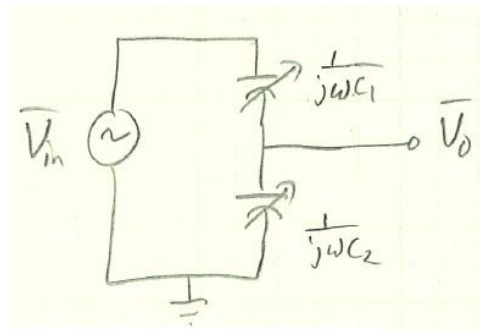


Capacitive Sensing

1. Capacitor Interface Circuitry: AC Voltage Division: Techniques to recover the amplitude

From last time:

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d_1} = \frac{\epsilon_0 \epsilon_r A}{d_0 + x(t)} \quad \text{and} \quad C_2 = \frac{\epsilon_0 \epsilon_r A}{d_2} = \frac{\epsilon_0 \epsilon_r A}{d_0 - x(t)}$$



$$\frac{\bar{V}_o}{\bar{V}_{in}} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1}{C_2 + C_1} \rightarrow \text{An AC voltage divider}$$

$$\text{Let } \bar{V}_{in} = V_1 \sin(\omega t)$$

$$\text{Therefore: } \bar{V}_o = V_1 \sin(\omega t) \left[\frac{C_1}{C_2 + C_1} \right] \rightarrow \text{select } \omega \gg \omega_{MEMS}$$

$$\text{Which could have this form: } \bar{V}_o(t) = V_1 \sin(\omega t) \left[0.5 \left(1 - \frac{x(t)}{d_0} \right) \right]$$

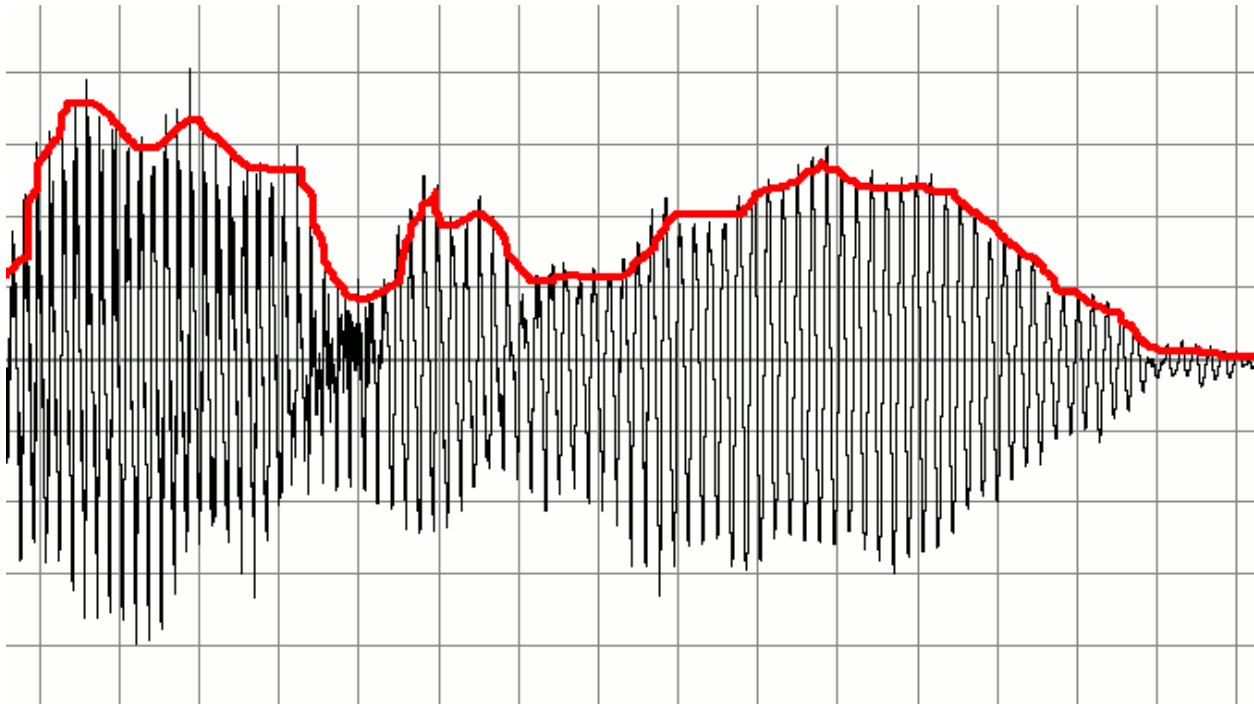
The amplitude of $\bar{V}_o(t)$ is a linear function of $x(t)$.

So, how do we recover the amplitude of $\bar{V}_o(t)$, V_1 ?

It is desirable to produce a DC voltage proportional to V_1 .

a. Envelope Detection

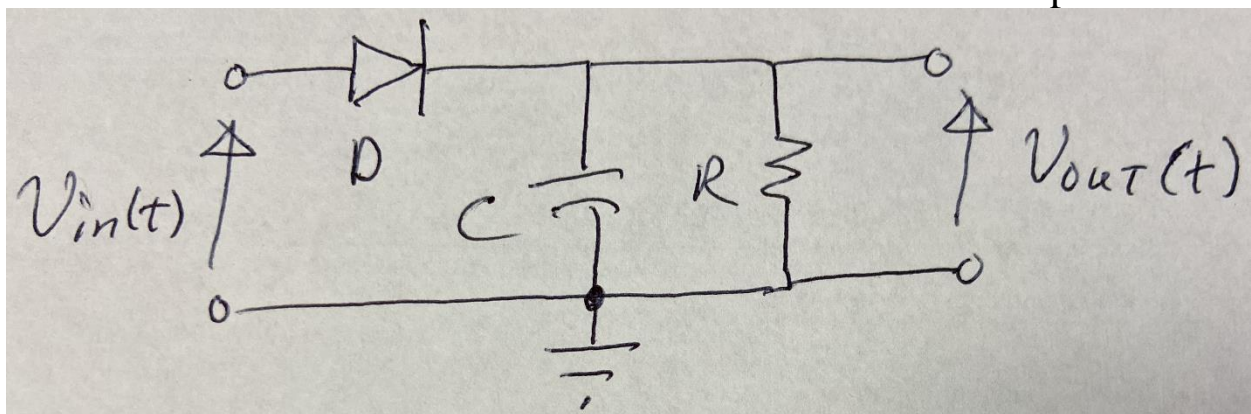
An envelope detector is a diode circuit that recovers the demodulated envelope of an AM modulated signal.



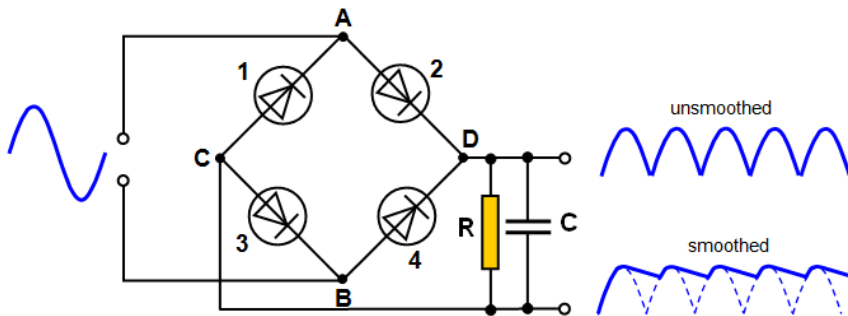
Courtesy: https://en.wikipedia.org/wiki/Envelope_detector#/media/File:C_Envelope_follower.png

The black waveform is the AM signal. The red waveform is the envelope or message we wish to recover.

A diode rectifier with a RC LPF circuit can be used to accomplish this:



More advanced rectifier circuits, such as using a four diode full wave bridge rectifier, or using an op amp based “super diode” could also be used for envelope detectors:



Courtesy: https://www.schoolphysics.co.uk/age16-19/Electronics/Semiconductors/text/Rectification_/index.html

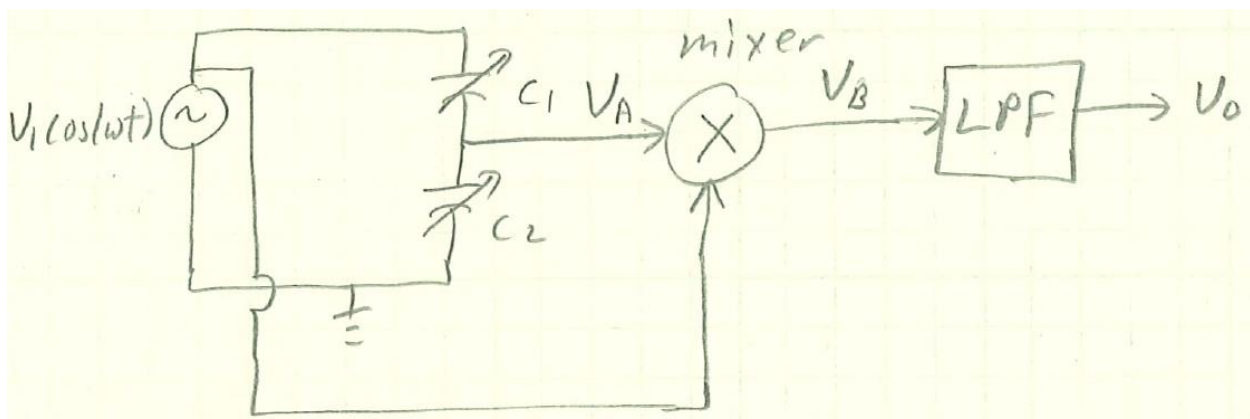
b. Synchronous Demodulator (Lock-in Amplifier)

Consider this:

$$\begin{aligned}
 A\cos(\omega t) \times B\cos(\omega t) &= 0.5AB[\cos(\omega t - \omega t) + \cos(\omega t + \omega t)] \\
 &= 0.5AB[\cos(0) + \cos(2\omega t)] \\
 &= 0.5AB + 0.5AB\cos(2\omega t)
 \end{aligned}$$

\uparrow \uparrow
 DC term AC term at $2\omega t$

So let's connect our differential capacitive sensor into a new circuit:



$V_A = V_1 \cos(\omega t) \left[0.5 \left(1 - \frac{x(t)}{d_o} \right) \right]$ → same form as with the AC voltage divider

$$V_B = V_A V_1 \cos(\omega t)$$

$$= 0.5 V_1^2 \left[0.5 \left(1 - \frac{x(t)}{d_o} \right) \right] + 0.5 V_1^2 \cos(2\omega t) \left[0.5 \left(1 - \frac{x(t)}{d_o} \right) \right]$$

↑
↑
 DC term
 AC term at $2\omega t$

The LPF attenuates the AC term so that:

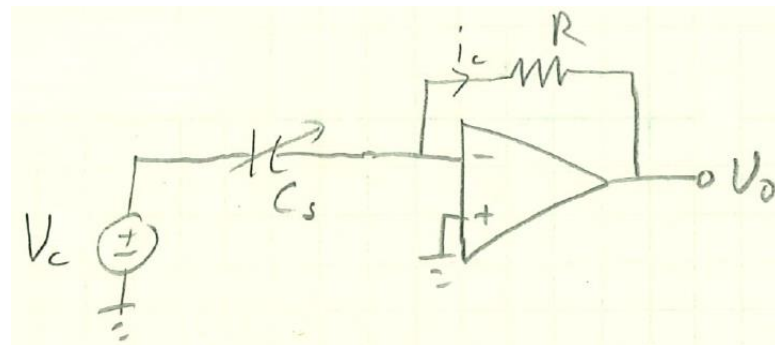
$$V_o \approx 0.5 V_1^2 \left[0.5 \left(1 - \frac{x(t)}{d_o} \right) \right] = V_c \left(1 - \frac{x(t)}{d_o} \right)$$

where V_c is a constant: $V_c = 0.5 V_1^2 (0.5)$

Now, V_o is a DC voltage that is a linear function of $x(t)$.

2. Capacitor Interface Circuitry: Transimpedance amplifier (TIA)

Consider:



Note: some op amps are not stable with the input tied to a capacitor, and the output will break into high frequency oscillation.

But assuming the op amp configuration is stable: $v_o = -i_c R$

Note: for this inverting amplifier circuit, since the input is a current and the output is a voltage, the gain is a resistance with units of Ω .

For a fixed capacitor: $i_c = C \frac{dv_c}{dt}$. Note: as a differentiator of v_c , the TIA is noisy: it amplifies high frequency noise.

However, the more general case is: $i_c = C \frac{dv_c}{dt} + v_c \frac{dC}{dt} = C \frac{dv_c}{dt} + v_c \frac{\partial C}{\partial x} \frac{dx}{dt}$

a. If v_c is a constant, V_c , then $i_c = V_c \frac{\partial C}{\partial x} \frac{dx}{dt}$

If the capacitor's electrodes are in relative motion, then $\frac{dx}{dt}$ is a velocity term. In steady state, the time varying $C_s(t)$ pumps i_c into the circuit.

b. If $V_c = V_A \sin(\omega t)$ and $\omega \gg \omega_{MEMS}$,

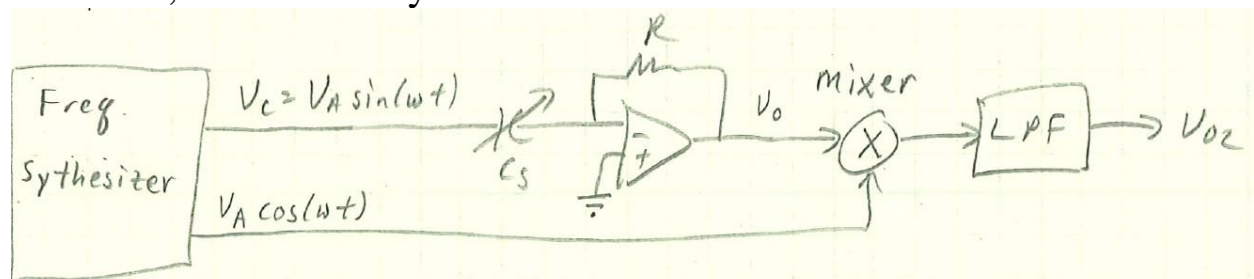
then for short time periods of several V_c cycles, C_s is nearly constant and

$v_c \frac{dC_s}{dt} \approx 0$ (i.e. a very small change during the measurement time)

So: $i_c \approx C_s \frac{dV_c}{dt} = C_s V_A \omega \cos(\omega t)$

And finally: $v_o \approx -C_s R V_A \omega \cos(\omega t)$ for quick measurements of C_s .

However, we can add synchronous demodulation here too:



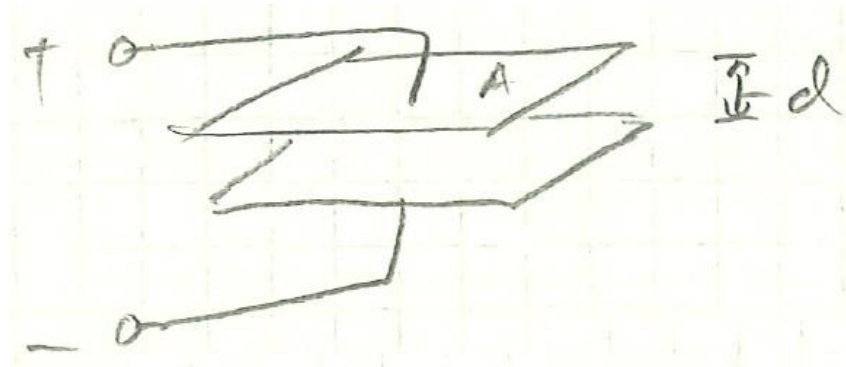
$$\therefore V_{o2} = LPF[V_o \times V_A \cos(\omega t)] = -0.5C_s R V_A^2 \omega = k C_s$$

where k is a constant: $k = -0.5 R V_A^2 \omega$

So once again, V_{o2} is a DC voltage proportional to C_s , our sensor's capacitance.

Capacitive Fringing Field Sensors

Consider:

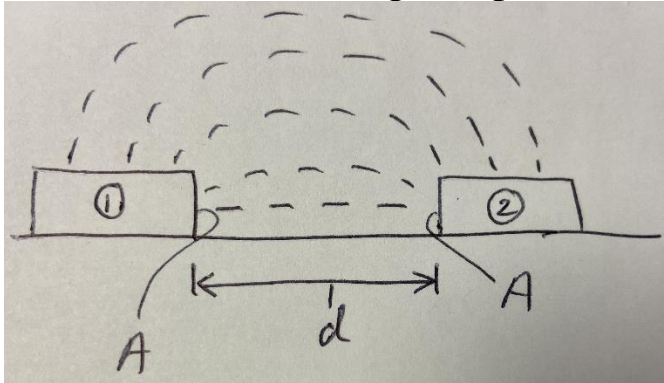


For $A \gg d^2$: $C \approx \frac{\epsilon_0 \epsilon_r A}{d}$, and fringing effects are small,

But $A \approx d^2$ or if $d^2 > A$, $C \neq \frac{\epsilon_0 \epsilon_r A}{d}$.

Actually now, $C > \frac{\epsilon_0 \epsilon_r A}{d}$ due to fringing effects.

Consider the case using two planar electrodes:

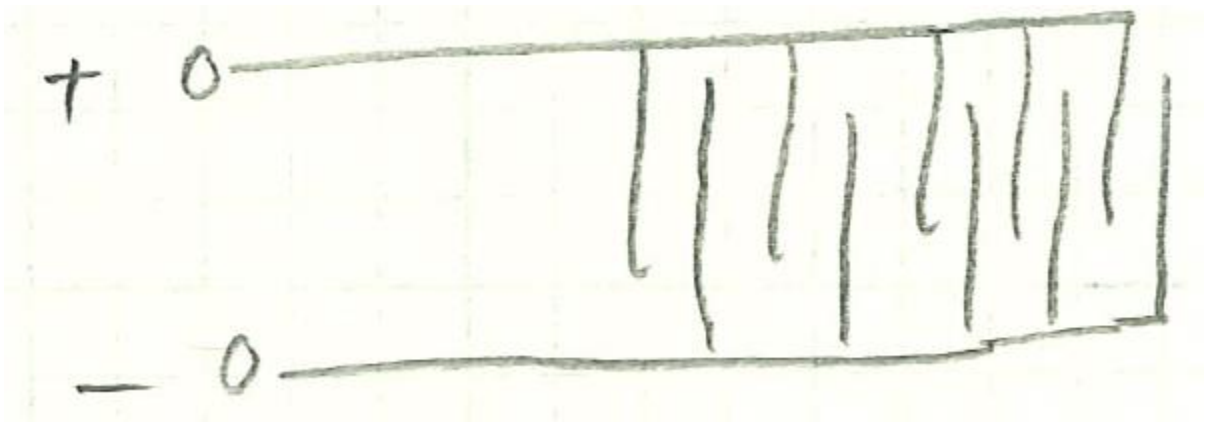


1 and 2 are the electrodes, d is the distance between them, and A is the area of each electrode facing each other. The dashed lines represent electric flux lines.

The capacitance between 1 and 2 can be modeled by:

$$C \approx \frac{\epsilon_0 \epsilon_r A \gamma}{d}, \text{ where } \gamma \text{ is a fringing scale factor and } \gamma > 1.$$

Often, the two electrodes are arranged in an interdigitated electrode (IDE) layout on a planar surface, realizing a capacitive fringing field sensor:



Let n = number of interdigitated fingers.

$$C \approx \frac{(n - 1) \epsilon_0 \epsilon_r A \gamma}{d}$$

Sometimes, the IDE is coated with a thin insulating layer, such as polyimide (PCB technology) or silicon dioxide (MEMS technology).

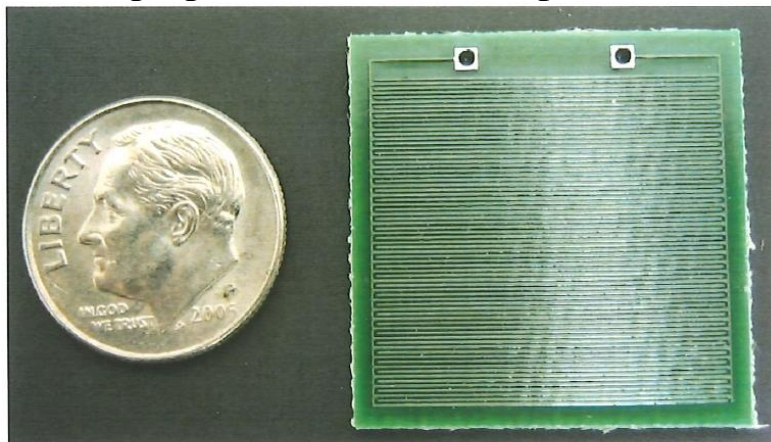
When the electrode width is equal to d , the sensing range above the sensor is approximately $1.25d$ to $1.5d$.

Applications for capacitive fringing field sensors:

1. Detecting the presence of liquid water ($\epsilon_{r|\text{air}} \approx 1$ and $\epsilon_{r|\text{water}} \approx 80$ at room temperature)
2. Measuring the moisture content of many materials
3. Measuring the level of water and other liquids
4. Detecting ice: above ~ 10 KHz, $\epsilon_{r|\text{water}} \gg \epsilon_{r|\text{ice}}$
5. Measuring relative humidity

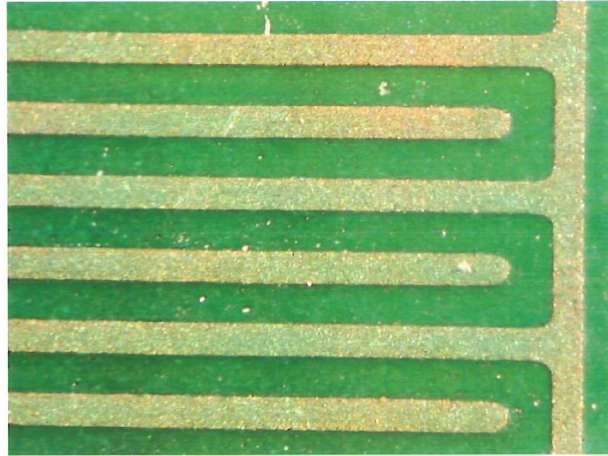
Meso scale versions, such as in PCB technology, can have relatively large capacitances \rightarrow 100s of pF.

PCB Capacitive Fringing Field Sensor Example:

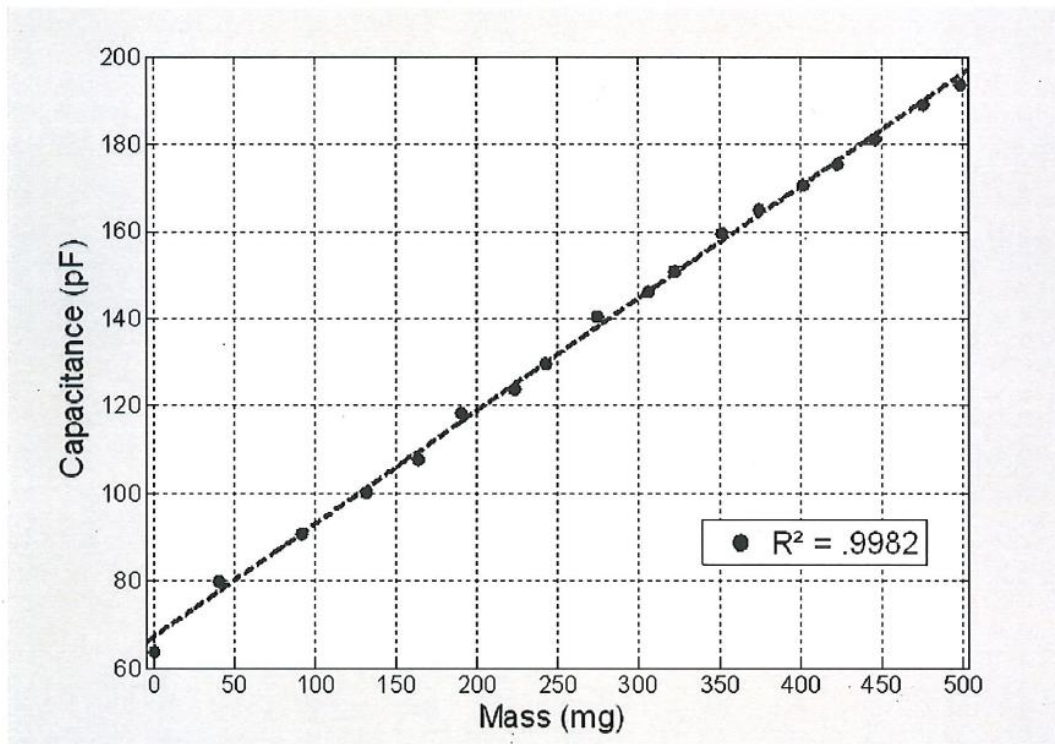


Photo

Device: 25.4 mm x 25.4 mm
70 interdigitated fingers ($\sim 150\mu\text{m}$ wide)
22.4 mm electrode overlap
63.9pF capacitance in air
321.3pF capacitance when submerged in water



Close up photo of electrode structure



Mass of water drop sensor response

