STAT 3600 SUMMER 2005 FINAL
SOLUTION

1)

a) (5 pts)

$$\int_0^\theta \frac{2(\theta-x)}{\theta^2}dx = \frac{2}{\theta^2}\int_0^\theta (\theta-x)dx = \frac{2}{\theta^2}\left(\theta x - \frac{x^2}{2}\right)\Big|_0^\theta$$

$$= \frac{2}{\theta^2}\left(\theta^2 - \frac{\theta^2}{2}\right) = 1 \checkmark$$

OK

b) (10 pts)

$$E(X) = \int_0^\theta \frac{2}{\theta^2}x(\theta-x)dx = \frac{2}{\theta^2}\int_0^\theta x(\theta-x)dx$$

$$= \frac{2}{\theta^2}\left(\frac{\theta x^2}{2} - \frac{x^3}{3}\right)\Big|_0^\theta = \frac{2}{\theta^2}\left(\frac{\theta^3}{2} - \frac{\theta^3}{3}\right)$$

$$= \frac{2}{\theta^2}\left(\frac{\theta^3}{6}\right) = \frac{\theta}{3}$$

$$\bar{X} = E(X) \Rightarrow \frac{\hat{\theta}}{3} = \bar{x} \longrightarrow \hat{\theta} = 3\bar{X}$$

$$B(\hat{\theta}) = E(\hat{\theta}) - \theta$$

$$= E(3\bar{x}) - \theta = 3E(\bar{x}) - \theta$$

$$= 3E(X) - \theta$$

$$= 3\left(\frac{\theta}{3}\right) - \theta = 0 \rightarrow \text{unbiased}$$

(5 pts)

c) $\bar{X} = 9.72$

$\hat{\theta} = 29.16$

2 - $E(X) = \bar{X} = \alpha\beta$

2 - $E(X^2) = V(X) + [E(X)]^2 = \alpha\beta^2 + \alpha^2\beta^2$

10 pts

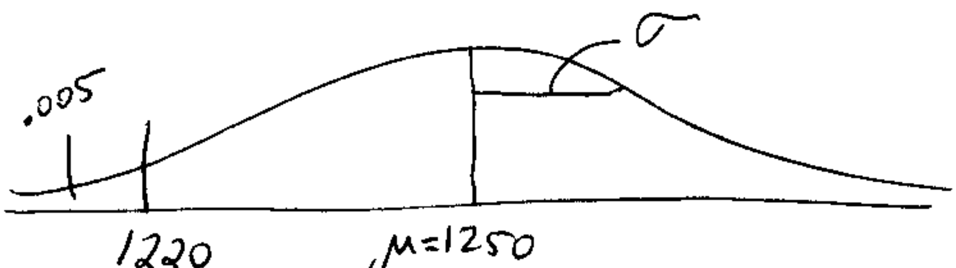
$\bar{X} = 63.1 \quad \hat{\alpha}\hat{\beta} = 63.1 \quad \hat{\alpha}^2\hat{\beta}^2 = 3981.61$

$\hat{\beta}(\hat{\alpha}\hat{\beta}) + \hat{\alpha}^2\hat{\beta}^2 = \frac{1}{10}\sum_{i=1}^{10} X_i^2 = 4419.7$

$\hat{\beta}(63.1) + 3981.61 = 4419.7$

$\hat{\beta} = 6.943 \quad \hat{\alpha} = 9.088$

3 -



$\left(z_{.995} = -2.575 \right)$

a) 10 pts

$\frac{1220 - \mu}{\sigma} = z_{.995} \longrightarrow \frac{1220 - 1250}{\sigma} = -2.575$

$\sigma = 11.65$

b) 5 pts

if $\sigma = 13.1$, since $13.1 > 11.65$, the standard deviation (or hence the variance) is higher, which would cause a $p > 0.01$

4) $\bar{X} = 1235$, $n = 10$

4 $\sum_{i=1}^{10} X_i = n \cdot \bar{X} = 12350$

$USS = \sum_{i=1}^{10} X_i^2 = 15{,}253{,}750$

$CF = \dfrac{(12350)^2}{10} = 15{,}252{,}250$

5 pts $CSS = 15{,}253{,}750 - 15{,}252{,}250 = 1500$

$S^2 = \dfrac{1500}{10-1} = 166.67 \quad S = 12.91$

a) 95% 2-sided CI for μ:

$\bar{X} \pm t_{.025;9} \cdot S/\sqrt{10}$

10 pts $1235 \pm (2.262)(12.91/\sqrt{10})$

$(1225.77, 1244.23)$

b) $0 < \sigma^2 < \dfrac{(n-1)S^2}{\chi^2_{.95;9}} \qquad \chi^2_{.95;9} = 3.325$

5 pts $0 < \sigma^2 < \dfrac{(9)(166.67)}{3.325}$

$0 < \sigma^2 < 451.14$

a) $Z_{.005} = 2.575$

5

99% NTL $\rightarrow \mu \pm Z_{0.005} \cdot \sigma$

10 pts

$12.00 \pm (2.575)(.60)$.

$(10.455, 13.545)$

b) $\mu_{new} = 12.02$ CSL $= (11.84, 12.16)$

10 pts

$\sigma^2_{new} = 0.36$

99% $NTL_{new} = 12.02 \pm (2.575)(.60)$

$(10.475, 13.565)$

NOT CONFORMING!

6) $n = 40$

$USS = 43,638$

$CF = \frac{1320^2}{40} = 43560$

$CSS = 78 \quad s^2 = \frac{78}{39} = 2 \quad s = \sqrt{2} = 1.414$

a) (10pts)

$0 < \sigma^2_{new} < \frac{(n-1)s^2}{\chi_{0.95;39}} \qquad \chi_{.95;39} = 25.695$

$0 < \sigma^2_{new} < \frac{(39)(2)}{25.695} \longrightarrow 0 < \sigma^2_{new} < 3.0356$

$3.0356 < 4$. therefore the new variance is smaller and better. The company will switch to the new system.

b) (10pts) $\gamma = 95\%$ $(1-\alpha) = 95\%$ 2-sided NTL $n = 40$

$k = 2.445$ from table A-6

$(95\%, 95\%)$ NTL of new system

$= \bar{x} \pm kS \Rightarrow \frac{1320}{40} \pm (2.445)(1.414)$

$(29.5428, 36,4572)$

Bonus

a) $E(X) = \int_0^1 x \left(\frac{\theta}{2}+1\right) . x^{\left(\frac{\theta}{2}\right)} dx$

1pt

$= \int_0^1 \left(\frac{\theta}{2}+1\right) x^{\left(\frac{\theta}{2}+1\right)} dx = \left[\frac{\left(\frac{\theta}{2}+1\right)}{\left(\frac{\theta}{2}+2\right)} x^{\left(\frac{\theta}{2}+2\right)}\right]_0^1$

$E(x) = \dfrac{\frac{\theta}{2}+1}{\frac{\theta}{2}+2}$

$\bar{X} = E(X)$

$\Rightarrow \dfrac{\frac{\hat{\theta}}{2}+1}{\frac{\hat{\theta}}{2}+2} = \bar{X}$

$\frac{\hat{\theta}}{2}+1 = \bar{x}\left(\frac{\hat{\theta}}{2}+2\right)$

$\frac{\hat{\theta}}{2}\left(1-\bar{x}\right) = 2\bar{x}-1$

$\boxed{\hat{\theta} = \dfrac{(2\bar{x}-1)}{(1-\bar{x})} \cdot 2}$

b) $\bar{X} = 0.8$ $\quad \hat{\theta} = \dfrac{(2 \cdot 0.8 - 1)}{(1-0.8)} \cdot 2 = \dfrac{0.6}{0.2} \cdot 2 = \frac{6}{1}$

1pt

$\boxed{\hat{\theta} = 6}$

6/7

Bonus b)

12

$E(X) = \mu = 1/\lambda$

$\bar{X} = 1/\lambda = \hat{\mu}$

$E(\hat{\mu}) = E(\bar{X}) = E(x) = \mu$

$\Rightarrow B(\hat{\mu}) = E(\hat{\mu}) - \mu$

$= \mu - \mu$

$= 0$ unbiased