Problem 1 **HW 7**

1^{st} moment of a sample = $\bar{X} = E(X)$

2^{nd} moment of a sample = $E(X^2) = \frac{1}{n}\sum_{i=1}^{n} x_i^2$

$\bar{X} = 60.13333 = \alpha\beta$

$E(X^2) = 3944.133 = \alpha(\alpha+1)\beta^2 = \alpha^2\beta^2 + \alpha\beta^2$

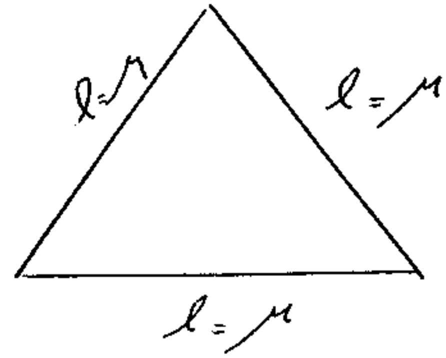
$(60.13333)^2 + \alpha\beta^2 = 3944.133 \Rightarrow \alpha\beta^2 = 328.1156$

$\beta = \frac{\alpha\beta^2}{\alpha\beta} = \frac{328.1156}{60.13333} = 5.456467$

$\alpha = \frac{\bar{X}}{\beta} = \frac{60.13333}{5.456467} = 11.02056$

Problem 2

$A = \text{Area} = \frac{\sqrt{3}\,\mu^2}{4}$



$$E(A) = E\left(\frac{\sqrt{3}}{4}\bar{x}^2\right) = \frac{\sqrt{3}}{4} E(\bar{x}^2)$$

$$= \frac{\sqrt{3}}{4}\left[V(\bar{x}) + \left(E(\bar{x})\right)^2\right]$$

$$= \frac{\sqrt{3}}{4}\,(V(\bar{x}) + \frac{\sqrt{3}}{4}\bar{x}^2$$

$$B(A) = B\left(\frac{\sqrt{3}}{4}\bar{x}^2\right) = \frac{\sqrt{3}}{4} V(\bar{x}) + \cancel{\frac{\sqrt{3}}{4}\bar{x}^2} - \cancel{\frac{\sqrt{3}}{4}\bar{x}^2} = \frac{\sqrt{3}}{4} V(\bar{x})$$

$B(A) = \frac{\sqrt{3}}{4} V(\bar{x}) = \frac{\sqrt{3}}{4}\frac{\sigma^2}{n} \neq 0$ so it is biased

and amount of bias is $\frac{\sqrt{3}}{4}\frac{\sigma^2}{n}$

3) a)

For a 2-sided CI for the population mean μ

For large samples : $\bar{X} \pm z_{\alpha/2} \cdot S/\sqrt{n}$

For small samples : $\bar{X} \pm t_{(n-1);\, \alpha/2} \cdot S/\sqrt{n}$
($n<40$)

In both cases, $\bar{X}$ is placed at the center of the CI. Therefore $\bar{X} = \dfrac{114.4 + 115.6}{2} = 115.0$

b) A higher level of confidence means smaller α, thus larger z or t values. Therefore a larger interval.

For a given sample, a higher confidence level will always result in wider confidence intervals.

4)

$n = 110 > 40$ → use z values from the standard normal distribution.

$\bar{X} = 0.81 \qquad S = 0.34$

The 99% CI for population mean is
(two-sided)

$$\bar{X} \pm z_{.01/2} \cdot S/\sqrt{n}$$

$$= \bar{X} \pm z_{.005} \cdot S/\sqrt{n}$$

$$z_{.005} = 2.575 \quad \left(\frac{2.57 + 2.58}{2} \text{ from table A.3} \right)$$

⇒ the 99% CI is

$$\left(0.81 - (2.575)\left(\frac{0.34}{\sqrt{110}}\right),\ 0.81 + (2.575)\left(\frac{0.34}{\sqrt{110}}\right) \right)$$

$$= (0.7265,\ 0.8935)$$

we are 99% confident that μ lies between

0.7265 sec. and 0.8935 secs.

5)

$n = 46$, > 40 so use standard normal dist.

$\bar{X} = 382.1$ min. $s = 31.5$ min.

the 95% upper CI or confidence bound for the true mean charge-to-tap time.

$$\bar{X} + z_{.05} \cdot s/\sqrt{n} \qquad z_{.05} = 1.645 \quad \left(\frac{1.64 + 1.65}{2}\right)$$

from table A-3

$$\Rightarrow \quad 382.1 + (1.645)\left(\frac{31.5}{\sqrt{46}}\right) = 389.74$$

We are 95% confident that the true time-to-tap is less than 389.74 minutes.

$(0, 389.74]$ is the 95% CI

6)

$n = 14$

$\bar{X} = 8.48$ MPa

$S = 0.79$ MPa

a) 95% lower confidence bound

$t_{.05,\,13} = 1.771$

95% lower confidence bound

$$= 8.48 - (1.771)\left(\frac{0.79}{\sqrt{14}}\right)$$

$$= 8.11.$$

We are 95% confident that the true proportional limit stress is larger than 8.11 MPa, or the true proportional limit stress lies within

$[8.11, \infty)$

b) We did not cover this, but from page 304, eq. 7.16,

A lower 95% prediction bound

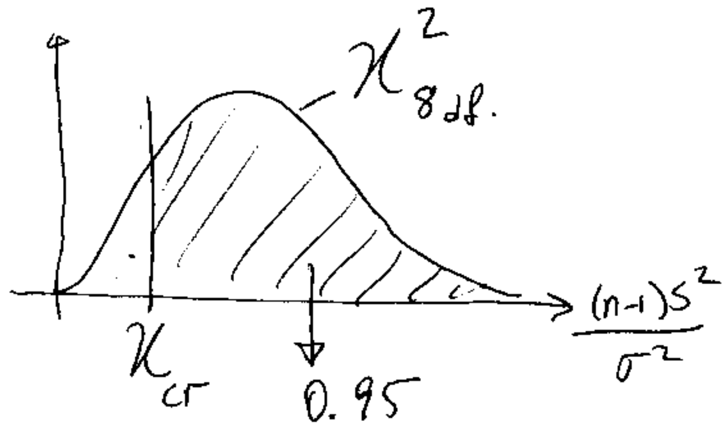
$$= 8.48 - (1.771)(0.79)\left[1 + \frac{1}{14}\right] = 7.03$$

Means we are 95% confident that a randomly selected single item from the population will have a proportional limit stress of 7.03 MPa.

7) $n = 9$

$s = 2.81$

$\% 95$ CI for σ^2



$$P\left(\frac{(n-1)s^2}{\sigma^2} \geq \chi_{cr}\right) = .95$$

$$\chi_{cr} = \chi_{.95,\,8} = 2.733$$

$$P\left(\sigma^2 \leq \frac{(n-1)s^2}{\chi_{cr}}\right) = .95$$

$$P\left(\sigma^2 \leq \frac{(8)(2.81)^2}{(2.733)}\right) = .95 \Rightarrow 0 < \sigma^2 \leq 23.1134$$

$$0 < \sigma \leq 4.808$$