Exam # 2  
.ELEC 5760/6760  
Wed 12/3/14

Constants:  \( \pi = 3.14159, \quad \varepsilon_0 = 8.854\text{pF/m}, \quad 1\text{G} = 9.8\text{m/s}^2, \quad 1\text{atm} = 101.325\text{kPa} \)

Equations:  
\[
PE = mgh, \\
KE = \frac{1}{2}mv^2, \\
P_s = \rho gh, \\
F_{PPA} = \frac{\varepsilon_0 \varepsilon_r A V^2}{2d^2} \\
V_{pl} = \sqrt{\frac{8kd_o^3}{27A\varepsilon_0 \varepsilon_r}}, \\
A_{circle} = (\pi)r^2, \\
d = a\left(\frac{m}{k}\right) = aS, \\
y(t) = \frac{2m\Omega^2}{c^2\omega_n^2}\cos(\omega_n t) \\
P_t = P_s + \frac{\rho v^2}{2}, \\
f_d = \frac{f_s}{1 + \frac{v_{object}}{v_{wave}}}, \\
V_{rms} = \frac{V_{amplitude}}{\sqrt{2}}, \\
C_{PPA} = \frac{\varepsilon_0 \varepsilon_r A}{d}
\]

Laplace Transforms:  
\[
\mathcal{L}\{\text{u}(t)\} = \frac{c}{s}, \\
\mathcal{L}\{\text{e}^{-at}\} = \frac{c}{s + a}, \\
\mathcal{L}\{\text{ce}^{-at}\} = \frac{c}{(s + a)^2}
\]

Problems:

1) An object fell onto a hard surface where its velocity then went to 0m/s in 100ms. The on-board shock sensor indicated an average acceleration of -500m/s² during the impact. If \( G = 9.8\text{m/s}^2 \), what height did the object fall from? (10 points)

\[
a \approx \frac{\Delta \text{V}}{\Delta t} \rightarrow \Delta \text{V} \approx a \Delta t = (-500)(0.1) = -50 \text{ m/s} \\
|\text{V}| = 50 \text{ m/s} \\
mgh = \frac{1}{2}mv^2 \rightarrow h = \frac{v^2}{2g} = \frac{50^2}{2(9.8)} = 127.55 \text{ m}
\]
Match the question with an answer by writing the letter of the answer in the blank next to the question. No answer is used more than once. (30 points)

Questions

1) This material property is what makes a thermocouple operate: __D__
2) Transmissibility can be measured using this optical instrument: __E__
3) When a reflected wave from a moving object has a different frequency: __I__
4) In a PPA, this pulls the electrodes toward each other: __N__
5) Heating of a resistive element due to current flow: __F__
6) A bimorph is made of two materials with different values of: __M__
7) The chemical we desire to sense: __B__
8) Measures pressure with respect to a full vacuum: __O__
9) The principle of buoyancy is also known as: __P__
10) Closed loop accelerometers and pressure sensors make use of: __C__
11) This is an inertial sensor: __H__
12) The acceleration of a typical mechanical shock event has a characteristic: __K__
13) MEMS gyroscopes make use of the: __G__
14) A temperature sensor that is compatible with IC technology: __A__
15) Used as a low-cost medical thermometer: __L__

Answers to choose from

A. PTAT
B. Analyte
C. Force Feedback
D. Seebeck Coefficient
E. Interferometer
F. Joule Heating
G. Coriolis Force
H. Gyroscope
I. Doppler Shift
J. Diaphragm
K. Half Sine Pulse
L. Thermistor
M. CTE
N. Electrostatic Force
O. Absolute Pressure Sensor
P. Archimedes’ Principle
2) An object is moving through a fluid with a density of 2g/cm³ at 10m/s. Its pressure sensor measures a total pressure of 105kPa using a gauge pressure sensor. What is the static pressure of the object in the fluid? (10 points)

\[ p = \left( \frac{2 \text{ g/cm}^3}{1 \text{ kg}} \right) \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 2000 \text{ kg/m}^3 \]

\[ p_t = p_s + \frac{\rho v^2}{2} \]

\[ p_s = p_t - \frac{\rho v^2}{2} = 105,000 - \frac{2000(10)^2}{2} = 5000 \text{ Pa} \]

\[ = 5 \text{ kPa} \]

3) A parallel plate actuator (PPA) consists of two square electrodes 100μm across, separated by 10μm, in a vacuum. If the spring constant is 50N/m, what is the pull-in voltage? (10 points)

\[ V_{PI} = \sqrt{\frac{8Kd_0^2}{27\varepsilon_0 \varepsilon_r}} \]

\[ = \sqrt{\frac{8(50)(10 \times 10^{-6})^3}{27(100 \times 10^{-6})^2 (8.854 \times 10^{-12})(1)}} \]

\[ = 409.05 \text{ V} \]
4) What is the amplitude of motion along the sense axis for a certain MEMS gyroscope that has a mass of 1µg, C=0.1x10^{-6}Kg/s, f_0=10KHz, and A_x=1µN when it experiences and angular rate of 180°/s? (10 points)

\[ Y = \frac{2m KL A_x}{c^2 \omega_n} = \frac{2(1 \times 10^{-9})(180)(\pi)(1 \times 10^{-6})}{(0.1 \times 10^{-6})^2 (2\pi)(10,000)} \]

\[ = \frac{(1 \times 10^{-9})(1 \times 10^{-6})}{(1 \times 10^{-7})^2 (1 \times 10^4)} \]

\[ = 1 \times 10^{-5} \mu m \]

\[ = 10 \mu m \]

5) A parallel plate actuator (PPA) consists of two round electrodes, 500µm in diameter, separated by 10µm, in a vacuum. If one electrode is connected to ground and the other electrode is connected to a high frequency AC voltage (100V amplitude), what is the average force produced by the PPA? (10 points)

\[ V_{rms} = \frac{100}{\sqrt{2}} \]

\[ A = \pi r^2 = \pi (250 \mu m)^2 \]

\[ f = \frac{E_0 \varepsilon_r A V^2}{2d^2} \]

\[ = \frac{8.854 \times 10^{-12} (1) \pi (250 \times 10^{-6})^2 (100)^2}{2(2)(10 \times 10^{-6})^2} \]

\[ = 4.346 \times 10^{-5} N \]

\[ = 43.46 \mu N \]
6) What does GCA stand for? **(5 points)**

\[ \text{Gap Closing Actuator} \]

7) What does CDA stand for? **(5 points)**

\[ \text{Combi Drive Actuator or Combsdrive Actuator} \]

8) What is the measurand for a carbon monoxide sensor? **(5 points)**

\[ \text{CO or carbon monoxide} \]

9) The proof mass for a certain open-loop MEMS accelerometer experiences a displacement of 10\(\mu\)m due to a 10m/s\(^2\) acceleration. What is the natural frequency of the accelerometer in Hz? **(5 points)**

\[
\begin{align*}
    a &= \Delta s = a \frac{m}{k} = \frac{a}{\omega_n^2} \\
    \omega_n^2 &= \frac{a}{d} \\
    \omega_n &= \sqrt{\frac{a}{d}} = \sqrt{\frac{10}{10 \times 10^{-6}}} = 1000 \text{ rad/s} \\
    f_n &= \frac{\omega_n}{2\pi} = \frac{1000}{2\pi} = 159.155 \text{ Hz}
\end{align*}
\]
**Bonus Question (10 points)**

A certain open loop MEMS accelerometer has the following transfer function, where the damping ratio is not equal to one:

\[ G(s) = \frac{1}{s^2 + 8s + 1}. \]

It is placed in a closed-loop controller (shown below) to adjust the damping ratio to one and the DC gain to one. Select values for the two gains, a and b, to achieve this goal for the closed loop system. Show all steps.

\[ a = 16 \]
\[ b = \frac{15}{16} \]

\[ 2 \beta \omega_n = 8 \]
\[ \beta = \frac{8}{2 \omega_n} = \frac{8}{2(4)} = \frac{8}{8} = 1 \]
Blank sheet for Calculations