NAME: KEY

Exam #2 ELEC 5820/6820 Wed 12/2/15

Constants: \[ \pi = 3.14159, \quad \varepsilon_0 = 8.854 \text{pF/m}, \quad 1 \text{G} = 9.8 \text{m/s}^2, \]

Equations: \[ k \approx \frac{N_{\text{leg}}}{N_{\text{zig}}} \frac{E \cdot W \cdot t^3}{L^2}, \quad \text{Power} = \frac{\text{Energy}}{\text{Time}}, \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d}, \quad \alpha = \frac{\Delta L}{L \Delta T}, \quad Q = CV \]
\[ E_c = \frac{C V^2}{2}, \quad F_{\text{CCA}} = \frac{n \varepsilon_0 \varepsilon_r A \beta V^2}{2} \left[ \frac{1}{(d_1 - x)^2} - \frac{1}{(d_1 + x)^2} \right], \quad F_{\text{PPA}} = \frac{\varepsilon_0 \varepsilon_r A V^2}{2(x_o - x)^2} \]
\[ V_p = \sqrt{\frac{8 k x_o^3}{27 A \varepsilon_0 \varepsilon_r}}, \quad A_{\text{circle}} = \pi r^2, \quad d = a \frac{m}{k} = a S, \quad F_{\text{EDA}} = \frac{n \varepsilon_0 \varepsilon_r \beta V^2}{d_o} \]

Problems:

1) Circle the item below that is NOT an application for a pn junction. (5 points):

Diode \underline{Thermal Bimorph} Photovoltaics Thermoelectric Cooler

2) The proof mass of an open-loop MEMS accelerometer with a sensitivity of 1 \( \mu \text{s}^2 \) experienced a displacement of 10\( \mu \text{m} \). What acceleration did it experience, in units of \( \text{m/s}^2 \)? (5 points):

\[ d = a S \Rightarrow a = \frac{d}{S} = \frac{10 \times 10^{-6}}{1 \times 10^{-6}} = 10 \text{ } \mu \text{s}^2 \]
Match the question with an answer by writing the letter of the answer in the blank next to the question. No answer is used more than once. (20 points)

Questions

1) This actuator can suffer from lateral instability:  

2) A temperature sensor often integrated into a MEMS device: 

3) A technique for aligning polarized crystallites in piezoelectrics:  

4) A MEMS pump that uses electro-osmotic flow:  

5) A commonly used SMA material in MEMS:  

6) For a negative feedback system, the oscillation condition where Aβ = -1:  

7) In dicing a wafer, the saw lane width:  

8) Membranes and diaphragms are used in MEMS:  

9) A 2-axis resonator is often used in MEMS:  

10) This is used to correct a distorted optical wavefront:  

Answers to choose from


3) Circle the item below that is NOT used as an energy harvesting device (5 points):

FlowFET, TEC, Rectenna, PN Junction
4) Consider the transmissibility plot for a MEM device with a 10mg proof mass shown below and answer the following questions with regard to this device:

(a) What is the resonant frequency, \( f_r \) (5 points)?

\[ f_r = 1500 \text{ Hz} \]

(b) What is the resonant frequency, \( \omega_n \) (5 points)?

\[ \omega = 2\pi f = 9424.77 \text{ rad/s} \]

(c) What is the mechanical quality factor, \( Q \) (5 points)?

\[ Q = 25 \]

(d) If the frame has a displacement amplitude of 1\( \mu \)m at the resonant frequency, what is the displacement amplitude of the proof mass at the resonant frequency (5 points)?

\[ 25 \mu \text{m} \]

(e) What is the magnitude of \( T(j\omega) \) at DC? (5 points)?

\[ 1 \]
5) Consider the transfer function of a MEMS spring-mass-damper with a 1g proof mass:

\[ T(s) = \frac{10s + 16}{s^2 + 10s + 16} \]

(a) What is the order of this system (5 points)?

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(b) What is the natural frequency, \( \omega_n \) (5 points)?

\[ \omega_n = \sqrt{16} = 4 \text{ rad/s} \]

(c) What is the system spring constant, \( k \) (5 points)?

\[ \omega_n = \sqrt{\frac{F}{m}} \rightarrow k = m \omega_n^2 = 1 \times 10^{-5} (16) = 1.6 \times 10^{-2} N/m \]

(f) What is the Quality factor, \( Q \) (5 points)?

\[ \frac{\omega_n}{Q} = 10 \rightarrow Q = \frac{\omega_n}{10} = \frac{4}{10} = 0.4 \]

(g) What is the damping coefficient, \( c \) (5 points)?

\[ \frac{c}{m} = 10 \rightarrow c = 10 m = 1 \times 10^{-2} \text{ Kg/s} \]
6) For the MEMS spring-mass-damper device drawn below, with all beams the same size, what is an expression for the system spring constant, k, in terms of beam (spring) dimensions (L, w and t) and the Young's Modulus (E)? (5 points)

\[ k = \frac{N_{sy}}{N_{ig}} \cdot \frac{Ew^3}{L^2} = \frac{3Ew^3}{L^3} \]

7) A parallel plate actuator (PPA) consists of two square electrodes, 1mm on a side, separated by 20\(\mu\)m, in a vacuum. The system spring constant is 25N/m and the movable electrode has a mass of 100\(\mu\)g.

a. What is the pull-in voltage for this actuator? (5 points)

\[ U_{PI} = \sqrt{\frac{8kx_0^2}{27AE_0elr}} = \sqrt{\frac{8(25)(20\times10^{-6})^3}{27(1\times10^{-3})^2 \times 8.854 \times 10^{-12}}} = 81.81V \]

b. What is the stable range of motion for this actuator when it is directly connected to a DC power supply? (5 points)

\[ 0 \leq x \leq \frac{20\times10^{-6}}{3} m \]

\[ l \]

\[ 6.67 \mu m \]
**Bonus Question (5 points)**

Consider the phase shift oscillator shown below, consisting of an inverting amplifier gain stage and 3 identical RC delay stages where \( R = 10k\Omega \) and \( C = 1nF \). What is the frequency of oscillation?

\[
\frac{V_o}{V_i} = \frac{1}{R + j \frac{1}{RC}} = \frac{1}{RSC + j}
\]

\[
\frac{V_o}{V_i} = \tan^{-1}(-RC\omega) = -60^\circ \text{ for oscillation}
\]

\[
-RC\omega = \tan(-60^\circ)
\]

\[
2\pi f RC = -\tan(-60^\circ)
\]

\[
f = \frac{-\tan(-60^\circ)}{2\pi RC} = \frac{-\tan(-60^\circ)}{2\pi (10 \times 10^3 \times 1 \times 10^{-9})} = 27.566 \text{ kHz}
\]