Continuous bivariate r.v.

Conditional pdf.

$$f(x_2 \mid x_1) = \frac{\boxed{f(x_1, x_2)}}{f_1(x_1)}$$

$$= \frac{\cancel{\tfrac{1}{8}}(6 - x_1 - x_2)\cdot\cancel{4}}{2\cancel{8}\cdot\left(\frac{3 - x_1}{\cancel{4}}\right)} = \underbrace{\frac{(6 - x_1 - x_2)}{2(3 - x_1)}}_{f(x_2 \mid x_1)}$$

$2 \leq x_2 \leq 4$

Conditional Expectation.

$$E(x_2 \mid x_1) = \int_2^4 x_2 \cdot \frac{(6 - x_1 - x_2)}{2(3 - x_1)} \cdot dx_2$$

$$\int_2^4 \frac{6x_2 - x_1 x_2 - x_2^2}{2(3 - x_1)}\, dx_2 = \frac{1}{2(3 - x_1)} \int_2^4 (6x_2 - x_1 x_2 - x_2^2)\, dx_2$$

$$= \frac{1}{2(3 - x_1)} \cdot \left(3x_2^2 - \frac{x_1}{2} x_2^2 - \frac{x_2^3}{3}\right)\Bigg|_2^4$$

$$\left(48 - 8x_1 - \frac{64}{3}\right)$$

$$-\left(12 - 2x_1 - \frac{8}{3}\right)$$

$$= \frac{1}{\cancel{2}(3-x_1)}\left(\overset{18}{\cancel{36}} - \overset{3}{\cancel{6}}x_1 - \frac{\overset{28}{\cancel{56}}}{3}\right)$$

$$= \frac{1}{(3-x_1)}\cdot\left(\frac{-9x_1 + 26}{3}\right) = \frac{26-9x_1}{3(3-x_1)}$$

$$E(X_2 \mid X_1) = \frac{26-9x_1}{3(3-x_1)}$$

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2 rvs. C = cost, R = revenue)

$$E(R \mid C) = 1 + (\pi/2)^{15} = 868.95 \rightarrow \text{Case } 1$$

$$\text{Case } 2 \Rightarrow E(R \mid C) = 2500 - C \Rightarrow \text{NOT independent}$$

in Case 1 ⇒ R & C are independent.

Ex. 5.12 page 219

# of seats separating the two.

$h(x_1, x_2) =$ # of seats separating the two.

1 3

5 2

$$h(X_1, X_2) = \underbrace{|x_1 - x_2| - 1}_{\text{function for the \# of seats between the two.}}$$

$h(x_1, x_2)$

x_1 \ x_2	1	2	3	4	5
1		0	1	2	3
2	0		0	1	2
3	1	0		0	1
4	2	1	0		0
5	3	2	1	0	

$$\sum_{x_2=1}^{5} \sum_{x_1=1,\ x_1 \neq x_2}^{5} h(x_1, x_2) \cdot p(x_1, x_2)$$

$$= \sum\sum h(x_1, x_2) \cdot \frac{1}{20}$$

$$E[h(x_1, x_2)] = \frac{1}{20} \sum_{1}^{5} \sum_{1}^{5} h(x_1, x_2)$$

$$= \frac{1}{20}(20) = 1$$

h	0	1	2	3
$p(h)$	0.4	0.3	0.2	0.1

$$E(h) = (0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1)$$

$$= 1.0 \checkmark$$

Covariance

$$Cov(X_1, X_1) = E[X_1 \cdot X_1] - \mu_{x_1} \cdot \mu_{x_1}$$

$$\mu_{x_1} = E(X_1) \Rightarrow Cov(X_1, X_1) = \underbrace{E[X_1^2] - [E(X_1)]^2}_{V(X_1)}$$

For the discrete ex.

$$Cov(X_1, X_2) = \underbrace{E(X_1 \cdot X_2)}\, - \mu_{x_1} \cdot \mu_{x_2}$$

$$h(x_1, x_2) = X_1 \cdot X_2$$

X_1 \ X_2	1	2	3
1	1 .01	2 .05	3 .0.04
2	2 .05	4 .10	6 .10
3	3 .10	6 .15	9 .10
4	4 .04	8 .15	12 0.11

$$E(X_1 \cdot X_2) = (1)(.01) + (2)(.05) + (3)(.04)$$
$$+ (2)(.05) + (4)(.10) + (6)(.10)$$
$$+ (3)(0.10) + (6)(.15) + (9)(.10)$$
$$+ (4)(.04) + (3)(.15) + (12)(.11)$$

$$E(X_1 \cdot X_2) = 6.11$$

$$Cov(X_1, X_2) = 6.11 - (2.85)(2.15)$$
$$= -0.0175$$

Covariance Matrix

$Cov(X_1, X_2) = \sigma_{12}$

$Cov(X_1, X_1) = \sigma_{11} = \sigma_{X_1}^2$

$\sigma_{11} \to \sigma_{X_1}^2$, $\sigma_{12} \to Cov(X_1, X_2)$, $\sigma_{21} \to Cov(X_2, X_1)$, $\sigma_{22} \to \sigma_{X_2}^2$

$$\sigma_{12} = \sigma_{21}$$

$$Cov(X) = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 0.9275 & -0.0175 \\ -0.0175 & 0.5275 \end{bmatrix}$$

$0.5275 \to \sigma_{22}$

$\sigma_{11} \to Var(X_1) = ?$

$E(X_1^2) - [E(X_1)]^2$

mpmf $p_1(x_1)$	x_1	x_1^2
0.10	1	1
0.25	2	4
0.35	3	9
0.30	4	16

$E(X_1^2) = (1)(0.10) + (4)(.25) + (9)(.35) + (16)(.30) = 9.05$

$\sigma_{11} = V(X_1) = 9.05 - (2.85)^2 = 0.9275$