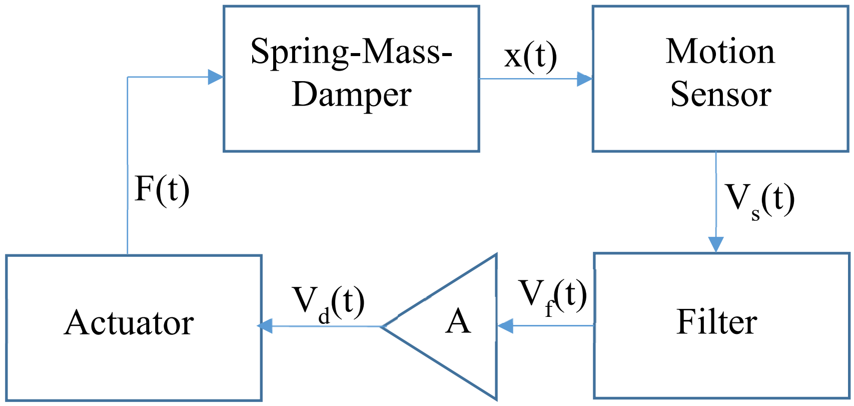
Tuesday 4/4/23

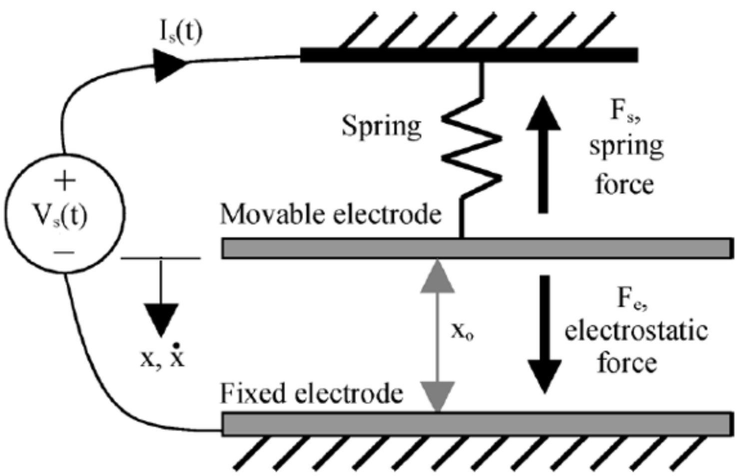
MEMS resonators can also be realized using linear oscillator implementations:



where all of the components are approximately linear and the system satisfies the Barkhausen criterion for oscillation. When examining the components, it is important that they also have (or approximately have) a linear response:

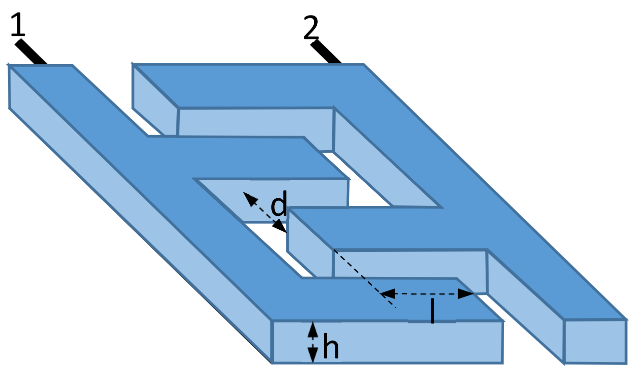
1) Actuator

Considering the PPA where $F = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)^2}$. Note, F is used here instead of F_{EL} so that the actuator block could represent any type of actuator.



Force is proportional to voltage squared and inversely proportional to electrode separation distance squared. The PPA's response is highly nonlinear.

So consider a different type of MEMS electrostatic actuator, the combdrive actuator or CDA:



It consists of two interdigitated comb-style electrodes. One electrode is considered as fixed in space. The other one is secured by a suspension system that only allows it to move such that its teeth can move in or out of the teeth of the other electrode.

For this device, its capacitance, C, is:

$$C = \frac{n\gamma 2\epsilon_o \epsilon_r hx}{d}$$

where n is the number of electrode tooth pairs, γ is a fringing field correction factor ($\gamma \geq 1$), ε_o is the permittivity of free space, ε_r is the relative permittivity of the gas between the electrodes, h is the electrode height, x (l in the drawing) is the overlapping distance between opposing teeth, and d is the separation between the facing sides of opposing teeth.

The energy stored in the capacitor is:

$$E(x) = \frac{1}{2}CV^2 = \frac{V^2 n\gamma\epsilon_o\epsilon_r hx}{d}$$

And the magnitude of the electrostatic force is then:

$$F = \frac{\partial E(x)}{\partial x} = \frac{V^2 n\gamma \epsilon_o \epsilon_r h}{d}$$

This force attempts to pull the movable comb deeper into the stationary comb. Observe that force is proportional to drive voltage squared, but is NOT a function of displacement!

Therefore:

$$F = V^2 B$$

where B is a constant. Force is still a nonlinear function of drive voltage though.

Consider a CDA drive voltage, V_D, of:

$$V_D = V_B + V(t)$$

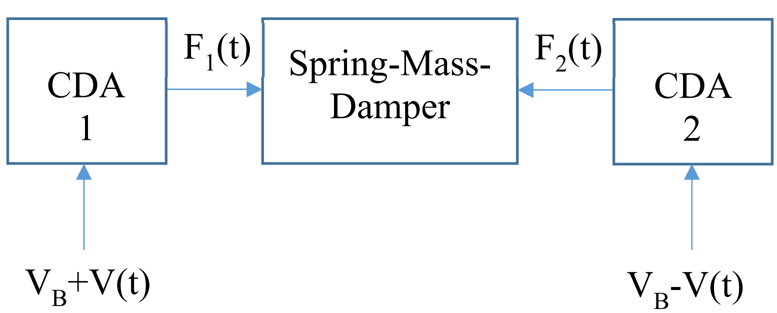
where V_B is a DC voltage and V(t) can vary over time. Therefore:

$$F = (V_B + V(t))^2 B$$

and

$$F = (V_B^2 + 2V_B V(t) + V(t)^2)B$$

Now consider this configuration with two identical CDAs:



$$F_1 = (V_B^2 + 2V_B V(t) + V(t)^2)B$$

$$F_2 = (V_B^2 - 2V_B V(t) + V(t)^2)B$$

The net force acting on the spring-mass-damper (SMD) is F_N, where:

$$F_N = F_1 - F_2 = 4V_B V(t)B$$

Now, the net applied force acting on the SMD is linearly proportional to the applied voltage, V(t).

2) Spring-Mass-Damper

Assuming the SMD has a linear spring constant and linear damping, the SMD can be modelled as:

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

Let's assume that: $f(t) = Acos(\omega t)$

And let's assume that x(t) has the form: $x(t) = x_o sin(\omega t)$

Therefore:

$$x(t) = x_o sin(\omega t)$$

$$\dot{x}(t) = x_o \omega cos(\omega t)$$

$$\ddot{x}(t) = -x_o \omega^2 sin(\omega t)$$

Therefore, the differential equation becomes:

$$-mx_o\omega^2 sin(\omega t) + cx_o\omega cos(\omega t) + kx_o sin(\omega t) = Acos(\omega t)$$

Equating cosine terms:

$$cx_o\omega cos(\omega t) = Acos(\omega t)$$

Leading to: $x_o = \frac{A}{c\omega}$

Notice that since x(t) is a function of sin(ωt) and force is a function of cos(ωt), x(t) lags the applied force by 90^o. This phase shift will be part of the -180^o phase shift needed to satisfy the Barkhausen criterion for oscillation.

3) Motion Sensor

Various types of sensing techniques can be used to measure the motion of the proof mass relative to the frame around it: resistive, capacitive, and optical. Let's examine resistive (piezoresistive) sensing.

Piezoresistors possess a resistance that is affected by mechanical strain, with a positive or negative sensitivity.

If we place two piezoresistors in the beam (spring element) where one experiences a tensile strain during motion while the other one experiences a compressive strain, one will increase in resistance while the other one decreases in resistance, ideally by the same amount of resistance.

Let R_1 be the piezoresistor experiencing an increase in resistance: $R_1 = R + \Delta R$

Let R_2 be the piezoresistor experiencing a decrease in resistance: $R_2 = R - \Delta R$

Put R_1 and R_2 in a voltage divider and drive the circuit with a DC voltage, V_{DC}:

$$V_s = V_{DC}\frac{R_2}{R_1 + R_2} = V_{DC}\frac{R - \Delta R}{2R}$$

Therefore, V_s has a term that is linearly proportional to ΔR, with is proportional to x(t).

4) Filter

The filter block provides the remaining phase delay needed to achieve oscillation. Typically, the MEMS resonator will be high Q and will be resonated at its natural frequency, ω_o:

$$\omega_o = \sqrt{\frac{k}{m}}$$

This allows for the maximum proof mass motion with the smallest amount of effort (energy applied). The SMD provides -90^o of phase delay. Other components will provide some phase delay. Therefore, the filter will provide close to -90^o of additional phase delay.

5) Amplifier

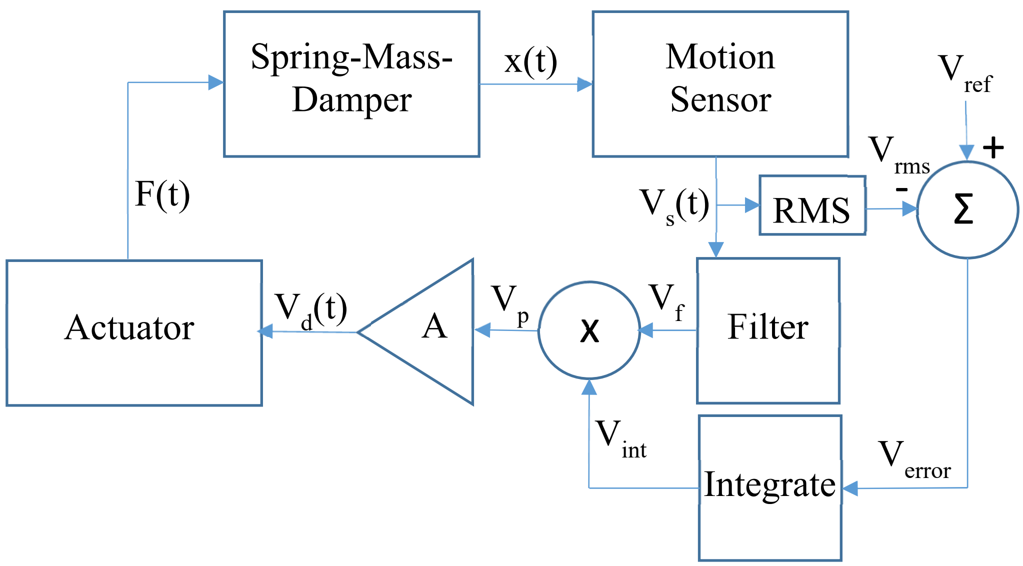
The amplifier does several things. Firstly, in provides the gain necessary to satisfy (or exceed) the Barkhausen criterion for oscillation.

Secondly, it provides the drive voltage level necessary to operate the CDAs. MEMS electrostatic oscillators tend to be fairly high voltage. 100 V is not uncommon for a drive voltage. This can be difficult to achieve with modern low-voltage electronic components. So, a power electronics backend may be required to handle the required voltage levels.

Also, $\pm V_{DRIVE}$ will be required to maintain a linear force being applied to the SMD device. Additionally, the DC voltage V_B will need to be added to $\pm V_{DRIVE}$.

Advanced Operation

An AGC will be necessary for controlling startup and maintaining a constant proof mass amplitude of motion:



The RMS block determines the root mean square value of the V_s signal, V_{rms}. This signal is compared with a reference voltage, V_{ref}, which generates an error signal if they are not equal. The integrator and multiplier attempt to reduce V_{error} to 0 V by keeping the amplitude of x(t) at the desired value. The integrator is necessary to reduce the error in the proof mass amplitude to zero.

The operations of summer, filter, integrator, and multiplier could all be performed digitally in a microcontroller. In that case, $V_s(t)$ would be fed into an A/D converter and $V_p(t)$ would be produced by a D/A converter.

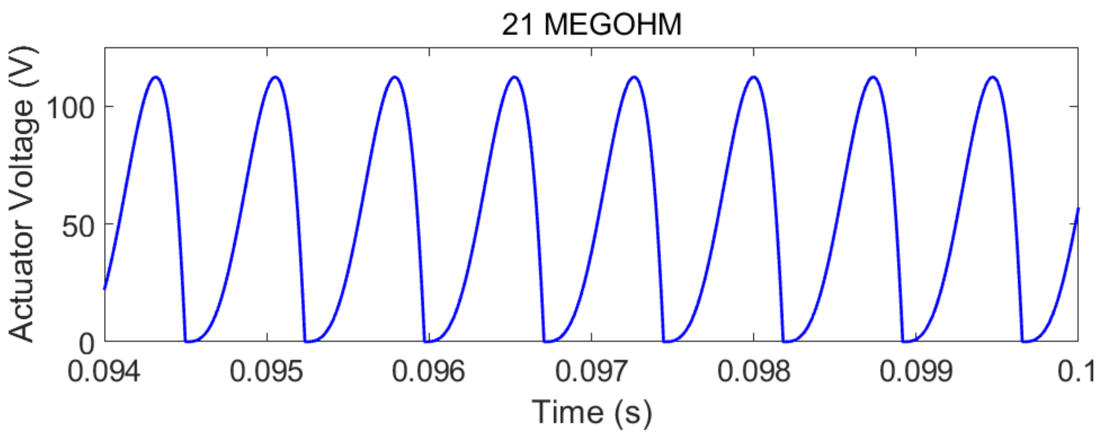
The microcontroller approach would allow for the filter to be tuned, which could allow the entire resonator to be able to lock its operation at the natural frequency of the SMD device as it changes (due to temperature, aging, mass loading, etc.).

From two x(t) measurements and the time in between when they were recorded, $\dot{x}(t)$ could be estimated. These two states could be used to feedback forces proportional to them in order to electrically control ω_o and Q of the SMD.

Applications for MEMS resonators:

1) Self-resonating MEMS PPA resonator

The voltage across the PPA swings from 0V to the pull-in voltage, which can be quite high. Below are the results of a Simulink simulation of a MEMS PPA resonator:



Since MEMS devices can operate over a much larger temperature range than semiconductor electronic devices, the DC source – resister – PPA MEMS resonator could be used to generate AC voltage signals in operating environments too hot or too cold for transistors to operate.

2) MEMS vibratory gyroscopes

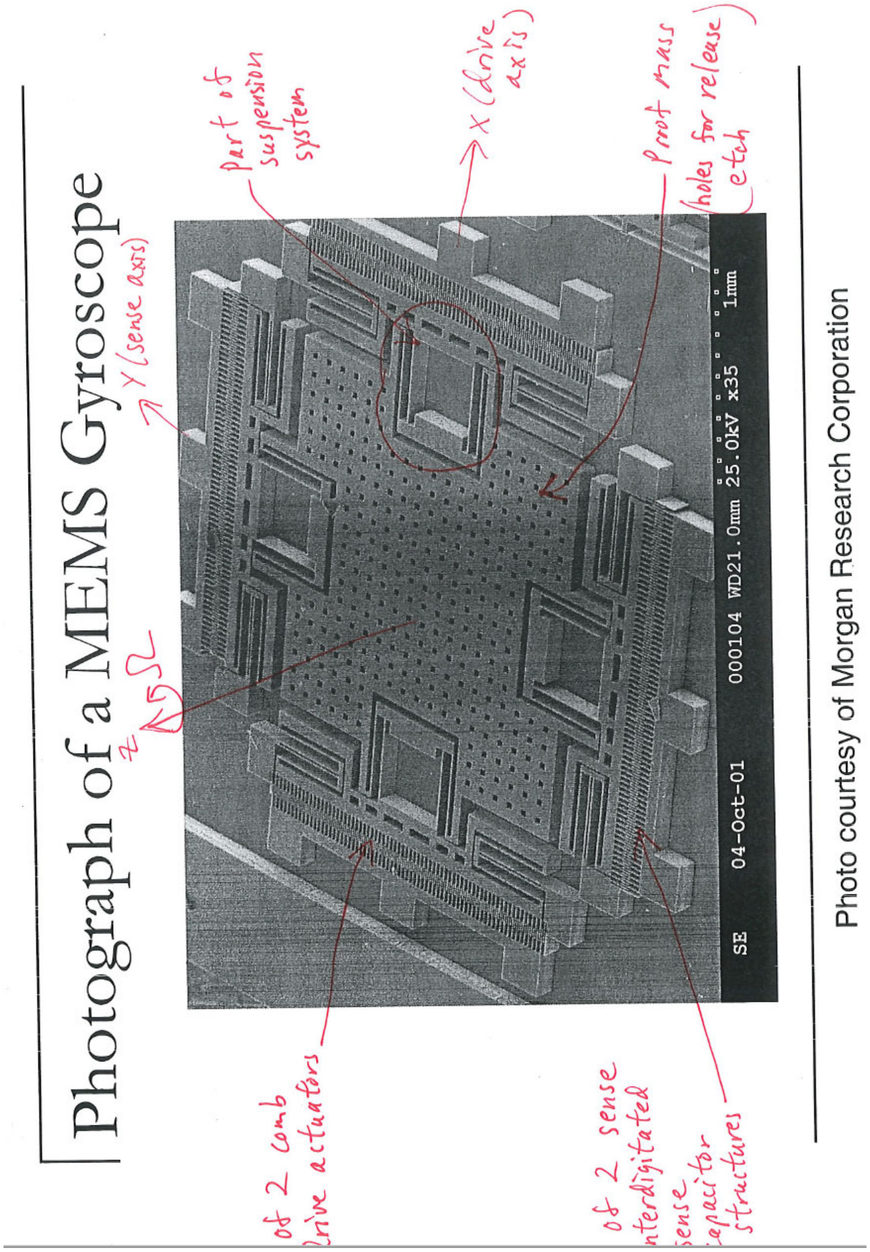
Assume that the plane of a MEMS device is along the x-y plane, and the proof mass is oscillating along the x-axis with a velocity vector of $\overrightarrow{V_x}$.

If the MEMS device is rotated about the z-axis at an angular rate of $\overrightarrow{\Omega_z}$,

then the proof mass experiences an acceleration along the y-axis, the Coriolis acceleration, according to:

$$\overrightarrow{a_y} = 2\overrightarrow{\Omega_z}X\overrightarrow{V_x}$$

Therefore, if the proof mass's suspension system allows the proof mass to also move along the y-axis, the amplitude of the y-axis motion can be measured to determine the angulate rate of rotation about the z-axis.



3) Mass sensor

A MEMS resonator that is oscillating at its natural frequency can be used to measure the mass of various substances that might adhere to the proof mass, since

$$\omega_o = \sqrt{\frac{k}{m}}$$

One example is dust. The resonator could be used to detect or measure the accumulation of dust.

Another example would be to detect specific species of bacteria. Researchers at the AU Vet School have developed chemical monolayers that are bacteria specific: only a certain species of bacteria will adhere to each type. By coating the proof mass with a certain species-specific monolayer, the MEMS resonator could be used to determine if that particular species is present in a solution.

Thermal actuators would typically need to be used to drive the proof mass instead of electrostatic actuators, since it would likely be immersed in a liquid. Thermal actuators usually have lower frequency operation compared to electrostatic actuators.

4) Pressure sensor

A proof mass with a relatively large surface area, where the motion is normal to that surface, can experience a mass loading effect from pushing the surrounding gas out of the way while it oscillates. This mass loading will change the effective mass, resulting in a corresponding change in the natural frequency. This effect typically increases with increasing pressure.

Also, the Q value typically decreases with an increase in gas pressure, due to increased fluidic damping, increasing the amount of energy required to oscillate the resonator at its natural frequency. The feedback system will detect this change for a constant amplitude of proof mass motion, and that signal could be used to determine the ambient gas pressure.

5) Strain Sensor

Mechanically straining the resonator's suspension system can increase the effective spring constant, resulting in a corresponding change in the natural frequency of the SMD and the resonator.

6) Temperature Sensor

Temperature will affect the material properties of the SMD, and possibly the support electronics, which can change the Q and/or the natural frequency. This change can be detected and used to determine ambient temperature. Many MEMS devices are extremely sensitive to temperature.