7\right]}_{F_X(1.5)}$

$= \frac{5}{9} - \frac{2.75}{9} = \frac{2.25}{9} = 0.25$

$P(a \leq X \leq b)$

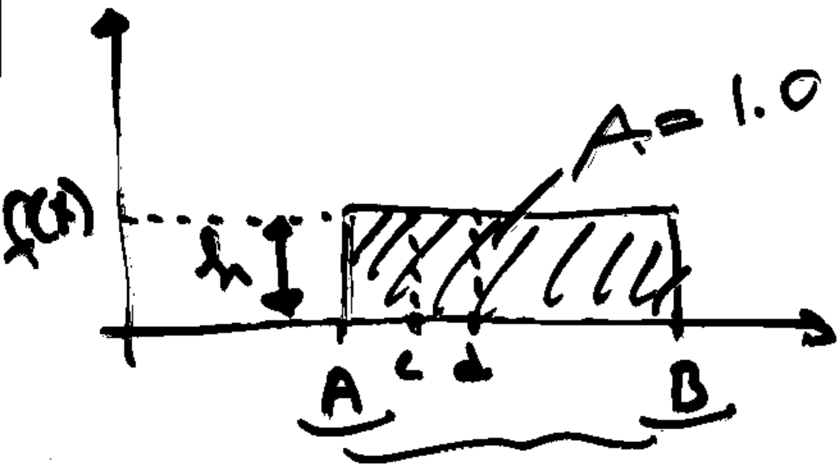
$B = \{ \text{Repair time} \geq 2 \}$

$A = \{ \text{Repair time} \geq 3 \}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

$P(A) = 1 - F_X(3)$ $\quad \frac{P(A)}{P(B)} = \frac{1 - 8/9}{1 - 5/9} = 0.25$

$P(B) = 1 - F_X(2)$



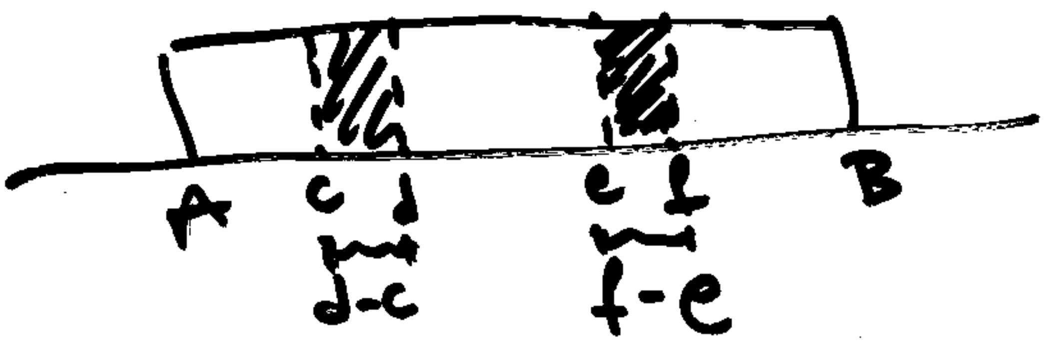
$$f(x) = h \qquad h \times (B - A) = 1.0$$

$$h = \frac{1}{B - A}$$

$$f(x) = \begin{cases} \frac{1}{B-A}, & A \leq x \leq B \\ 0, & \text{otherwise.} \end{cases}$$

$$P(c \leq x \leq d) = P(e \leq x \leq f)$$

$$\text{if } (d - c) = (f - e)$$



$E(X)$, if X is a cont. r.v

$$E(x) = \int_{-\infty}^{+\infty} x \cdot f(x)\,dx$$

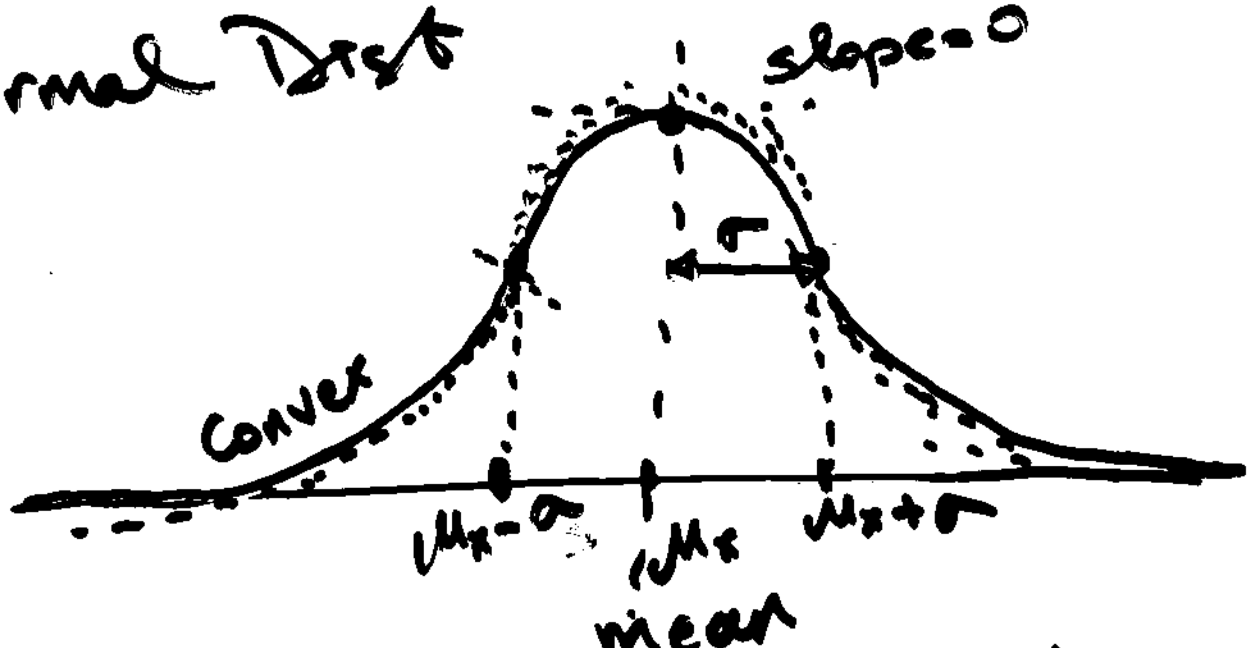
for discrete

$$E(X) = \sum X_i \cdot P(X_i)$$

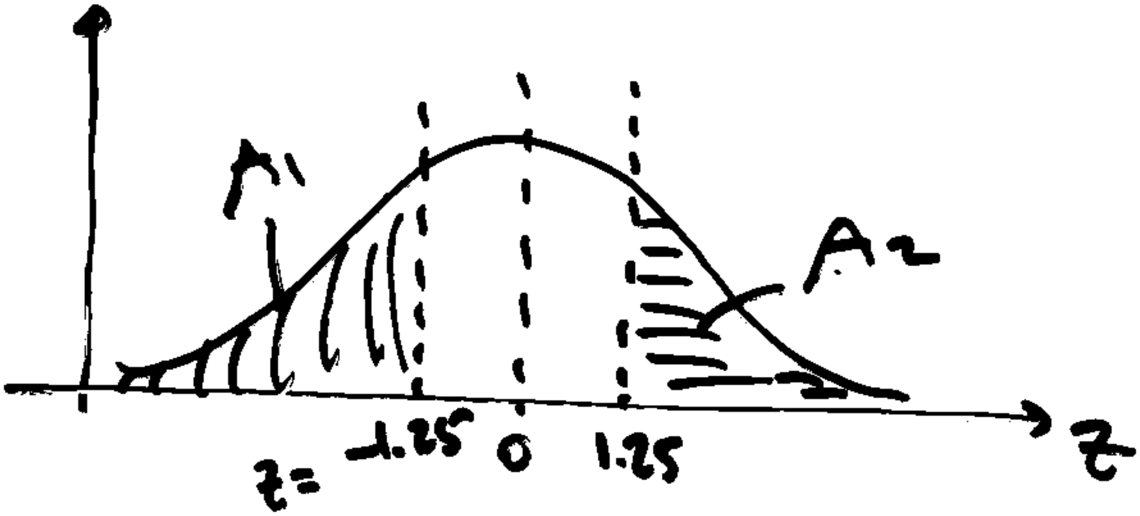
If $h(X)$ is a function of the cont. r.v X, then $E[h(X)]$ is:

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) \cdot f(x) \cdot dx$$

Normal Dist



$$P(a \le X \le b) = \int_a^b f(x)\,dx$$



$$A_1 = P(z \le -1.25)$$

$$A_2 = P(z \ge 1.25) = 1 - \underbrace{P(z \le 1.25)}_{\text{look up } \checkmark \; 0.8944}$$

$$A_2 = 1 - 0.8944$$
$$= 0.1056$$

$$A_1 = A_2 = 0.1056$$