Correlation

X_1 = # of products produced.

Price of 1 product = 100\$

$Rev = 100 * X_1$

$Corr(X_1, Rev) = 1.0.$

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2 B = # of breakdowns of a pipeline system.

C = amount of oil transported.

$$C = 500 - 50 \cdot B$$

$Corr(B, C) = -1.0$

Statistics & Their distributions

Ex Lets take AU students.

Parent population = The entire set of AU students.

$\mu = 20.5$ yrs. (Average age)

$\sigma^2 = 4\,yr^2$ ($\sigma = 2$ yrs)

Take a sample of size 20 $n = 20$

(Assuming 20 is large enough for normal distribution approximation of T)

$T = X_1 + X_2 + \dots + X_{20}$ where X_i is the age of ith student in the sample.

$\mu_1 = \mu_2 = \dots = \mu_{20} = \mu = 20.5$

$\sigma_1^2 = \sigma_2^2 = \dots = \sigma_{20}^2 = 4$

RV $T \sim N(n.\mu, n.\sigma^2)$

$T \sim N(410, 80)$

$\frac{T}{n} = \sim N\left(\frac{\cancel{n}.\mu}{\cancel{n}}, \frac{1}{\cancel{n^2}n}.\cancel{n}.\sigma^2\right)$

$\underbrace{T/n}_{\text{Sample Avg.}}$ $\bar{X} \sim N(\mu, \frac{1}{n}\sigma^2)$

rv $Y \to$ mean μ_Y $\quad$ $c.Y \to$ mean $c.\mu_Y$

$\to$ variance σ_Y^2 $\quad$ $\to$ var $c^2.\sigma_Y^2$

Ex

Lec 9, slide 3, part b.

4 pipes are assembled end-to-end.

 $N(12.00, 0.0016)$

Taking a sample of size $4 = n$

$n = 4 \quad P(\bar{X} \geq 12.04) \quad \bar{X} \sim N(\mu, \tfrac{1}{n} \cdot \sigma^2)$

$$\bar{X} \sim N\left(12.00, \frac{0.0016}{4}\right)$$

$$P(\bar{x} \geq 12.04) = P\left(z \geq \frac{12.04 - 12.00}{0.02}\right)$$

$$= P(z \geq 2.0) = 1 - \Phi(2) = 0.0228$$

if I take a sample of size 10

$$\bar{X} \sim N\left(12.00, \frac{1}{10} \cdot 0.0016\right)$$

Lec 12 Examples

X = outcome of the 1^{st} die.
Y = outcome of the 2^{nd} die.

$X \backslash Y$	1	2	3	4	5	6	$P_X(x)$
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	.	.	.	.	.	.	1/6
3	.	.	.	.	.	.	1/6
4	.	.	.	.	.	.	1/6
5	.	.	.	.	.	.	1/6
6	.	.	.	.	.	.	1/6
							$\Sigma = 1.0$
$P_Y(y)$	1/6	1/6	1/6	1/6	1/6	1/6	$\Sigma = 1.0$

$$Cov(X,Y) = E(X \cdot Y) - E(X) \cdot E(Y)$$

$$E(X) = (1)\cdot(1/6) + (2)(1/6) \ldots\ldots$$
$$= 1/6\,(1+2+3+4+5+6) = \frac{21}{6} = 7/2$$

$$E(Y) = 7/2$$

$$E(X \cdot Y) = \sum_{j=1}^{6} \sum_{i=1}^{6} x_i \cdot y_j \cdot P(x_i, y_j)$$
$$= 1/36\,(1+2+3+4+5+6) +$$
$$1/36 \cdot 2 \cdot (1+2+\ldots\ldots+6) +$$
$$1/36 \cdot 3 \cdot (1+2+\ldots+6) +$$
$$1/36 \cdot 6 \cdot (1+2+\ldots\ldots+6) +$$

$$E(X \cdot Y) = \frac{1}{36}\left(21 + 2 \times 21 + 3 \times 21 + 4 \times 21 + 5 \times 21 + 6 \times 21\right)$$

$$= 12.25$$

$$Cov(X, Y) = (12.25) - \underbrace{\left(\frac{7}{2}\right)\left(\frac{7}{2}\right)}_{12.25}$$

$$Cov(X, Y) = 0 = \sigma_{XY} = \sigma_{12}$$

$$\sigma_{11} = \sigma_{XX} = \sigma_X^2$$

$$\sigma_X^2 = E(X^2) - [E(X)]^2$$

x^2	mpmf $= p_X(x)$
1	1/6
4	1/6
9	⋮
16	⋮
25	1/6
36	

$$E(X^2) = \frac{1}{6} \cdot (1 + 4 + 9 + \dots + 36)$$

$$= 15.167$$

$$E(Y^2) = 15.167$$

$$\sigma_{11}^2 = \sigma_X^2 = 15.167 - \left(\frac{7}{2}\right)^2 = 2.917$$

$$\sigma_{22} = \sigma_Y^2 = 2.917$$

$$Cov = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

$$Cov = \begin{bmatrix} 2.917 & 0 \\ 0 & 2.917 \end{bmatrix}$$

$$p(X \mid Y=y) = \frac{p(x,y)}{p_Y(y)}$$

$$p(X \mid Y=4) = \frac{1/36}{1/6} = \frac{6}{36} = 1/6 \text{ for all } x \in 1 \ldots 6$$

$$E(X \mid Y=4) = 1/6\,(1+2+3+4+5+6) = 7/2 = 3.5$$

Problem 2

a) The next 5 computers. Parent population $N(35.50, 25)$

$\Rightarrow$ Sample size $= 5$

$\mu_{\bar{X}}, \sigma_{\bar{X}}^2$ $\quad \bar{X}_a \sim N(\underbrace{35.50}_{\mu_{\bar{X}}}, \underbrace{\tfrac{25}{5}}_{\sigma_{\bar{X}}^2})$

b) $n = 20$

$$\bar{X}_b \sim N(35.50, \tfrac{25}{20})$$

c) $p(35 \leq \bar{X}_a \leq 36) = \frac{35 - 35.5}{\sqrt{5}} \leq z \leq \frac{36 - 35.5}{\sqrt{5}}$

$-0.22 \leq z \leq 0.22$

$\Rightarrow \phi(0.22) - \phi(-0.22)$

$\Rightarrow 0.5871 - 0.4129 = 0.1742$

$$P(35 \leq \bar{X}_b \leq 36)$$

$$\frac{35 - 35.5}{\sqrt{5/4}} \leq z \leq \frac{36 - 35.5}{\sqrt{5/4}}$$

$$-0.45 \leq z \leq 0.45$$

$$\phi(0.45) - \phi(-0.45)$$

$$0.6736 - 0.3264 = 03472$$

for a) $P(35 \leq \bar{X}_{5\,\text{computers}} \leq 36) = .1742$

$$P(35 \leq \bar{X}_{20\,\text{comp.}} \leq 36) = .3472$$