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STAT 3600
Homework 6

1. (a) Chart should equal 1
$a = 1 - 0.97$
$a = 0.03$

(b)

X \ Y	1	2	3	4	5	$P_X(X)$
1	0.10	0.04	0.08	0.08	0.06	0.36
2	0.09	0.05	0.03	0.05	0.02	0.24
3	0.02	0.05	0.02	0.05	0.04	0.18
4	0.03	0.02	0.07	0.04	0.06	0.22
$P_Y(Y)$	0.24	0.16	0.20	0.22	0.18	

(c) X and Y are not independent because
$P(3,4) = 0.05 = ?\ P_X(3) \cdot P_Y(4) = (0.18)(0.22)$
$0.05 \neq 0.0396$

(d) $\mu_x = E(X) = 1(0.36) + 2(0.24) + 3(0.18) + 4(0.22)$
$= 0.36 + 0.48 + 0.54 + 0.88$
$= 2.26$

$\mu_y = E(Y) = 1(0.24) + 2(0.16) + 3(0.20) + 4(0.22) + 5(0.18)$
$= 0.24 + 0.32 + 0.60 + 0.88 + 0.90$
$= 2.94$

$E(X^2) = 1(0.36) + 4(0.24) + 9(0.18) + 16(0.22)$
$= 0.36 + 0.96 + 1.62 + 3.52$
$= 6.46$

$\sigma_x^2 = E(X^2) - [E(X)]^2$
$= 6.46 - 5.1076$
$\sigma_{11} = 1.3524$
$\sigma_x = 1.1629$

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$E(Y^2) = 1(0.24) + 4(0.16) + 9(0.20) + 16(0.22) + 25(0.18)$

$= 0.24 + 0.64 + 1.80 + 3.52 + 4.5$

$= 10.70$

$\sigma_y^2 = E(Y^2) - [E(Y)]^2$

$= 10.70 - 8.6436$

$\sigma_{22} = 2.0564$

$\sigma_y = 1.4340153442$

$E(XY) = 0.10 + 2(.04) + 3(.08) + 4(.08) + 5(.06) + 2(.09) +$
$4(.05) + 6(.03) + 8(.05) + 10(.02) + 3(.02) + 6(.05) +$
$9(.02) + 12(.05) + 15(.04) + 4(.03) + 8(.02) + 12(.07) +$
$16(.04) + 20(.06)$

$= 6.90$

$\sigma_{12} = 6.90 - (2.26)(2.94) \rightarrow COV(X,Y) = E(X,Y) - \mu_x \mu_y$

$6.90 - 6.6444$

$= 0.2556$

$$COV(X,Y) = COV\left(\begin{bmatrix} X \\ Y \end{bmatrix}\right) = \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 1.3524 & 0.2556 \\ 0.2556 & 2.0564 \end{bmatrix}$$

$$\rho = \frac{\sigma_{12}}{\sqrt{\sigma_{xx}\sigma_{yy}}} = \frac{0.2556}{(1.1629)(1.434015342)} = 0.153272655$$

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(c) $p(x|y=4)$

$\frac{p(X,y)}{p_y(Y)} \rightarrow$

X	P(X,y)/Py(Y)
1	.08/.22
2	.05/.22
3	.05/.22
4	.04/.22
	1.0

$E(X|Y=4) \Rightarrow 1(0.363636364)+2(0.227272727+$
$3(0.227272727)+4(0.181818182)$

$\Rightarrow (0.363636364)+(0.454545455)+$
$(0.681818182)+(0.727272727)$

$\Rightarrow 2.227272728$

2. $f(x,y) = C(10-x-y) \quad 2 \le X \le 4,\ 0 \le Y \le 3$ Alexis D. Jaylor

a) $\int_0^3 \int_2^4 C(10-x-y)\,dx\,dy = \int_0^3 C\left(10x - \frac{x^2}{2} - xy\right)\Big|_2^4 dy$

$C\int_0^3 \left[\left(10\cdot 4 - \frac{4^2}{2} - 4y\right) - \left(10\cdot 2 - \frac{2^2}{2} - 2y\right)\right] dy$

$C\int_0^3 (40 - 8 - 4y - 20 + 2 + 2y)\,dy = C\int_0^3 14 - 2y\,dy$

$C(14y - y^2)\Big|_0^3 \rightarrow C(14\cdot 3 - 3^2) \Rightarrow 33C = 1$

$C = \frac{1}{33}$

$C = 0.0303030$

b) $f_X(x) = \int_0^3 \frac{1}{33}(10-x-y)\,dy = \frac{1}{33}\left[10y - xy - \frac{y^2}{2}\right]_0^3$

$\frac{1}{33}\left[10\cdot 3 - 3x - \frac{3^2}{2}\right] = \frac{1}{33}\left(30 - 3x - \frac{9}{2}\right),\ 2 \le x \le 4$

Check: $\int_2^4 \frac{1}{33}\left(30 - 3x - \frac{9}{2}\right) dx \rightarrow \frac{1}{33}\left[30x - \frac{3x^2}{2} - \frac{9x}{2}\right]_2^4$

$\frac{1}{33}\left[\left(30\cdot 4 - \frac{3\cdot 4^2}{2} - \frac{9\cdot 4}{2}\right) - \left(30\cdot 2 - \frac{3\cdot 2^2}{2} - \frac{9\cdot 2}{2}\right)\right]$

$\frac{1}{33}(120 - 24 - 18 - 60 + 6 + 9) = \frac{33}{33} = 1$ ✓

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① $E(X) = \int_2^4 x \frac{1}{33}\left(30 - 3x - \frac{9}{2}\right)dx$

$\frac{1}{33}\int_2^4 \left(30x - 3x^2 - \frac{9x}{2}\right) dx$

$\frac{1}{33}\left[\frac{30x^2}{2} - \frac{3x^3}{3} - \frac{9x^2}{4}\right]_2^4$

$\frac{1}{33}\left[\left(\frac{\cancel{30}^{15}\cdot 4^2}{\cancel{2}} - \frac{\cancel{3}\cdot 4^3}{\cancel{3}} - \frac{9\cdot 4^{\cancel{2}}}{\cancel{4}}\right) - \left(\frac{\cancel{30}^{15}\cdot 2^2}{\cancel{2}} - \frac{\cancel{3}\cdot 2^3}{\cancel{3}} - \frac{9\cdot \cancel{2^2}}{\cancel{4}}\right)\right]$

$\frac{1}{33}(240 - 64 - 36 - 60 + 8 + 9) = \frac{1}{33}97 = 2.939393939$

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1 $f_y(Y) = C\int_2^4 (10-x-y)\,dx$

$C\left(10x-\frac{x^2}{2}-xy\right)_2^4 = C(40-8-4y-20+2+2y)$

$C(14-2y) = f_y(Y)$

$f(x|y) = \frac{f(x,y)}{f_y(Y)} = \frac{C(10-x-y)}{C(14-2y)}$

$E(X|Y=2) \to \int_2^4 x\,\frac{(10-x-y)}{(14-2y)}\,dx \Rightarrow \int_2^4 x\,\frac{(10-x-(2))}{(14-2(2))}\,dx$

$\int_2^4 x\,\frac{(8-x)}{10}\,dx = \frac{1}{10}\int_2^4 8x-x^2\,dx = \frac{1}{10}\left(4x^2-\frac{x^3}{3}\right)_2^4$

$\frac{1}{10}\left[\left(4(4)^2-\frac{(4)^3}{3}\right)-\left(4(2)^2-\frac{(2)^3}{3}\right)\right] = \frac{1}{10}\left[\left(64-\frac{64}{3}\right)-\left(16-\frac{8}{3}\right)\right]$

$\frac{1}{10}\left[\frac{128}{3}-\frac{40}{3}\right] = \frac{1}{10}\cdot\frac{88}{3} = \frac{88}{30} = 2.9333$

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3. $X \sim N(125.50 \text{ mins}, 49 \text{ min}^2)$

ⓐ sample size = 7 cars parent pop = (125.50, 49)
$\bar{X} \sim N(125.50, 49/7) \rightarrow (125.50, 7)$

ⓑ sample size = 14 cars parent pop = (125.50, 49)
$\bar{X} \sim N(125.50, 49/14) \rightarrow (125.50, 7/2)$

ⓒ P(122 < average repair time < 129)

$P(122 < \bar{X}_a < 129) \rightarrow \frac{122-125.50}{\sqrt{7}} < Z < \frac{129-125.50}{\sqrt{7}}$

$-1.32 < Z < 1.32$

$\Rightarrow \Phi(1.32) - \Phi(-1.32)$
$\Rightarrow 0.9066 - 0.0934$
$\Rightarrow 0.8132$

$P(122 < \bar{X}_b < 129) \rightarrow \frac{122-125.50}{\sqrt{7/2}} < Z < \frac{129-125.50}{\sqrt{7/2}}$

$-1.87 < Z < 1.87$

$\Rightarrow \Phi(1.87) - \Phi(-1.87)$
$\Rightarrow .9693 - .0307$
$\Rightarrow 0.9386$