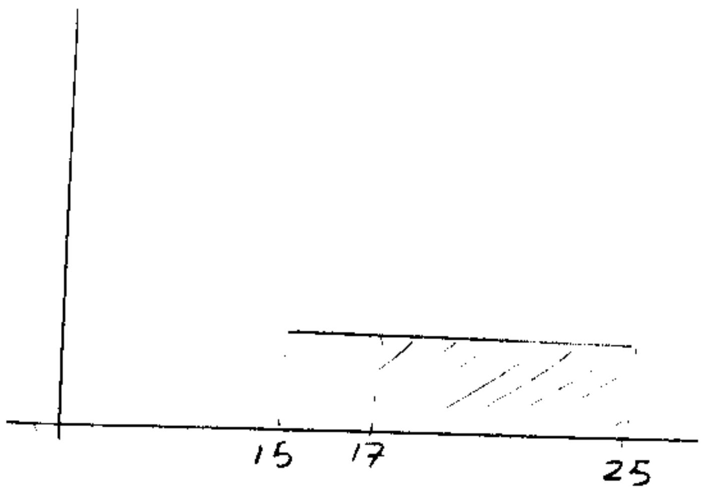
Problem 1

$$f(x, 15, 25) = \begin{cases} \dfrac{1}{25-15} = .1 & 15 \leq x \leq 25 \\ 0 & x < 15 \text{ or } x > 25 \end{cases}$$



$Pr = .8$ $\qquad P(X \geq 17) = \dfrac{8}{10}$

Problem 2

ⓐ P(conforming)

$$P(6.80 \leq X \leq 7.20) = P\left(\frac{6.80-6.98}{.07} \leq Z \leq \frac{7.20-6.98}{.07}\right) =$$

$$P(-2.5714 \leq Z \leq 3.1428) = \Phi(3.1428) - \Phi(-2.5714) =$$

$$= .9992 - .0051 = .9941$$

$$P(\text{nonconforming}) = 1 - P(\text{conforming}) = 5.9 \times 10^{-3}$$

ⓑ $\sigma = \frac{V(y)}{\sqrt{n}} = \sqrt{\frac{.0049}{15}} = .018074$

$$Pr(x \geq 6.95) \Rightarrow P\left(Z \geq \frac{6.95-6.98}{.018074}\right) = P(Z \geq -1.659)$$

$$= 1 - \Phi(-1.66) = 1 - .0485 = .9515$$

one tail

ⓒ $Z = 2.33 \Rightarrow \frac{y-6.98}{.07} = 2.33 \Rightarrow y = 7.1431$

99^{th} percentile;

$Z_{.01} = 2.33$ $\quad y = 99^{th}$ percentile of $X = 6.98 + (2.33)\cdot(0.07)$

$$= 7.1431$$

Problem 3

X \ Y	1	2	3	4	
1	0.06	0.04	0.15	0.09	.34
2	0.12	0.05	0.08	0.14	.39
3	0.07	0.10	0.02	**a** = .08	.27
$P_2(x_2)$	.25	.19	.25	.31	1

a. $a = .08$

b: $E(X) = \mu_1 = (1)(.34) + 2(.39) + 3(.27) = 1.93$

$E(Y) = \mu_2 = (1)(.25) + 2(.19) + 3(.25) + 4(.31) = 2.62$

$E(X^2) = (1)(.34) + (2)^2(.39) + 3^2(.27) = 4.33$

$E(Y^2) = (1)(.25) + (2)^2(.19) + 3^2(.25) + 4^2(.31) = 8.22$

$\sigma_{xx}^2 = V(X) = E(X^2) - [E(X)]^2 = 0.6051 \Rightarrow \sigma_x = 0.777882$

$\sigma_{yy}^2 = V(Y) = E(Y^2) - [E(Y)]^2 = 1.3556 \Rightarrow \sigma_y = 1.164302$

$E(X \cdot Y) = (1)(1)(.06) + 1(2)(.04) + \cdots = 4.94$

$\sigma_{xy} = 4.94 - (1.93)(2.62) = -0.1166$

$$COV = \begin{bmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{xy} & \sigma_y \end{bmatrix} = \begin{bmatrix} 0.6051 & -0.1166 \\ -0.1166 & 1.3556 \end{bmatrix}$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sqrt{\sigma_{xx}\sigma_{yy}}} = \frac{-.1166}{(.7778)(1.164302)} = -0.128755256$$

(c)

$$P(x \mid Y=1) = \begin{cases} \frac{.06}{.25} & x=1 \\ \frac{.12}{.25} & x=2 \\ \frac{.07}{.25} & x=3 \end{cases} = \begin{cases} .24 & x=1 \\ .48 & x=2 \\ .28 & x=3 \end{cases}$$

$$E(x \mid y=1) = \left(\frac{6}{25}\right)(1) + \left(\frac{12}{25}\right)(2) + \left(\frac{7}{25}\right)(3) = \boxed{\frac{51}{25}} = 2.04$$

d. They have $\rho = -.128755$ so they are negetively corrolated and dependent

Problem 4

(a)

$$R(60000\ ;\ 40000,\ 4,\ 80{,}000) = e^{-\left(\frac{\overset{20000}{60000-40000}}{\underset{40000}{80000-40000}}\right)^4} = 0.939413$$

(b) $F(60000, 40000, 4, 80000) = 1 - .939413 = 0.060587$

expected No. of failure $= 1{,}000{,}000 \times .060587 = 60{,}587$

expected cost $= 300 \times 60{,}587 = 18{,}176{,}100 < 25{,}000{,}000$

So it can be done by budget of \$25 000 000

Problem 5

$Y = 4x = 4(15) = 60$

$V(y) = 4V(x) = 4(4) = 16$

(a:) $P(Y < 55) = P\left(\frac{Y - \mu_y}{\sigma_y} < \frac{55-60}{4}\right) = P(Z \leq -1.25) = .1056$

$\Rightarrow P(y < 55) = 10.56\%$

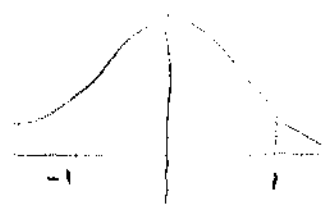
(b) $E(Y) = 15$

$V(Y) = \left(\frac{1}{4}\right)^2(4) + \dots + \left(\frac{1}{4}\right)^2(4) = 4(4)\left(\frac{1}{4}\right)^2 = \boxed{1}$

$Y = N(15, 1)$

$P(14 < Y < 16) = P\left(\frac{14-15}{1} < Z < \frac{16-15}{1}\right) = P(-1 < Z < 1)$

$= 2(\underbrace{.5 - P(Z < -1)}_{\text{sym}}) = .6826 = 68.26\%$



BONUS

(a) mean $= E(T) = \mu = \dfrac{1}{\lambda} = \dfrac{1}{.1\ (1/\text{min})} = 10.00$ Min

(b) cdf $= F(t;\ \overset{\lambda}{.1}) = \begin{cases} 0 & t < 0 \\ 1 - e^{-.1t} & t \geq 0 \end{cases}$

$- \ P(T > 20^{\text{min}}) = 1 - P(T \leq 20^{\text{min}}) = 1 - \left(1 - e^{\overset{\lambda}{(-.1)}\overset{t}{(20)}}\right)$

$= e^{-2} = 0.13534$

(c) $2.5 \times 60 = 150$ min

$= 1 - F(150, \lambda) = 0.000000305902$