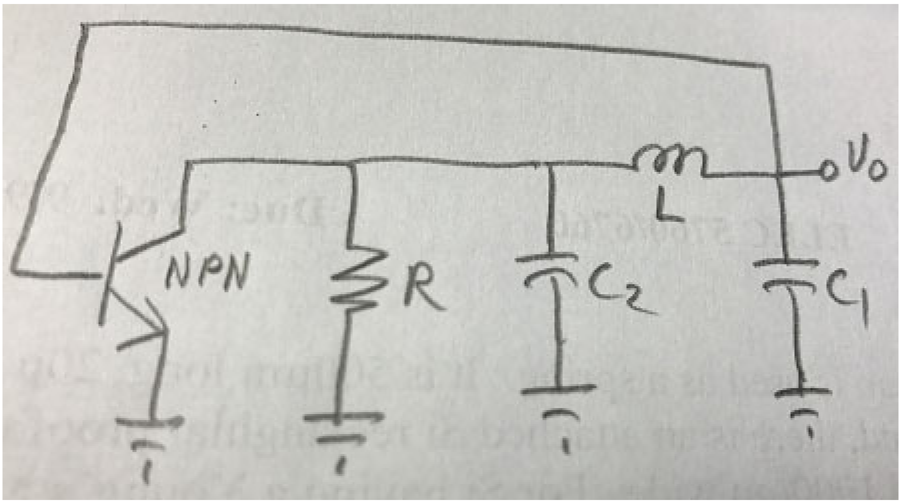
Tuesday 9/23/25

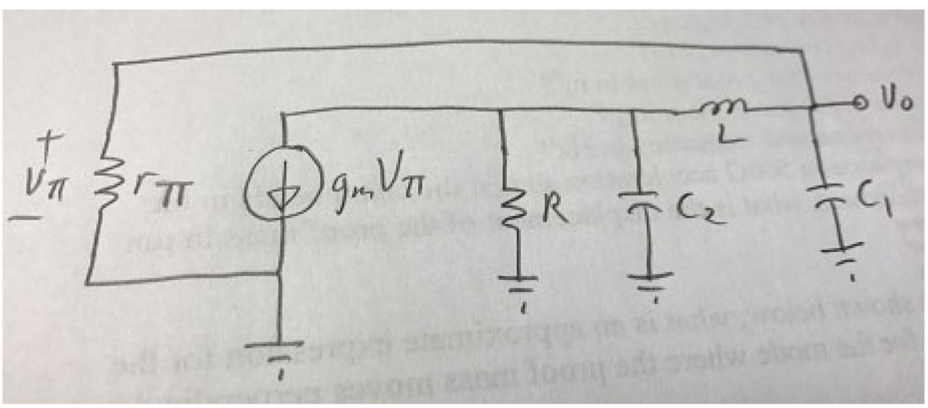
Simplified Circuit Analysis of BJT Colpitts and Hartley Oscillators

a. Colpitts Oscillator

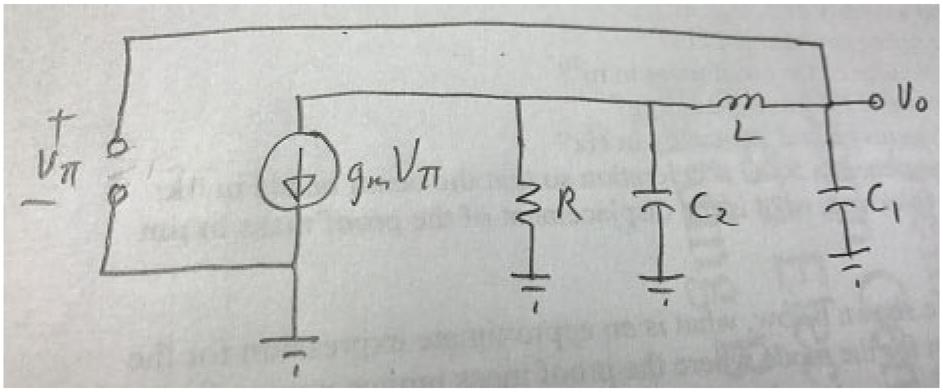


Notice that the biasing resistors are not shown in this simplistic circuit.

Simplistic Small signal model:



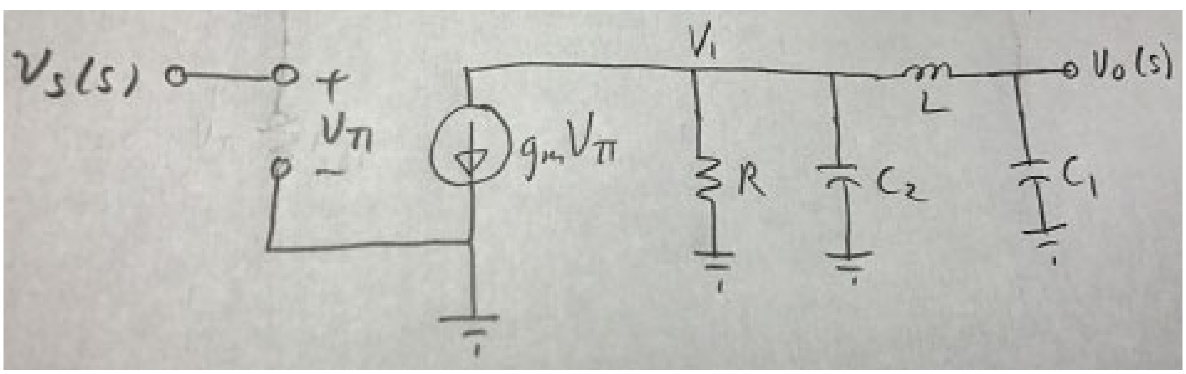
Let's further simplify by assuming that r_π is so large that we can approximate r_π as an open circuit:



Here, $V_\pi = V_o$

To evaluate the conditions for which the circuit will oscillate, one approach is to break the circuit at the v_π positive terminal, and evaluate T(s) for the condition that $T(j\omega) = 1|\underline{0^o}$, modelled as a positive feedback oscillator.

Therefore, define the circuit as:



$$\mathrm{T(s)} = \frac{V_{out}}{V_{in}}(s) = \frac{V_o}{V_s}(s)$$

$$V_o\left(sC_1 + \frac{1}{sL}\right) - V_1\left(\frac{1}{sL}\right) = 0$$

$$V_o(s^2LC_1 + 1) = V_1 \qquad (1)$$

$$V_1\left(\frac{1}{R}+sC_2+\frac{1}{sL}\right)-V_o\left(\frac{1}{sL}\right)=-g_mV_\pi=-g_mV_s$$

$$V_1\left(\frac{sL}{R}+s^2LC_2+1\right)-V_o=-g_msLV_s \qquad (2)$$

(1)→(2)

$$V_o(s^2LC_1+1)\left(\frac{sL}{R}+s^2LC_2+1\right)-V_o=-g_msLV_s$$

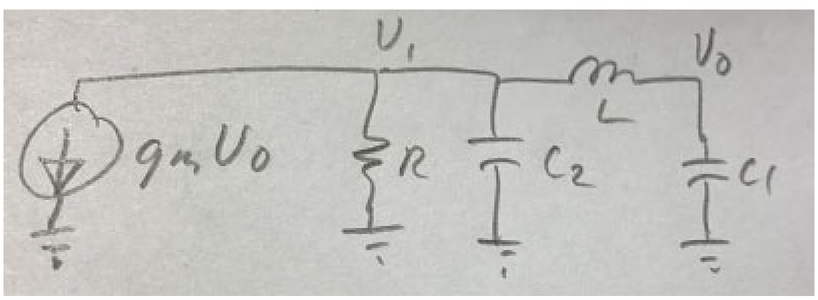
Then calculate T(s) and T(jω)

For oscillation: find the constraints so that $T(j\omega)=1|\underline{0^o}$.

Obviously, there are s^4 terms, making this a 4^{th} order system.

Much arithmetic will be required to obtain $T(j\omega)=1|\underline{0^o}$.

Another approach may be simpler.



From this tank circuit configuration, we know that: $\omega_o=\sqrt{\frac{C_1+C_2}{LC_1C_2}}$

For oscillation, the energy supplied by g_mV_o must be equal to or greater than the energy dissipated by R:

$E|_{g_mV_o}\geq E|_R$

$$V_o\left(sC_1+\frac{1}{sL}\right)-V_1\left(\frac{1}{sL}\right)=0$$

$$V_o(s^2LC_1+1)=V_1$$

$$E|_{g_mV_o}=(-g_mV_o)V_1t$$

$$E|_{g_mV_o}=-\frac{g_mV_1^2t}{1+s^2LC_1}$$

$E|_{g_mV_o}=-\frac{g_mV_1^2t}{1-\omega^2LC_1}$ → evaluate at $\omega = \omega_o$

$$E|_{g_mV_o}=-\frac{g_mV_1^2t}{1-\frac{C_1+C_2}{C_2}}$$

$$E|_{g_mV_o}=-\frac{g_mV_1^2t}{-\frac{C_1}{C_2}}$$

$$E|_{g_mV_o}=\frac{g_mV_1^2tC_2}{C_1}$$

$$E|_R=\frac{V_1^2}{R}t$$

Therefore:

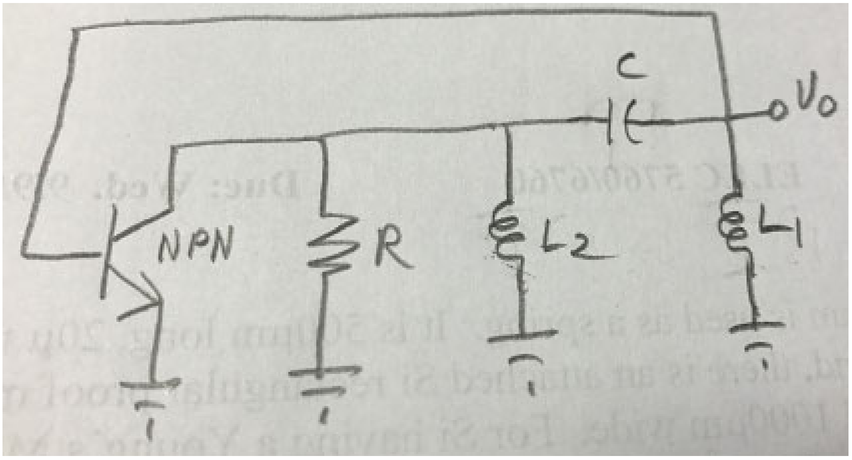
$$\frac{g_mV_1^2tC_2}{C_1}\geq\frac{V_1^2}{R}t$$

$$\frac{g_mC_2}{C_1}\geq\frac{1}{R}$$

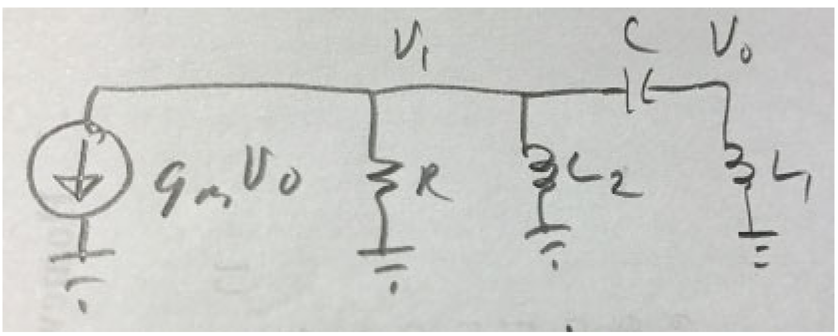
$g_mR\geq\frac{C_1}{C_2}$ This is the condition for oscillation.

This makes sense because an $R \to \infty$ Ω is the condition here for a lossless system.

b. Hartley Oscillator



Simplified small signal model:



Here, the tank circuit has $\omega_o = \frac{1}{\sqrt{C(L_1+L_2)}}$

$$V_o\left(\frac{1}{sL_1} + sC\right) - V_1(sC) = 0$$

$$V_o(1 + s^2LC_1) = V_1 s^2 LC_1$$

Using the energy balance relationship:

$$E|_{g_mV_o} \geq E|_R$$

$$E|_{g_mV_o} = (-g_mV_o)V_1t$$

$$E|_{g_m V_o} = -\frac{g_m V_1^2 t s^2 C L_1}{1 + s^2 C L_1}$$

$$E|_{g_m V_o} = \frac{g_m V_1^2 t \omega^2 C L_1}{1 - \omega^2 C L_1}$$ → evaluate at ω = ω_o

$$E|_{g_m V_o} = \frac{(g_m V_1^2 t)\frac{L_1}{L_1 + L_2}}{1 - \frac{L_1}{L_1 + L_2}}$$

$$E|_{g_m V_o} = \frac{(g_m V_1^2 t) L_1}{L_1 + L_2 - L_1}$$

$$E|_{g_m V_o} = \frac{(g_m V_1^2 t) L_1}{L_2}$$

$$E|_R = \frac{V_1^2}{R} t$$

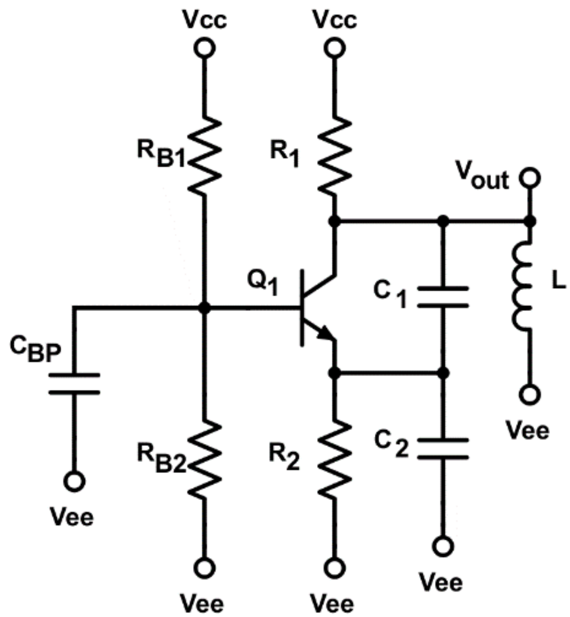
$$\frac{(g_m V_1^2 t) L_1}{L_2} \geq \frac{V_1^2}{R} t$$

$$\frac{g_m L_1}{L_2} \geq \frac{1}{R}$$

$$g_m R \geq \frac{L_2}{L_1}$$ This is the condition for oscillation.

c. A more thorough analysis of a BJT Colpitts oscillator

Consider the schematic diagram of this NPN BJT Colpitts oscillator:



C_{BP} is a bypass capacitor (assume an open for DC analysis and a short for small signal AC analysis)

R_{B1} and R_{B2} set the bias point for the transistor. First perform a DC analysis of the BJT operating mode and calculate a value for I_c. Then calculate values for g_m and r_π.

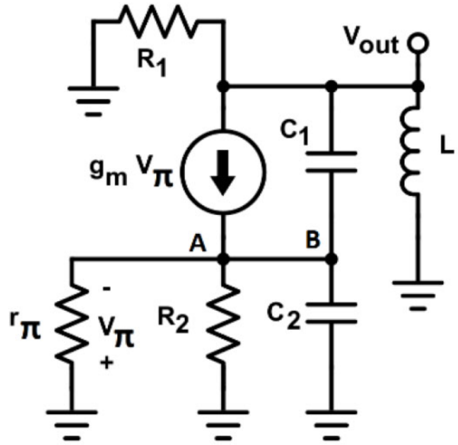
$$g_m = \frac{I_c}{V_t},$$

where V_t is the thermal voltage (known for the operating temperature).

$$r_\pi = \frac{\beta}{g_m}$$

where β is the common emitter current gain (known for the BJT).

Then perform a small signal analysis using hybrid-pi small signal model for the BJT.



We will break the loop between nodes A and B to calculate the loop gain, AB. The BSC is satisfied when:

$$A \cdot B = 1\underline{|0°} \ ,$$

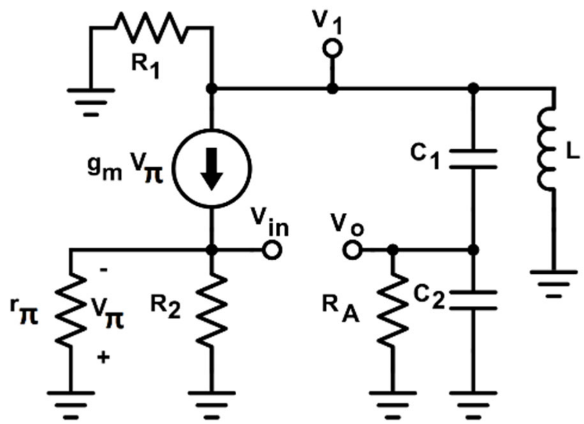
and the circuit will have negative damping when:

$$|A \cdot B| > 1 \ .$$

After breaking the loop, node A will be the input and node B will be the output. Node B will need to include the impedance looking into Node A, and we will call this impedance R_A, where R_A is calculated to be:

$$R_A = \frac{1}{\frac{1}{r_\pi}+\frac{1}{R_2}+g_m}.$$

Therefore, the circuit model becomes:



The three voltage nodes, V_{in}, V_o, and V_1, are used to calculate the loop gain, AB. A is therefore defined as:

$$A = \frac{V_1}{V_{in}} = \frac{g_m}{\frac{1}{R_1}+\frac{1}{4R_A}} \ .$$

In order to define B, the two capacitors, C_1 and C_2, with C_1 equal to C_2, are assumed to approximately act as a voltage divider between the voltage nodes V_1 and V_o at the tank circuit's natural frequency, yielding an equation for B of:

$$B = \frac{V_o}{V_1} = \frac{C_1}{C_1+C_2} = \frac{1}{2} \ .$$

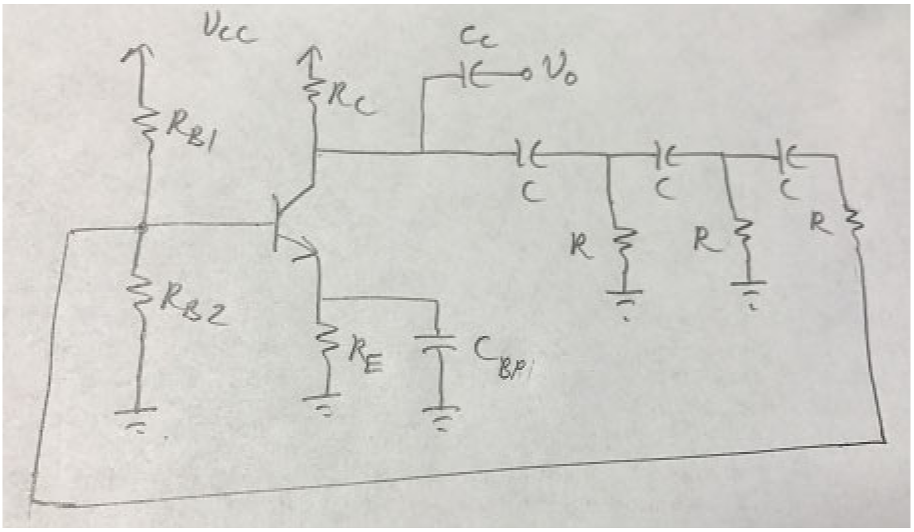
Combining the equations for A, B and R_A results in:

$$AB = \frac{g_m R_1}{2\left[1+\frac{R_1}{4}\left(\frac{1}{r_\pi}+\frac{1}{R_2}+g_m\right)\right]} \ .$$

As long as AB > 1, oscillations will grow in amplitude.

Other Transistor Oscillators

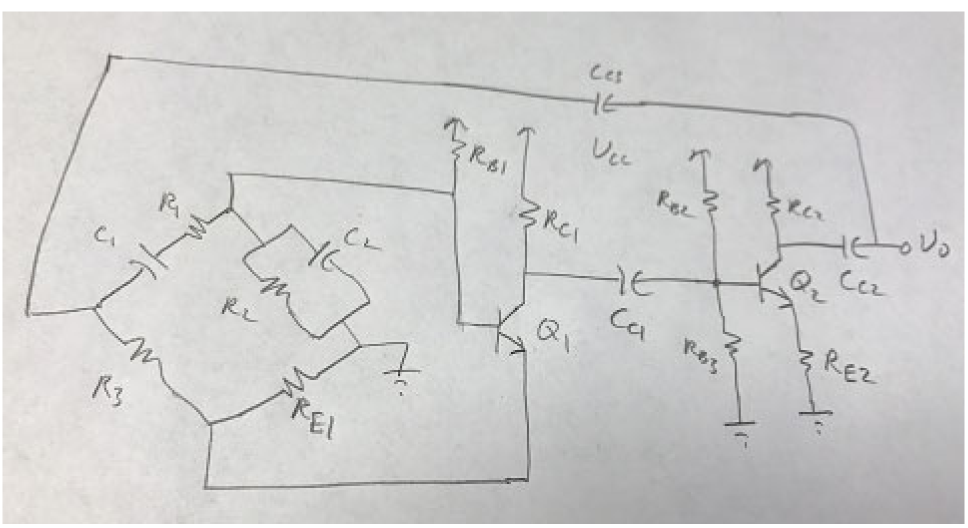
a. Phase Shift Oscillator



This circuit uses a common emitter (large negative gain) amplifier. The oscillator uses negative feedback to satisfy the BSC: $1|\underline{-180^{o}}$.

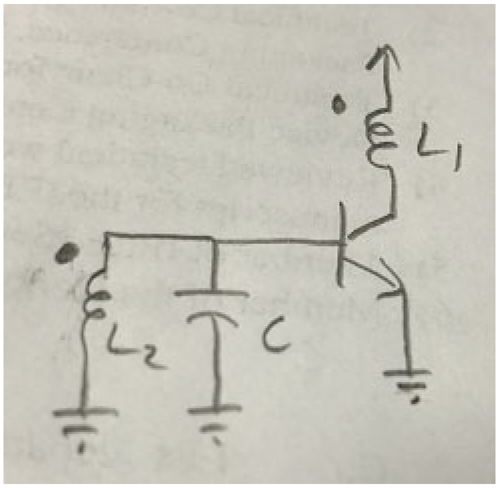
C_{BP} is a large capacitance bypass capacitor. C_C is a large capacitance coupling capacitor. R_{B1}, R_{B2}, R_C and R_E bias the transistor and determine the amplifier's gain.

b. Wien Bridge Oscillator

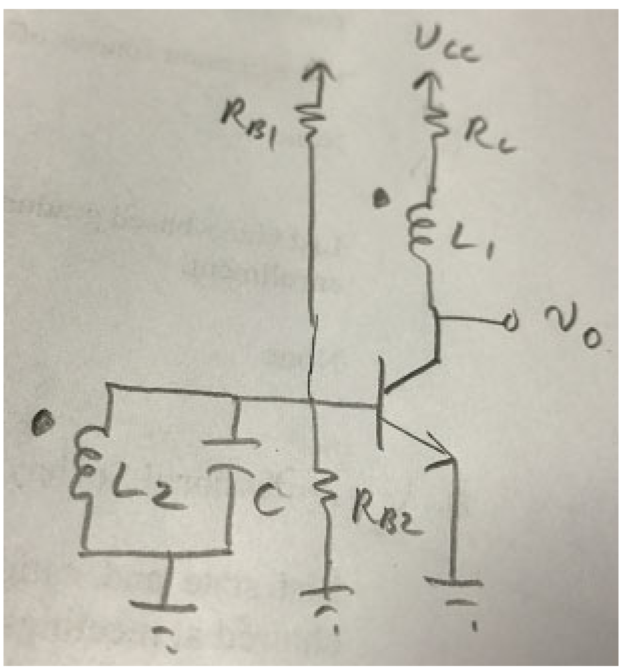


This circuit uses a two stage amplifier (two negative gain common emitter amplifiers) to achieve a positive gain for the Wein bridge oscillator.

c. Armstrong (1912 – U.S) or Meissner (1913 - Austria) Oscillator



Without biasing resistors shown. The transformer provides feedback from the output of the common emitter amplifier back around to the base. A transformer can easily provide a 180^o phase shift. Transformer's inductance with C determines the oscillation frequency.



Same oscillator with possible biasing resistors shown.