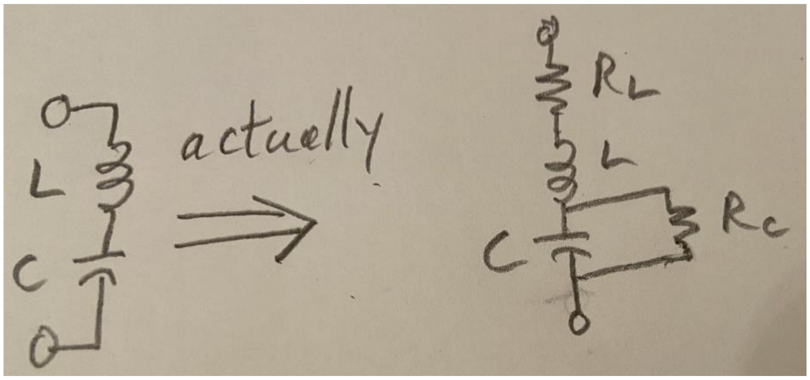
Lecture 9/2/22

Damping

All physical systems are lossy or dissipative (i.e. they have energy loss mechanisms).

Circuit example:



R_L represents the resistance of the wire used to make L.

R_C represents the leakage path through C.

In mechanical systems, energy losses are modelled by a Damping Coefficient, c.

[c] = Kg/s

Damping Force $\equiv F_D = cv = c\frac{dy}{dt} = c\dot{y}$

In the macro word (our world), friction is often the most important mechanical energy loss mechanism.

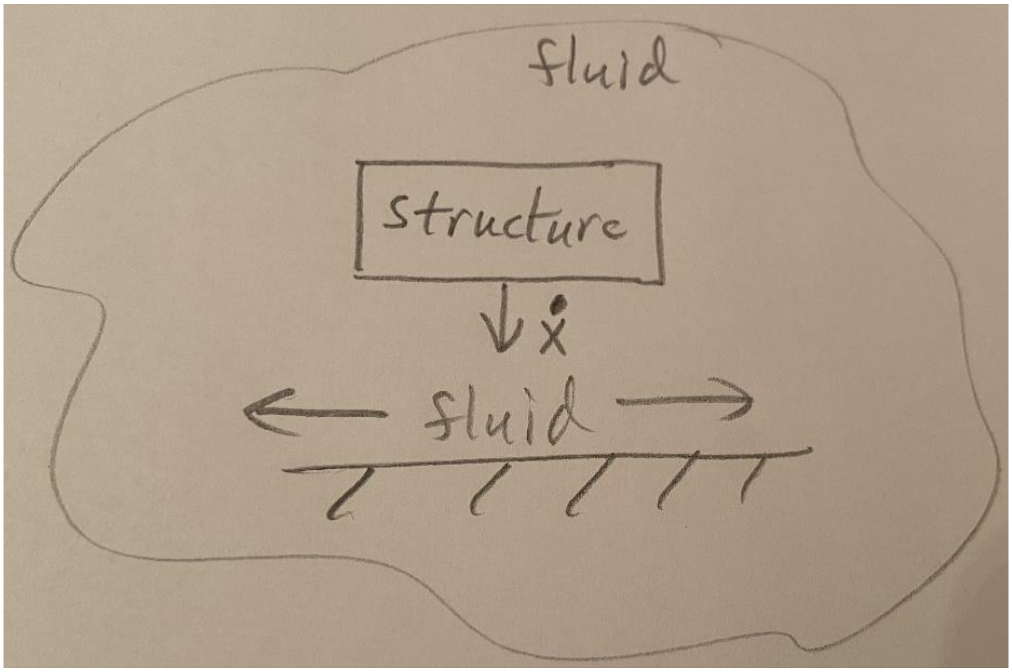
In the micro world, there are both internal and external energy loss mechanisms to consider:

Internal Sources

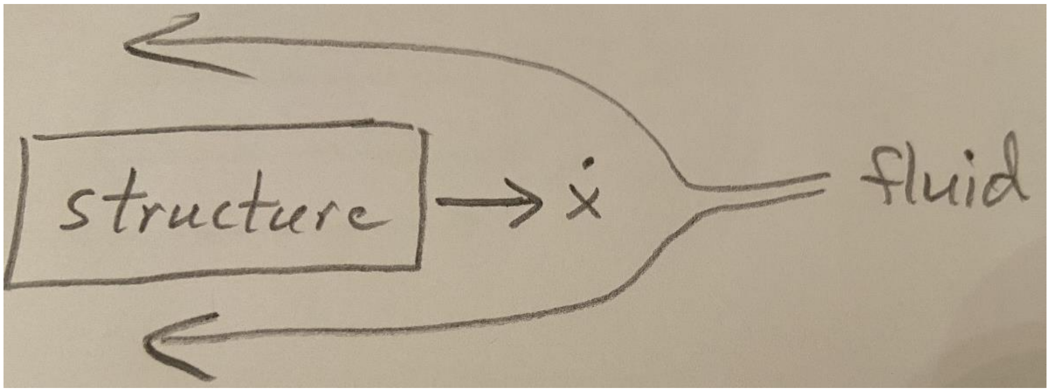
1) Thermoelastic Damping: an internal coupling of mechanical stress/strain and heat flow in a material. Some of the energy used to deform the beam gets converted to heat.

External Sources

1) Friction
2) Impact
3) Eddy current damping: a DC magnetic field in a moving conductor creates a drag force that resists that motion.
4) Interaction with a surrounding fluid: fluidic damping
   a) Squeeze-Film Damping: from the compression of a surrounding fluid – the fluid is forced out by compression



b. Shear-Resistance Damping: from a resistance to shearing of a fluid as an object moves through it



With microstructures, a gas is the fluid. Gases are compressible.

c = f(geometry, μ), where μ is gas viscosity.

For gas pressures > few hundred Pa: μ is not proportional to P

1 atm = 760 Torr = 101,325 Pa

→ 200 Pa ≈ 1.5 Torr [Mars' atmosphere ≈ 5.03 Torr]

For pressures < few hundred Pa: $\mu \propto P \rightarrow c \propto P$

10^{-3} Torr (0.133 Pa) ~ low vacuum

10^{-7} Torr (1.33×10^{-5} Pa) ~ high vacuum

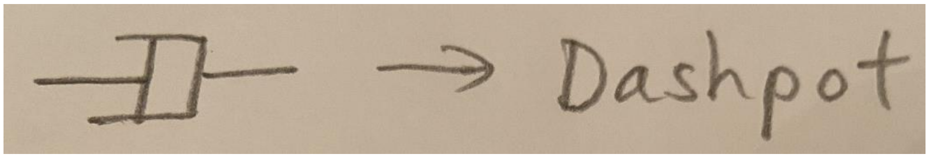
When a MEMS device is not packaged in a vacuum environment: fluidic damping >> thermoelastic damping

Therefore, MEMS devices are often sealed in a low pressure inert or dry gas to set the damping to a desired range.

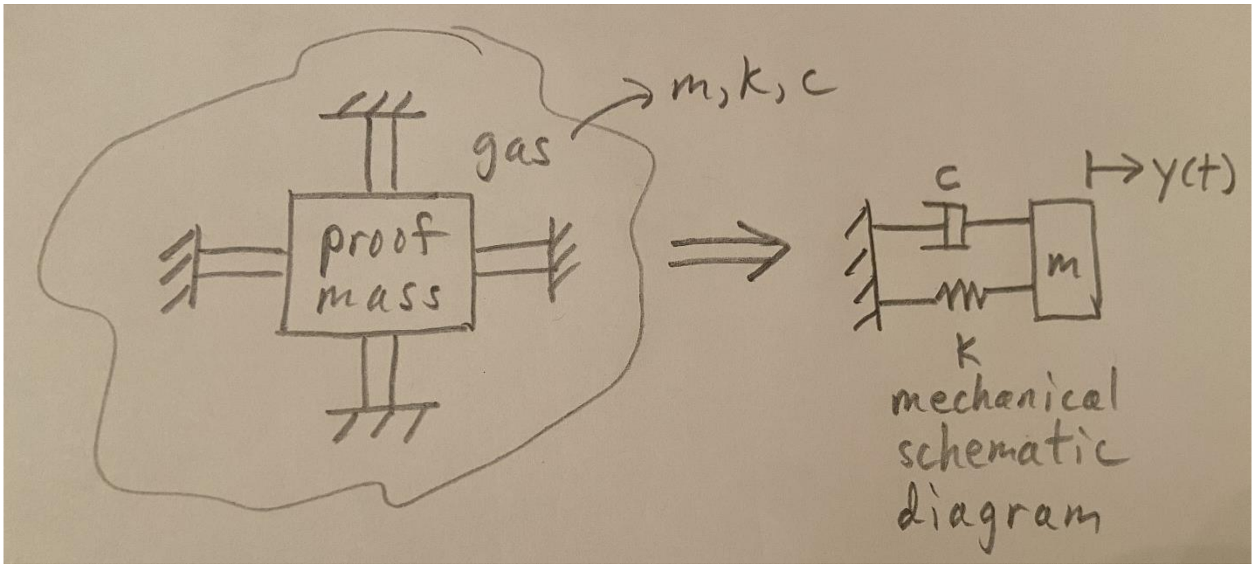
“Desired Range” → c varies with temperature.
→ all packages leak: can use “getters” to trap small amounts of gases leaking into the package.

A getter is a material the binds residual gas (typically only certain gases) in a vacuum sealed package or a vacuum system in attempt to maintain a high vacuum environment. The getter is often activated by heat after package assembly.

Schematic Symbol for Damping: c



Therefore, our spring-mass-damper system becomes:



System Dynamics with Damping Included

$F_I + F_D + F_S = 0 \rightarrow m\ddot{x} + c\dot{x} + kx = 0$ Note: using x or y for displacement changes nothing.

Let's rewrite the equation as: $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$

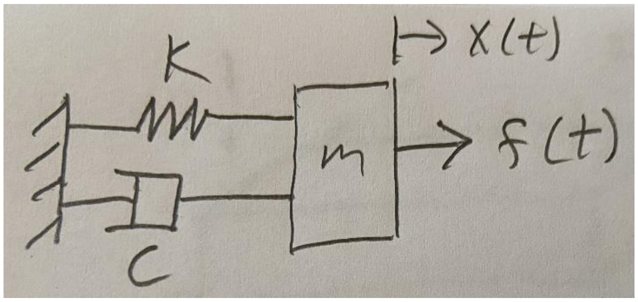
Use $\omega_n = \sqrt{\frac{k}{m}}$ ≡ natural frequency

ζ ≡ damping ratio and Q ≡ mechanical quality factor, where:

$\frac{c}{m} = 2\zeta\omega_n = \frac{\omega_n}{Q} \rightarrow Q = \frac{1}{2\zeta}$: High Q = low damping

$\therefore\ \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$: another form of our system differential EQ.

Let's apply an external force, f(t), to our MEMS device:



Now: $m\ddot{x} + c\dot{x} + kx = f(t)$

Using Laplace transforms: $ms^2X(s) + csX(s) + kX(s) = F(s)$

Or: $X(s)[ms^2 + cs + k] = F(s)$

We can define a mechanical transfer function: T(s), where:

$$T(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} = \frac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

So what does this mean?

At DC, f(t) would be a constant force producing a linear acceleration of the proof mass.

At AC, f(t) is a sinusoidal force causing the proof mass to oscillate back and forth at a frequency, f, (i.e. it vibrates).

T(s) is a second order function with a low pass frequency response.

Therefore, our spring-mass-damper is a mechanical 2nd order low pass filter that filters mechanical vibrations.

If our device has very low damping (high Q), which is very common for MEMS devices, it will have a mechanical gain near f_n (small force: large proof mass motion in the vicinity of f_n).

Reasonable Answers with MEMS Problems

Always think about your numerical answers to see if they are reasonable.

1) Reasonable mass

Consider a “large” Si chip for a MEMS device: 1 cm x 1 cm x 500 μm.

Since $\delta_{Si} = 2.3\ g/cm^3$, the entire chip can only have a mass of 115 mg.

Mass of the proof mass << 115 mg.

2) Reasonable natural frequency

$\omega_n = 2\pi f_n \rightarrow f_n$ is usually in the audio range: 20 Hz to 20 kHz.

3) Reasonable proof mass displacements

Reasonable proof mass displacements << chip width for lateral motion or << chip thickness for vertical motion: 0.1 to 10 μm is reasonable.

Note: displacements cannot exceed gap distances!

4) Reasonable capacitance values

Reasonable MEMS capacitors ~ 1 pF: 10 pF → a really large MEMS cap

Note: MEMS associated capacitances can be much smaller than this, even less than 1 fF (femto Farad: $1\text{x}10^{-15}$ F), particularly if you are considering a change in capacitance.

5) Reasonable voltages and currents

Reasonable voltages can be up to a few 100 V.

What is the approximated current from continuously fully charging and discharging a 10 pF MEMS capacitor with 100 V at 20 kHz:

Let $I = \frac{dQ}{dt} \approx fCV = 20{,}000 * 10X10^{-12} * 100 = 20\mu A$ << 1A.

Example unreasonable answers to MEMS problems:

1 m proof mass displacement
10 kg proof mass
V = 10,000 V
I = 10 A
F_n = 2 MHz
C = 10 μF