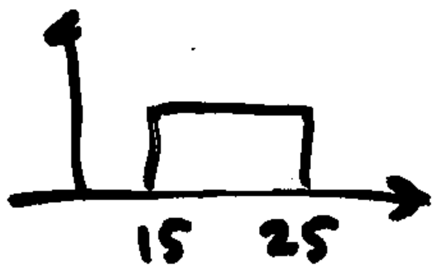
1) <u>pdf = ?</u> Uniform (15, 25)

$$f(x) = \frac{1}{B-A} = \frac{1}{25-15} = \frac{1}{10}$$



$$\cancel{\int_{15}^{25} \frac{1}{10} = pdf}$$

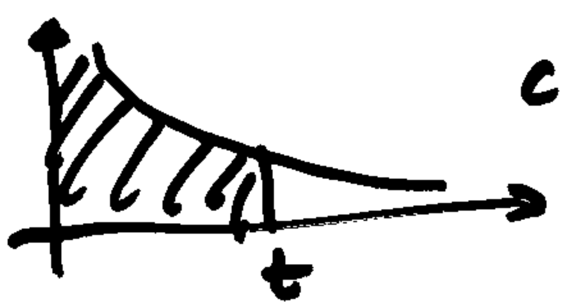
Bonus Test 2

Intervening time T has an expo. distr.

$\lambda = 0.10$ accident/hr.

a) $$\mu = \frac{1}{\lambda} = \frac{1}{.10} = 10.00 \text{ min.}$$

b) $$F(t;\lambda) = 1 - e^{-\lambda t} = 1 - e^{-0.1t}$$

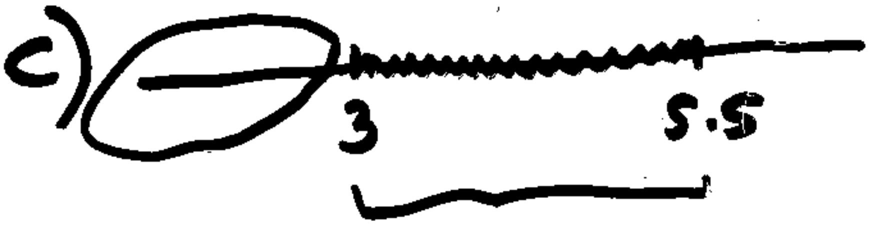


cdf = $P(T \leq t)$

$1 - F(20;\lambda)$

$$P(T > 20) = 1 - \left[1 - e^{-.1(20)}\right] = 0.13534$$

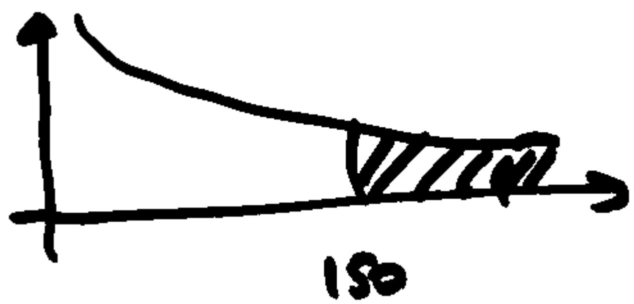
Test 2 Bonus

c) 

Timeline. (hrs)

P(no accidents in the next $\underline{2.5}$ hrs)

2.5 hrs = 150 minutes.



$$1-\left[1-e^{-0.1(150)}\right]$$

$$= e^{-15}$$

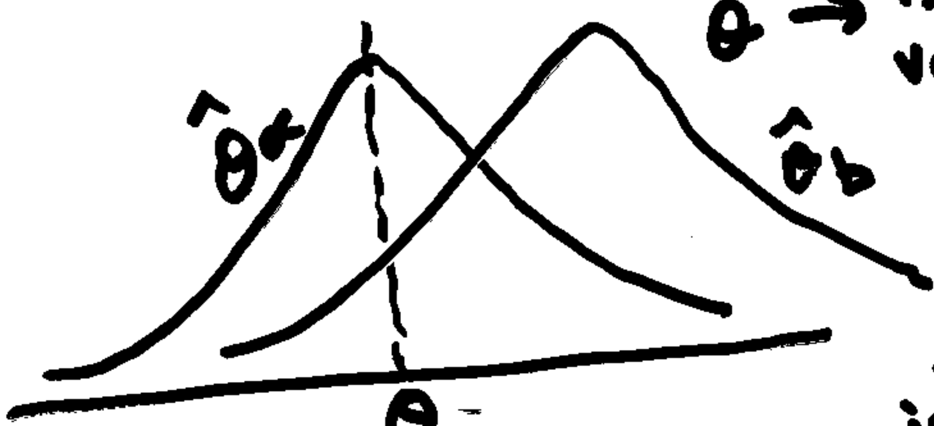
Point estimation

$\theta \rightarrow$ real value of the population parameter.

$\theta \rightarrow$ unknown

$\hat{\theta} \rightarrow$ estimated value.

$\hat{\theta} \rightarrow$ is a random variable.



if $\hat{\theta}a$ and $\hat{\theta}b$ have the same variance, $\hat{\theta}a$ is a better estimator.

MSE = mean square(d) error

$$E\left[(\hat{\theta}-\theta)^2\right]$$

Ex. 1 $\theta=2$

$$MSE(\hat{\theta}_1) = (0-2)^2(1/8) + (6-2)^2(1/8) + (1-2)^2(2/8) + (2-2)^2(4/8)$$

$$= 2.75$$

$$MSE(\hat{\theta}_2) = (1-2)^2(2/5) + (2-2)^2(2/5) + (3-2)^2(1/5)$$

$$= 0.60$$

$$\underbrace{N = 200,000}$$

$$\underbrace{22.5}\ \text{yrs.}$$

if you take a sample of size $n = 200,000$

$\bar{x} = 22.5$

$$\underbrace{s^2 = \frac{1}{(n-1)} \cdot CSS}$$

Sample variance $= \frac{1}{n} \cdot CSS$

$$\frac{s^2}{\text{Sample variance}} = \frac{n}{n-1}$$

Since $\hat{Q}$ is a rv.

$E(\hat{\theta}) - \theta$ = amount of bias $= B(\hat{\theta})$

Exmpe 1 cont'd.

$E(\hat{\theta}_1) = (0)\,1/8 + (6)\,1/8 + (1)\,2/8 + (2)\,4/8$

$= 2.0$

$B(\hat{\theta}_1) = E(\hat{\theta}_1) - \theta$

$= \underbrace{\phantom{E(\hat{\theta}_1)}}_{2.0} - 2.0$

$= 0 \Rightarrow \hat{\theta}_1$ is an unbiased estimator.

$E(\hat{\theta}_2) = 1.80$

$B(\hat{\theta}_2) = 1.80 - 2.0 = -0.20$

$\hat{\theta}_2$ is a biased estimator.

$\underbrace{\text{MSE}}_{\downarrow}$

$\hat{\theta} \to$ a rv $\to$ it has a
mean
variance
(bias)

$\to E(\hat{\theta}) = m$

$$mse(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$$

$$= E[[(\hat{\theta} - m) + \underbrace{(\underset{\downarrow}{m} - \theta)}_{\underbrace{E(\hat{\theta})}_{B}}]^2]$$

$$= E[[(\hat{\theta} - m) + B]^2]$$

$$= E[[\hat{\theta}^2 - 2\hat{\theta}m + m^2 + 2.(\hat{\theta} - m)(B) + B^2]]$$

$$= E[[\hat{\theta}^2 - 2\hat{\theta}m + m^2 + \cancel{2\hat{\theta}B - 2Bm} + B^2]]$$

$$= E[(\hat{\theta} - m)^2 + 2B(\hat{\theta} - m) + B^2]$$

$$= E[(\hat{\theta} - m)^2] + \cancel{2BE(\hat{\theta} - m)} + B^2$$

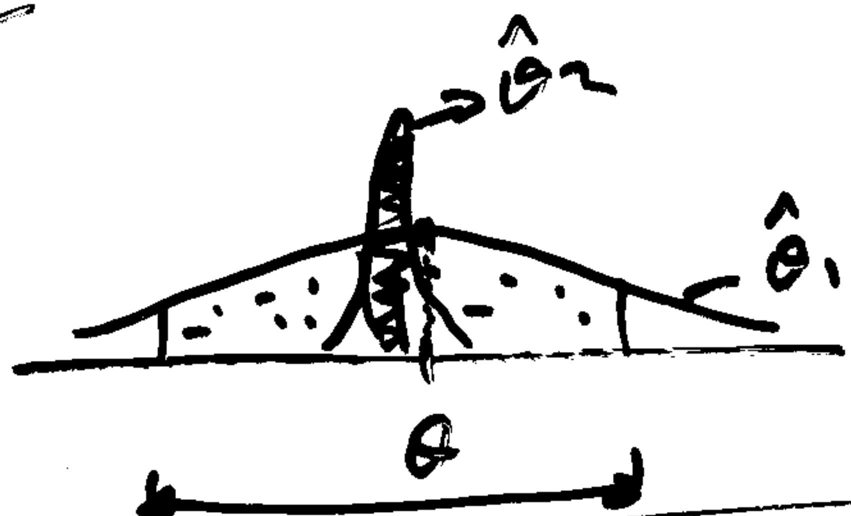
$$0 \quad E(\hat{\theta}) - m$$

$$MSE = \underbrace{E\left[(\hat{\theta} - m)^2\right]} + B^2$$

$$MSE = V(\hat{\theta}) + B^2$$

For any RV $\quad E(X - \mu) = 0$

Ex



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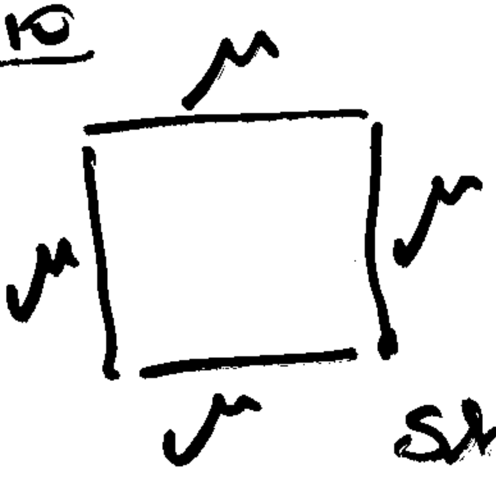
Ex. 10

Area $= \mu^2$



$X_1 \;\; X_2 \;\; X_3 \; \ldots \; X_n$

mean μ and var σ^2

Show that

$$E(\bar{X}^2) - \mu^2 \neq 0$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Parent dist. mean μ
var σ^2

mean of $\bar{X} = E(\bar{X}) = \mu$

variance of $\bar{X} = V(\bar{X}) = \frac{\sigma^2}{n}$

$\bar{X}^2$ estimator for μ^2
biased or not?

for any r v, $E(Y^2) = V(Y) + [E(Y)]^2$

$$E(\bar{X}^2) = V(\bar{X}) + [E(\bar{X})]^2$$

$$E(\bar{X}^2) = \frac{\sigma^2}{n} + \mu^2$$

$$B(\bar{X}^2) = \underbrace{E(\bar{X}^2)} - \mu^2$$

$$= \frac{\sigma^2}{n} + \cancel{\mu^2} - \cancel{\mu^2}$$

$$B(\bar{X}^2) = \frac{\sigma^2}{n} \neq 0$$