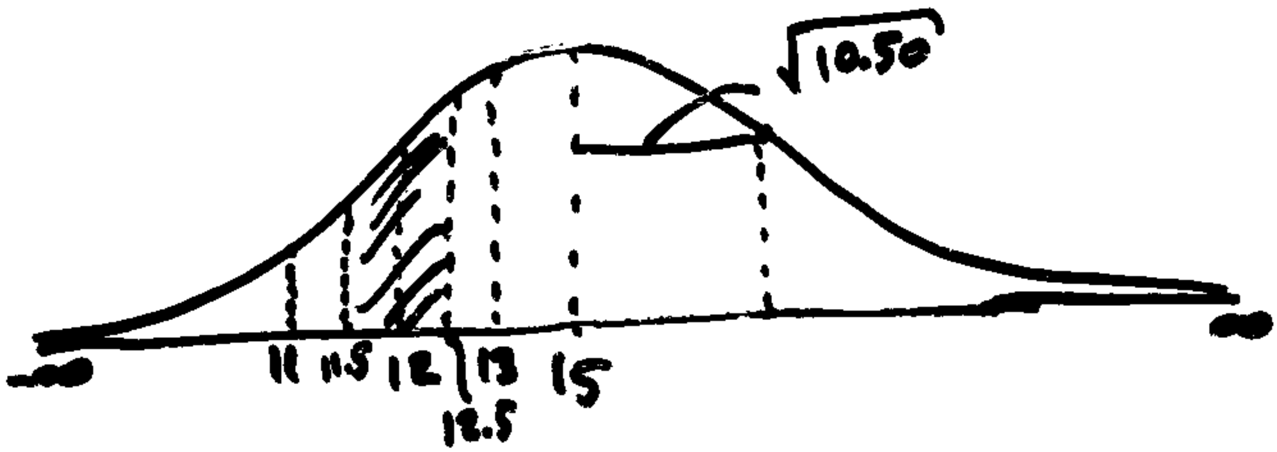
Normal to Binomial Approx.

Ex $n=50$ $p=.30$ = P of success.

X_D = number of successes in 50 trials.

$X_D = \{0, 1, 2, 3, \ldots, 50\}$

X_C = the continuous approximation for X_D. $X_C \sim N(15, 10.50)$



$X_D = 12$ $P(X_D = 12) = ?$ Find using the X_C.

$P(11.5 \leqslant X_C \leqslant 12.5) = P(X_D = 12)$

For $P(X_D = 15)$ Binomial pdf $b(15; 50, .30)$

X = reaction time

Standard gamma ($\beta = 1$) with $\alpha = 2$

$P(3 \leq X \leq 5) = ?$

$= F(5) - F(3)$

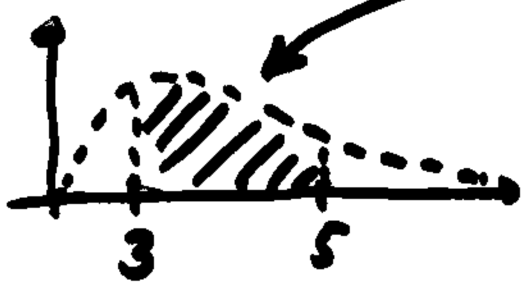
$\Rightarrow F(5) = F_{gamma}(5; \alpha) = F(5; 2)$

$\Rightarrow F(3) = F(3; 2)$

$F(5; 2) = 0.960$

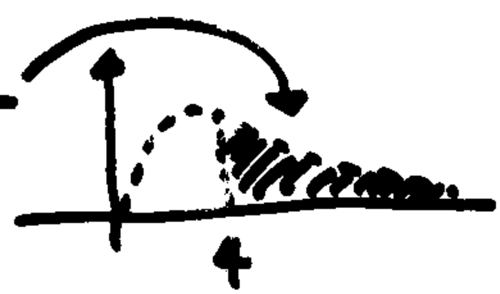
$F(3; 2) = 0.801$

$P(3 \leq X \leq 5) = 0.159$



$P(X > 4) = 1 - F(4; 2)$

$= 1 - 0.908 = 0.092$



Non-standard gamma.

X has gamma pdf with $\alpha = 8$, $\beta = 15$

X → survival time.

$E(X) = (\alpha)(\beta) = 8 \times 15 = 120$ wks.

$V(X) = (\alpha)(\beta)^2 = 1800 \,(\text{week})^2$

$\sigma_X = \sqrt{1800} = 42.43$ week

$P(60 \leq X \leq 120)$ using $F(x;\alpha,\beta)$

$F(120;8,15) - F(60;8,15)$

$\downarrow$

$F\left(\frac{120}{15};8\right) - F\left(\frac{60}{15};8\right)$

From page 742

$\downarrow$

$F(8;8) - F(4;8)$

$0.547 - 0.051$

$P(60 \leq X \leq 120) = 0.496$

$P(X \geq 30) = ?$ $\quad 1 - F(30;8,15)$

$1 - P(X \leq 30) \rightarrow 1 - F(2;8)$

$= 1 - 0.001 = 0.999$

Exponential Dist

Ex $X \to$ # of accidents per 1 day

Poisson dist. $\lambda = 1$

$T \to$ the number of days between two consecutive accidents.

T will have an exponential dist.

$\alpha = 1 \quad \beta = 1/\lambda$

$$E(T) = \frac{1}{\lambda} = (\underset{\substack{\downarrow \\ 1}}{\alpha})(\beta) \to 1/\lambda = \frac{1}{\lambda}$$

for exponential

$$E(T) = 1/1 = 1 \text{ day.}$$

$$V(T) = 1/\lambda^2 = 1 \text{ day}^2 \quad \sigma = \sqrt{\frac{1}{\lambda^2}} = \frac{1}{\lambda}$$

For exponential dist mean = st. dev.

a) $P(T \geq 5)$ $\quad X \to$ # of accidents in a certain period of time.

$$P(T \geq 5) = P(X \text{ (in 5 days)} = 0)$$

$$p(0; \lambda t) = p(0; 1.5) = 0.0067$$

OR $P(T \geq 5) = 1 - F(5)$

$\downarrow$ exponential dist.

$1 - P(T \leq 5)$

$$1 - \left[1 - e^{-\lambda t}\right] = 1 - \left[1 - e^{-5}\right]$$

$$= e^{-5}$$

$$= 0.0067$$

b) $P(T \leq 4) = 1 - e^{-\lambda t} = 1 - e^{-4} = 0.981$

$\delta = 10$

Weibull Dist. Example

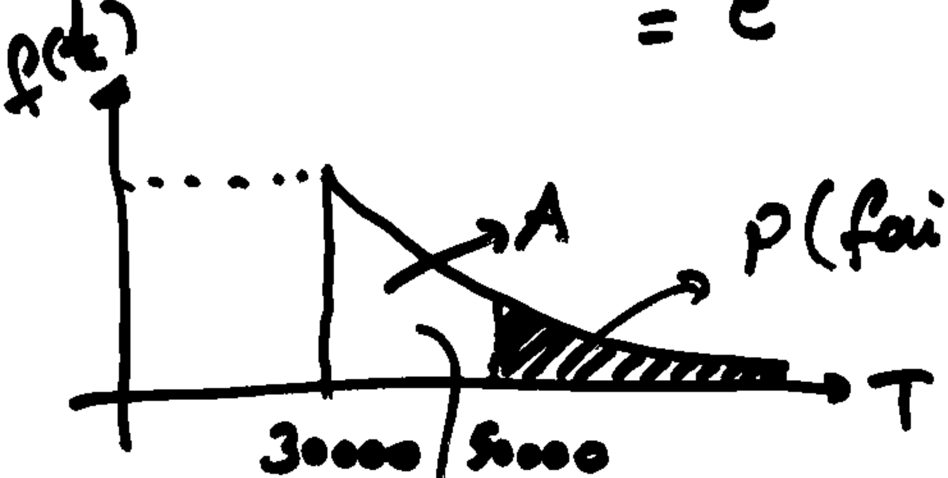
Water pump of a car

$\delta = 30000$ miles $\beta = 65000$ miles $\alpha = 4$

R(50000 miles)

$$R(t;\delta,\alpha,\beta) = e^{-\left(\frac{50000-30000}{65000-30000}\right)^4}$$

$$= e^{-\left(\frac{20}{35}\right)^4} = 0.898865$$



p(failure is after 50000)

p(failure is < 50000 miles)

$$= P(T \leq 50000) = 1 - R(50000)$$

$$= 1 - 0.898865$$

$$= 0.101135$$