CI for μ (population mean) when σ^2 (population var) is not known:

Ex

Random sample of size $n=13$

$\bar{X} = 59.7$ $\quad S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \bar{X})^2$

$S^2 = 17.64$

$X =$ loudness measure STB

$$T = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

we will construct an upper 1-sided CI.

$\alpha = 0.05 \Rightarrow 1-\alpha = 0.95$

$0 < \mu < x_L$

$n = 13$

$$T = \frac{59.7 - \mu}{\sqrt{17.64}/\sqrt{13}}$$

has a t-dist. with $\nu = 13 - 1 = 12$ d.f.

$$0 < \mu \leq \bar{X} + \underbrace{t_{.05,12}} \cdot \underbrace{S/\sqrt{13}}$$

$$0 < \mu \leq 59.7 + 1.782 \cdot (4.2)/3.6$$

$$0 < \mu \leq 61.78 \rightarrow \text{upper 1-sided CI for } \mu.$$

---

what if X was LTB?

we need to construct a lower 1-sided

CI $\rightarrow L_L < \mu < \infty$

$$\underbrace{\bar{X} - t_{.05,12} \cdot S/\sqrt{n}} < \mu < \infty$$

$$57.70 < \mu < \infty \rightarrow \text{lower-1 sided.}$$

---

what if X was NTB

$$\bar{x} - t_{\alpha/2} \cdot S/\sqrt{n} \leq \mu \leq \bar{x} + t_{\alpha/2} \cdot S/\sqrt{n}$$

$$t_{0.025,12} = 2.179$$

$$59.7 - 2.179 \times \frac{4.2}{3.6} \leq \mu \leq 59.7 + 2.179 \times \frac{4.2}{3.6}$$

$$57.14 \leq \mu \leq 62.22$$

The CI for the variance (σ^2) of a Normal Process:

$$\frac{(n-1) \cdot S^2}{\sigma^2} \sim \chi^2 \text{ distribution.}$$

(Chi squared)

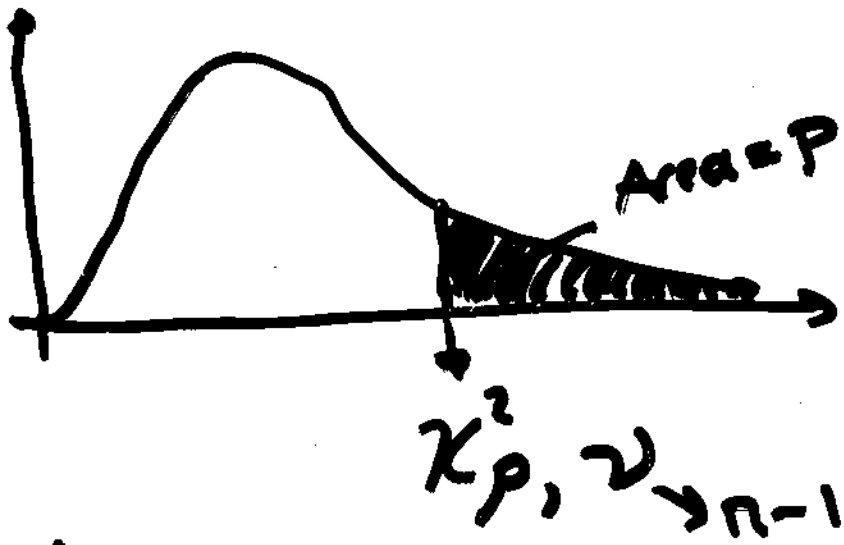

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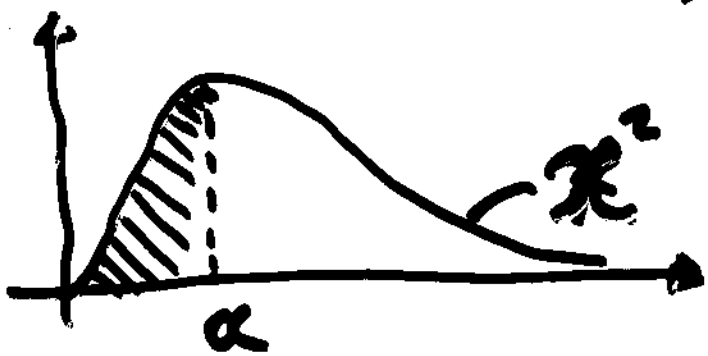
table A.7.

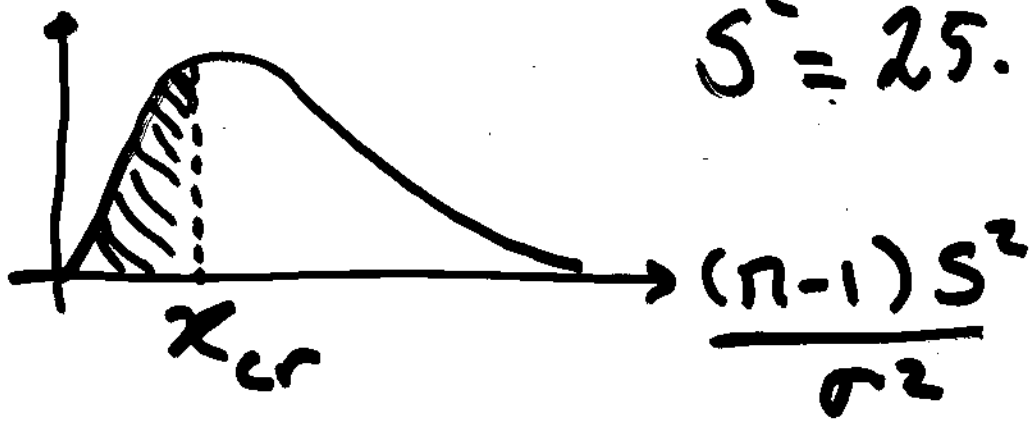


$P(X \leq \alpha) =$ Shaded Area.

Example

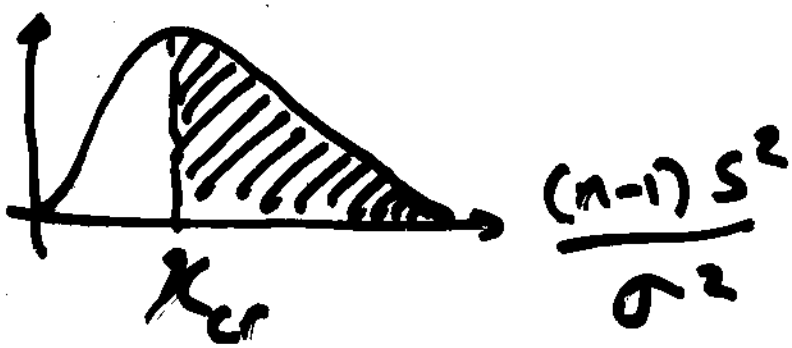
$\alpha = 95\%$, upper 1-sided CI

$S^2 = 25.368$



$$P\left(\frac{(n-1)S^2}{\boxed{\sigma^2}} \leq \chi_{cr}\right) = \text{Area.}$$

$$P\left(\frac{(n-1)S^2}{\chi_{cr}} \leq \sigma^2\right) \times$$



$$P\left(\frac{(n-1)S^2}{\sigma^2} \geq \chi_{cr}\right) = \text{Area}$$

$\chi_{.95,21}$

$$P\left(\frac{(n-1)S^2}{\chi_{cr}} \geq \sigma^2\right) = \text{Area.}$$

$1-\alpha = 0.95$

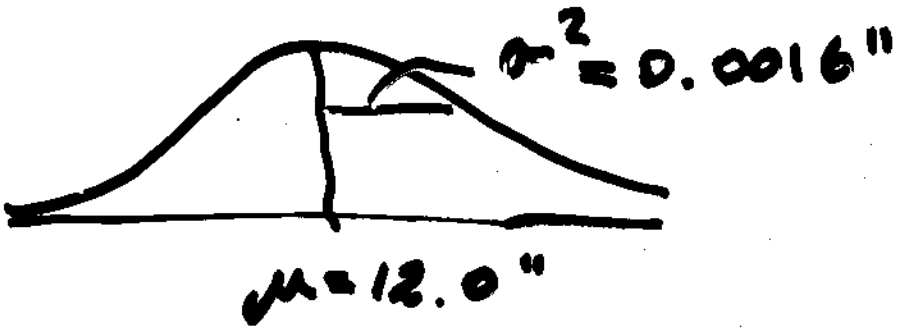
$$\chi_{.95,21} = 11.591$$

$$P\left(\frac{(21)(25.368)}{11.591} \geq \sigma^2\right) = 0.95$$

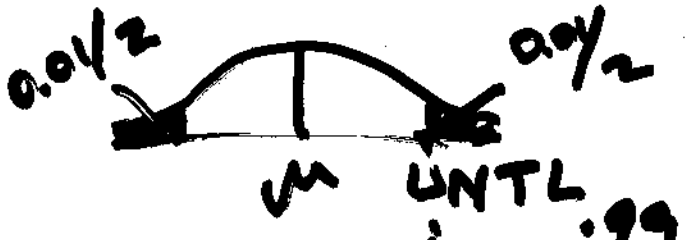
$$0 < \sigma^2 \leq 45.9605$$

$$S^2 = \frac{1}{(n-1)} \sum (x_i - \bar{x})^2 = \frac{CSS}{n-1}$$

TOLERANCE INTERVALS



The 99% Natural Tolerance Interval.



$$\frac{UNTL_{.99} - \mu}{\sigma} = Z_{.01/2}$$

$$\phi(Z_{.01/2}) = .995$$

From table A3 on pg. 741

$Z_{.01/2}$ is between 2.57 and 2.58

$$Z_{.005} = 2.575$$

$$UNTL_{.99} = \mu + Z_{.005} \cdot \sigma$$
$$= 12.0 + (2.575) \cdot (0.04)$$
$$= 12.103$$

$$LNTL_{.99} = \mu - Z_{.005} \cdot \sigma$$
$$= 11.897$$

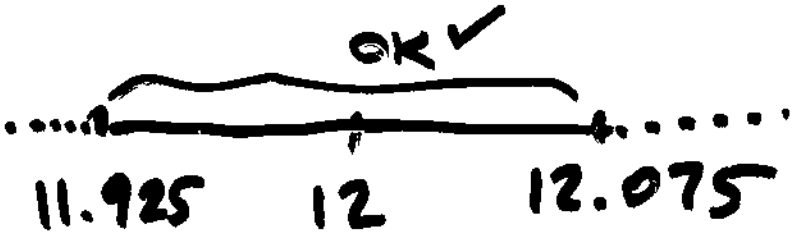
99% of all the produced elements will have a length l

$$11.897 \leqslant l \leqslant 12.103$$

DESIGN TOLERANCE LIMITS

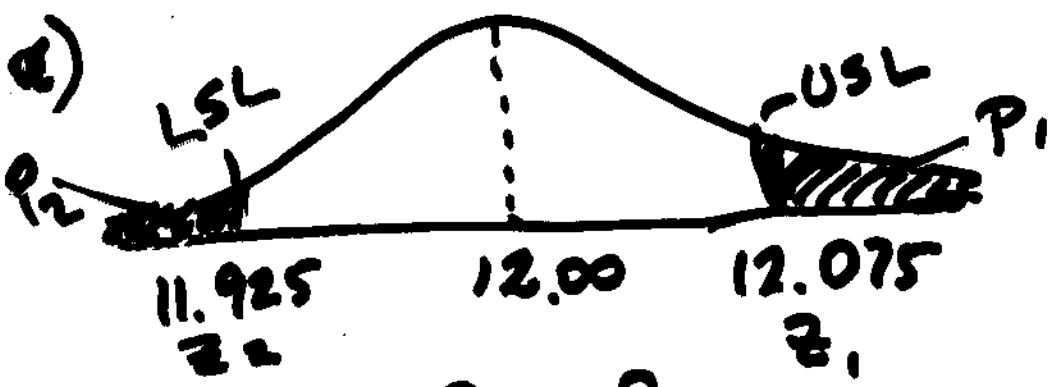
Design specifications

$$12.00'' \pm 0.075''$$



Production process $\sim N(\underset{\mu}{12.00''}, \underset{\sigma^2}{0.0016})$

$\sigma = 0.04$



$$FNC = P_1 + P_2$$

$$z_1 = \frac{12.075 - 12.00}{0.04} = 1.875$$

$$z_2 = \frac{11.925 - 12.00}{0.04} = -1.875$$

$$P_1 = 1 - \phi(z_1) \qquad P_2 = \phi(z_2)$$

$P_1 = 1 - .9699 \qquad P_2 = 0.0301$

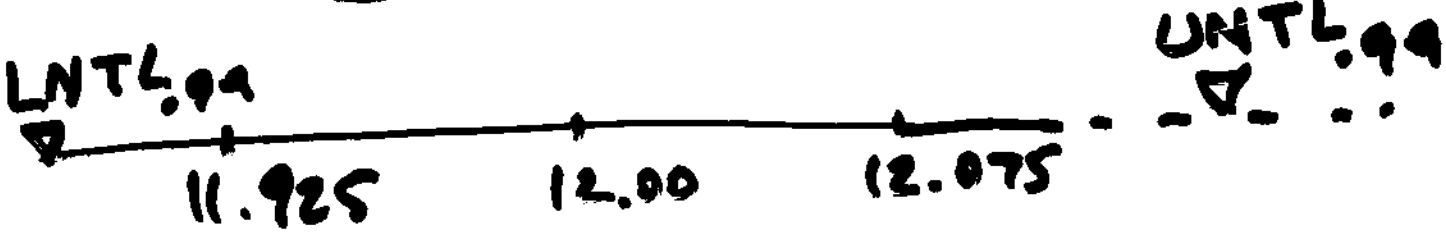
$P_1 = 0.0301$

$FNC = 0.0602 = p$

2) 99% natural tolerance level

$\alpha = 0.01$ since $FNC = p > \alpha$

the process is not capable of meeting the design spec. limits.



a)

$z_{.005} = 2.575$

$$\frac{12.075 - 12.00}{\sigma} = 2.575 \qquad \sigma = \frac{.075}{2.575} = .029$$

$$PCR = \frac{USL - LSL}{(2 \times 2.575)\,\sigma_{new}} = \frac{0.15}{(5.15)(.029)} = 1.004$$

if μ & σ^2 not known

$(\gamma, 1-\alpha)$.

the confidence level about the statement that $(1-\alpha)$ fraction of the entire production will be within $LNTL_{(1-\alpha)}$, $UNTL_{(1-\alpha)}$

Ex $n=25$ $\gamma = 0.95$, $1-\alpha = 99\%$.

$K = 3.457$

The 99% NTL with $\gamma = 95\%$

$\bar{x} \pm K.S$

$75.10 - (3.457)(0.070) = LNTL_{\gamma=.95, 1-\alpha=.99}$

$75.10 + (3.457)(0.07) = UNTL_{.99}$ with $\gamma = .95$

$\alpha\beta = 60.13$ $\qquad$ $\alpha(1+\alpha)\beta^2 = 3944.13$

$\beta^2 = \dfrac{60.13^2}{\alpha^2}$ $\qquad$ $\dfrac{\cancel{\alpha}(1+\alpha)(60.13)^2}{\alpha^{\cancel{2}}} = 3944.13$

$$(1+\alpha)\ 3615.02 = 3944\alpha$$

$$3615.02 + 3615.02\alpha = 3944.13$$
$$3615.02$$

$$3615.02 = 329.11\,\alpha$$

$$\alpha \cong 11$$