

KEY

NAME:

Exam #2

ELEC 5820/6820

Wed 4/28/10

Constants: $\pi = 3.14159$, $\epsilon_0 = 8.854 \text{pF/m}$, $1G = 9.8 \text{m/s}^2$,

Equations: $k \approx \frac{N_{leg}}{N_{zig}} \frac{Ewt^3}{L^3}$, $Power = \frac{Energy}{Time}$, $C = \frac{\epsilon_0 \epsilon_r A}{d}$, $\alpha = \frac{\Delta L / L}{\Delta T}$, $Q = CV$

$$E_c = \frac{CV^2}{2}, \quad F_{GCA} = \frac{n\epsilon_0 \epsilon_r ABV^2}{2} \left[\frac{1}{(d_1 - x)^2} - \frac{1}{(d_1 + x)^2} \right], \quad F_{PPA} = \frac{\epsilon_0 \epsilon_r AV^2}{2(x_o - x)^2}$$

$$V_p = \sqrt{\frac{8kx_o^3}{27A\epsilon_0 \epsilon_r}}, \quad A_{circle} = (\pi)r^2, \quad d = a\left(\frac{m}{k}\right) = aS, \quad F_{CDA} = \frac{n\epsilon_0 \epsilon_r b\beta V^2}{d_o}$$

Problems:

1) What does "MOEMS" stand for? (5 points):

Micro Optical Electromechanical Systems

2) An open-loop MEMS accelerometer has a natural frequency of 1KHz. If the sensor experiences a 100G acceleration in the direction of measurement, what is the magnitude of the proof mass displacement in μm ? (5 points):

$$d = a \frac{m}{k} = \frac{a}{\omega_n^2} = \frac{100(9.8)}{(2\pi 1000)^2} = 2.482 \times 10^{-5} \text{ m} \\ = 24.82 \mu\text{m}$$

Match the question with an answer by writing the letter of the answer in the blank next to the question. No answer is used more than once. (20 points)

Questions

- 1) This actuator is only stable over 1/3 of its rest gap distance: K
- 2) An actuator made of two bonded materials of different CTE's: M
- 3) Another name for a MOEMS deformable mirror: A
- 4) A beam clamped on one end and free on the other: I
- 5) Results in CDA teeth snapping into contact if the voltage is too high: E
- 6) For a negative feedback system, the condition where $A\beta = -1$: P
- 7) A photolithography mask alignment mark: G
- 8) Nitinol, austenite and martensite refer to this type of actuator: H
- 9) Liquid is to this device as current is to a transistor: J
- 10) This uses Joule heating to move fluid: B

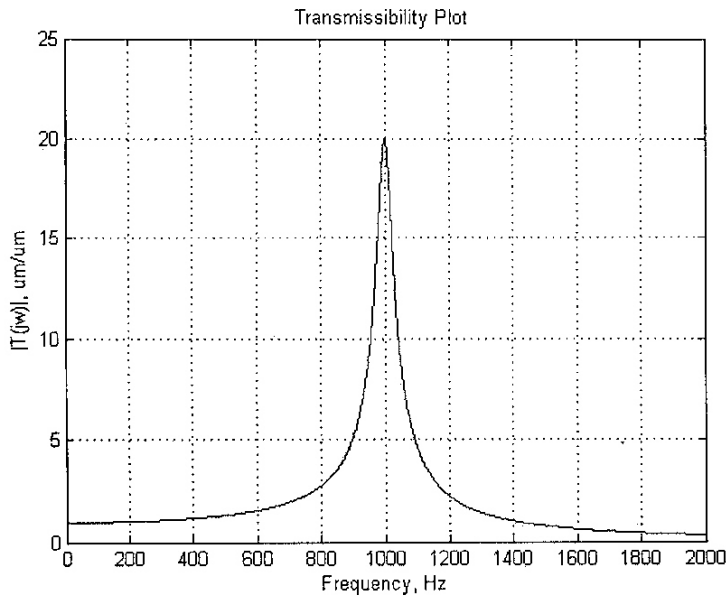
Answers to choose from

- | | |
|---------------------------------------|------------------------------------|
| A) Spatial Light Modulator | X) Cantilever |
| B) Thermal Pump | Y) FlowFET |
| C) CDA | K) PPA |
| D) Fulcrum | L) Poisson's Ratio |
| E) Lateral Instability | M) Bimorph |
| F) Varactor | N) Magnetic Flux |
| G) Fiducial | O) Squeeze-film |
| H) SMA | P) Barkhausen Criterion |

3) Circle the item below that is NOT a source of mechanical damping (5 points):

Squeeze-film, Thermoelastic, Electrostatic-breakdown, Shear-resistance

- 4) Consider the transmissibility plot for a MEM device with a $100\mu\text{g}$ proof mass shown below and answer the following questions with regard to this device:



- a. What is Q ? (5 points)

$$20$$

- b. What is the natural frequency in KHz? (5 points)

$$1 \text{ KHz}$$

- c. What is the damping ratio? (5 points)

$$\zeta = \frac{1}{2Q} = \frac{1}{40} = 0.025$$

- d. What is the spring constant? (5 points)

$$\frac{k}{m} = \omega_n^2 \rightarrow k = m \omega_n^2 = 100 \times 10^{-9} (2\pi 1000)^2 = 3.948 \text{ N/m}$$

- e. If the device is excited with a sinusoidal input at its natural frequency with an amplitude of $0.2\mu\text{m}$, what is the amplitude of the proof mass displacement at that frequency? (5 points)

$$X_{A_0} = Q X_{A_{in}} = 20(0.2) = 4 \mu\text{m}$$

- f. What is the damping coefficient? (5 points)

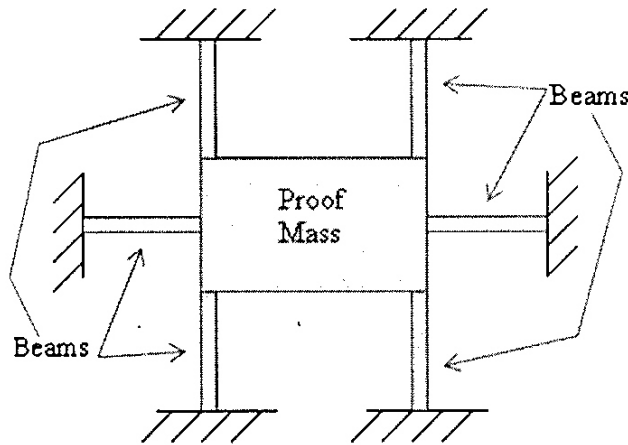
$$\frac{c}{m} = \frac{\omega_n}{Q} \rightarrow c = \frac{m \omega_n}{Q} = \frac{(100 \times 10^{-9})(2\pi 1000)}{20} = 3.14 \times 10^{-5} \text{ kg/s}$$

g. What is the expression for $T(s)$ for this device? (5 points)

$$\frac{c}{m} = \frac{\omega_n}{\alpha} = \frac{2\pi 1000}{20} = 314.159 \quad T(s) = \frac{\frac{L\omega_n}{\alpha} s + \omega_n^2}{s^2 + \frac{\omega_n}{\alpha} s + \omega_n^2} = \frac{314.159s + 3.948 \times 10^7}{s^2 + 314.159s + 3.948 \times 10^7}$$

$$\omega_n^2 = (2\pi 1000)^2 = 3.948 \times 10^7$$

5) For the MEMS spring-mass-damper device drawn below, with all beams the same size, what is an expression for the system spring constant, k , in terms of beam (spring) dimensions (L , w and t) and the Young's Modulus (E)? (5 points)



$$N_{leg} = 6$$

$$N_{z/y} = 1$$

$$K \approx 6 \frac{Ewt^3}{L^3}$$

6) Consider a micromachined beam with these dimensions: $500\mu\text{m}$ long, 50μ wide and $10\mu\text{m}$ thick. If the beam is made of a material that has a CTE of $10 \times 10^{-6}/^\circ\text{C}$, and the temperature increases by 120°C , how long does the beam become due to the temperature rise? (5 points)

$$L_{new} = L + \Delta L = L + \alpha L \Delta T$$

$$= 500 + 10 \times 10^{-6} (500)(120) = 500.6 \mu\text{m}$$

7) A parallel plate actuator (PPA) consists of two square electrodes, $500\mu\text{m}$ on a side, separated by $10\mu\text{m}$, in a vacuum. The system spring constant is 20N/m and the movable electrode has a mass of $10\mu\text{g}$.

a. What is the pull-in voltage for this actuator? (5 points)

$$U_p = \sqrt{\frac{8kx_0^3}{27A\epsilon_0\epsilon_r}} = \sqrt{\frac{8(20)(10 \times 10^{-6})^3}{27(500 \times 10^{-6})^2(8.854 \times 10^{-12})(1)}} = 51.74\text{V}$$

b. What is the natural frequency of this device in Hz? (5 points)

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{20}{10 \times 10^{-9}}} = 7117.63\text{ Hz}$$

c. What happens if a voltage greater than the pull-in voltage is applied to the actuator? (5 points)

The two electrodes will snap in contact because $F_E > F_S$

d. What maximum value of capacitance can be used for the series capacitor to fully increase the stable range of PPA motion using the series capacitance method? (5 points)

$$C_s = \frac{1}{2} C_{PPA} = \frac{1}{2} \frac{(8.854 \times 10^{-12})(500 \times 10^{-6})^2}{10 \times 10^{-6}} = 1.107 \times 10^{-13}\text{ F}$$

$$= 0.1107\text{ pF}$$

Bonus Question (5 points)

A MEMS electrostatic vibration energy scavenging device consists of a spring-mass-damper variable capacitor with 10V applied across it at its maximum capacitance. The rest capacitance is 1pF. In an application where it is vibrated at its natural frequency, 10KHz, (i.e. energy is scavenged each cycle) if the electrode motion is such that the distance between the electrodes has a range of 0.5 and 1.5 times the rest electrode distance, how much power in μW is scavenged, assuming 100% energy scavenging efficiency?

$$C_{\text{rest}} = 1 \text{ pF} \quad Q = C_{\text{max}} V_{\text{min}} = C_{\text{min}} V_{\text{max}}$$

$$C_{\text{max}} = 2 \text{ pF} \quad V_{\text{max}} = \frac{V_{\text{min}} C_{\text{max}}}{C_{\text{min}}} = \frac{10(2)}{(1/1.5)} = 30 \text{ V}$$

$$C_{\text{min}} = \frac{1}{1.5} \text{ pF}$$

$$E_{\text{min}} = \frac{1}{2} C_{\text{max}} V_{\text{min}}^2 = \frac{1}{2} (2) (10)^2 = 100 \text{ pJ}$$

$$E_{\text{max}} = \frac{1}{2} C_{\text{min}} V_{\text{max}}^2 = \frac{1}{2} \left(\frac{1}{1.5}\right) (30)^2 = 300 \text{ pJ}$$

$$\Delta E = 200 \text{ pJ}$$

$$P = \frac{\Delta E}{T} = \Delta E f_n = 200 \times 10^{-12} (10,000) = 2 \mu\text{W}$$