Thursday 3/30/23

Introduction to MEMS

MEMS: acronym for "Microelectromechanical Systems"

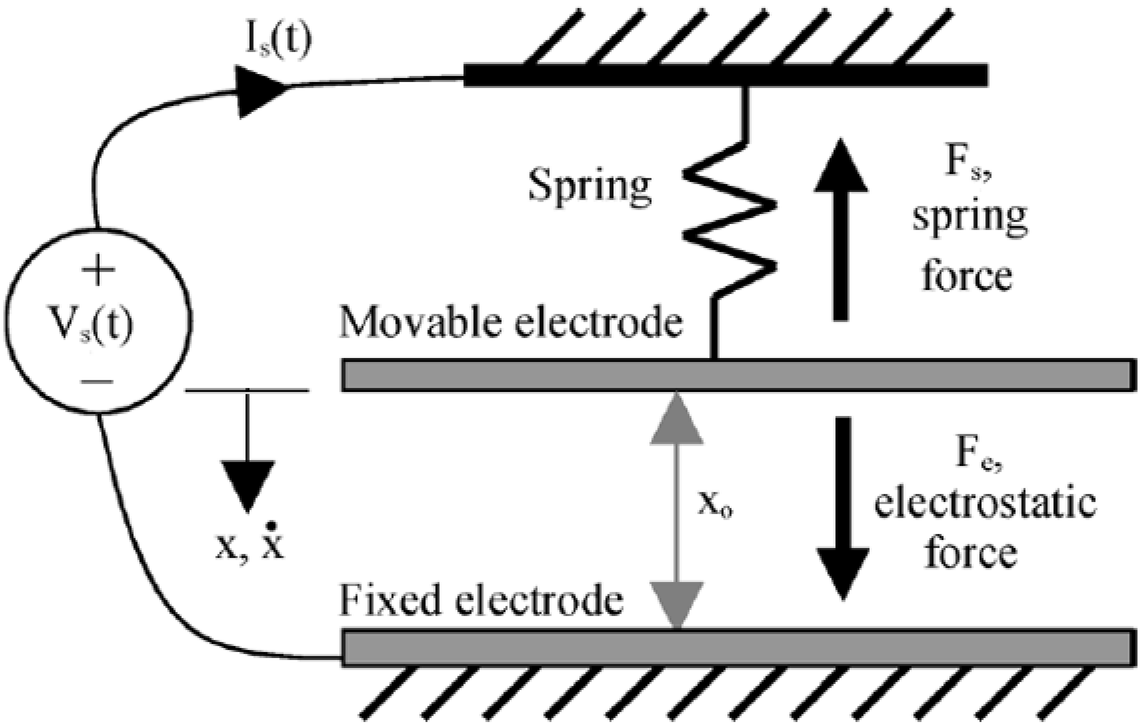
Definition: Any device or system partially or fully manufactured using microfabrication techniques.

Traditional MEMS: manufactured from Si using additive and subtractive micromachining techniques originating from integrated circuit manufacturing.

This may include sensors and actuators, mechanical structures, antennas, waveguides, optical components, integrated electronics, batteries, photovoltaic cells, etc., integrated onto the same substrate or on multiple separate substrates that are bonded together.

A. The Parallel Plate Actuator (PPA)

Consider this illustration of two parallel electrodes (i.e. plates) separated by a rest gap, x_o. The bottom electrode is fixed in place, while the upper electrode is allowed to move up and down relative to the bottom electrode. Its motion is constrained by a suspension system, modelled by a linear spring, k. Damping, c, is also present.



The two electrodes form a parallel plate capacitor, C(t), where:

$$C(t) = \frac{\varepsilon_o \varepsilon_r A}{x_o - x(t)}$$

where A is the overlapping electrode area, x(t) is the motion of the upper electrode toward the bottom electrode, ε_o is the permittivity of free space, and ε_r is the relative permittivity of the dielectric material between the two electrodes. Observe that C(t) varies as x(t) varies with time.

It is assumed that $A \gg (x_o)^2$, so that fringing effects can be neglected. Otherwise, C(t) with x(t) = 0 will be larger than estimated by the equation above.

Also, $x(t) \leq x_o$.

When a DC voltage, V, is applied across the two electrodes, charge builds up on them. The energy stored in the capacitor, E(t) is:

$$E(t) = \frac{1}{2}C(t)V(t)^2 = \frac{V(t)^2 \varepsilon_o \varepsilon_r A}{2(x_o - x(t))}$$

From a physics perspective, if x(t) ↑, C(t) ↑ and E(t) ↑.

The stored charge results in an electrostatic force, F_{EL}, between the two plates, that attempts to bring them into contact (i.e. the upper plate moves closer to the lower plate. To find the magnitude of F_{EL}, we will assume that V(t) is a fixed DC voltage, V, and we will represent x(t) as the variable x. Therefore:

$$E(x) = \frac{1}{2}C(x)V^2 = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)}$$

and the magnitude of F_{EL} is then:

$$F_{EL} = \frac{\partial E(x)}{\partial x} = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)^2}$$

Notice that the force is proportional to drive voltage squared and is also inversely proportional to electrode separation distance squared.

Therefore, the differential equation of motion for this system is:

$$m\ddot{x} + c\dot{x} + kx = F_{EL}(t) = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)^2}$$

This is a second order nonlinear system. It is nonlinear, but it cannot go chaotic (less than 3rd order), unless the system is forced.

Notice that the PPA is a square law device: same force regardless of the polarity of the drive voltage.

What if we slowly increase the V(t) from 0 V to some small nonzero voltage?

At steady state, the upper electrode will move closer to the bottom electrode until the spring force balances out the electrostatic force:

$$kx = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)^2}$$

The spring force is linearly proportional to displacement, while the electrostatic force is roughly proportional to displacement squared: i.e. the incremental increase in electrostatic force for an incremental increase in drive voltage is greater than the corresponding increase in spring force.

At a voltage called the pull-in voltage, V_{PI}, the electrostatic force becomes greater than the spring force, and the upper electrode then accelerates into contact with the bottom electrode.

$$V_{PI} = \sqrt{\frac{8kx_o^3}{27\varepsilon_o \varepsilon_r A}}$$

The corresponding stable range of the actuator's motion is:

$$0 \leq x < \frac{x_o}{3}$$

This unstable condition is called pull-in or snap-in.

By just varying the drive voltage, the displacement of the movable electrode can only be controlled over this range of x.

This system can be modeled as a mechanical system experiencing spring softening.

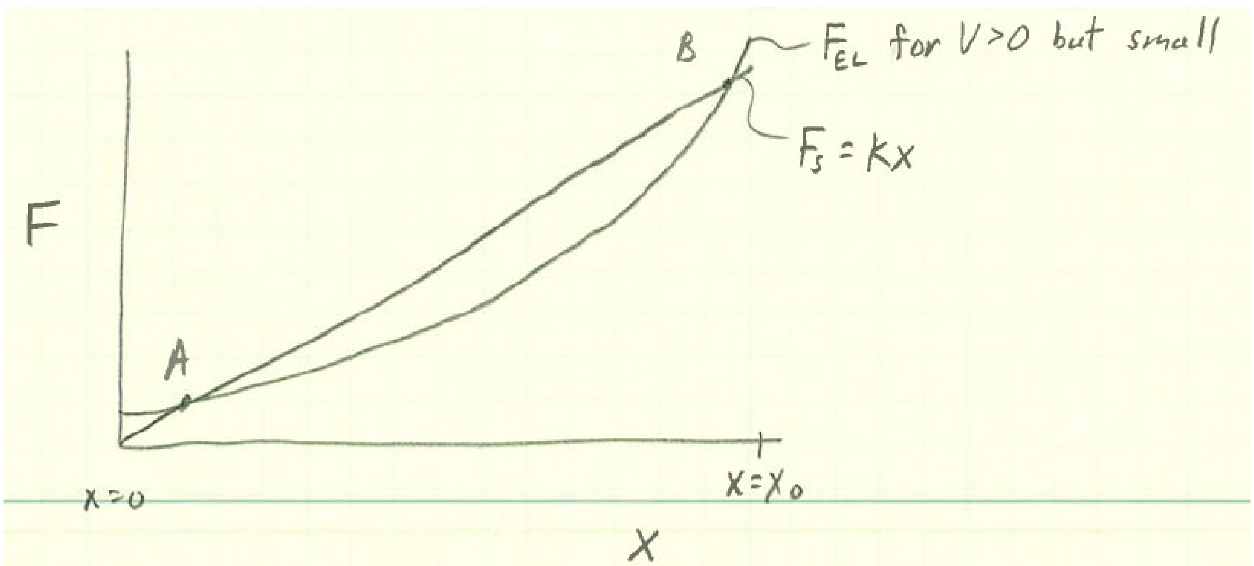
Note, a similar relationship exists with an electromagnetic solenoid for the actuator (typically macro scale, not micro scale):

$$F_M = \frac{(NI)^2 \mu_o A}{2(x_o - x)^2}.$$

Going back to the force balance equation for the PPA:

$$kx = \frac{V^2 \varepsilon_o \varepsilon_r A}{2(x_o - x)^2}$$

This equation can be solved graphically:



Two solutions exist: A and B

A is a stable equilibrium point, w.r.t. perturbation.
B is an unstable equilibrium point, w.r.t. perturbation.

Increase V slowly and the F_{EL} trace moves up with respect to the F_S (spring force) trace, and A and B move closer together.

When $|V| = V_{PI}$, A and B will have merged into a single unstable equilibrium point.

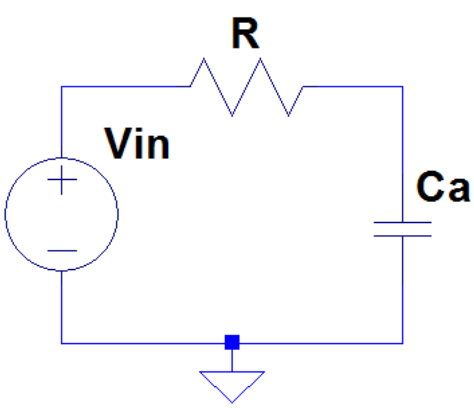
There are no solutions for when $|V| > V_{PI}$, and the electrodes will snap into contact.

B. Using the Parallel Plate Actuator

It is straight forward to use a PPA: connect a voltage source to it. But, there are some issues:

1) If $|V| \geq V_{PI}$, snap-in occurs.
2) For any drive voltage $< V_{PI}$, if the movable electrode somehow gets moved to the unstable equilibrium point, or further, snap-in will occur.
3) Transient events or even noise could result in moving the movable electrode to where snap-in occurs.
4) If snap-in occurs, the electrodes are usually shorted (unless a mechanical stop is employed), which can damage or destroy the PPA, or the amplifier circuit driving the PPA, or both.

So, a small resistor is often placed between the actuator drive voltage source (often an amplifier) and the PPA:



C_a above is the electrical model for the PPA capacitance, C(x) or C(t).

If snap-in occurs now, R limits the current flowing through the PPA to avoid or minimize damaging it or the power supply circuitry.

C. Snap-in Resonator

What if R above is very large? Let V_c be the voltage across the PPA (C_a):

$$i = \frac{V_{in} - V_c}{R} = V_c \frac{dC_a}{dt} + C_a \frac{dV_c}{dt}$$

Where i is the current through R and C_a.

Substituting in for C_a and rearranging:

$$\frac{dV_c}{dt} + V_c\left(\left(\frac{x_o - x}{R\varepsilon_o\varepsilon_r A}\right) + \frac{dx}{dt}\left(\frac{1}{x_o - x}\right)\right) - \frac{V_{in}(x_o - x)}{R\varepsilon_o\varepsilon_r A} = 0$$

Also:

$$m\ddot{x} + c\dot{x} + kx = F_e(t) = \frac{V^2\varepsilon_o\varepsilon_r A}{2(x_o - x)^2}$$

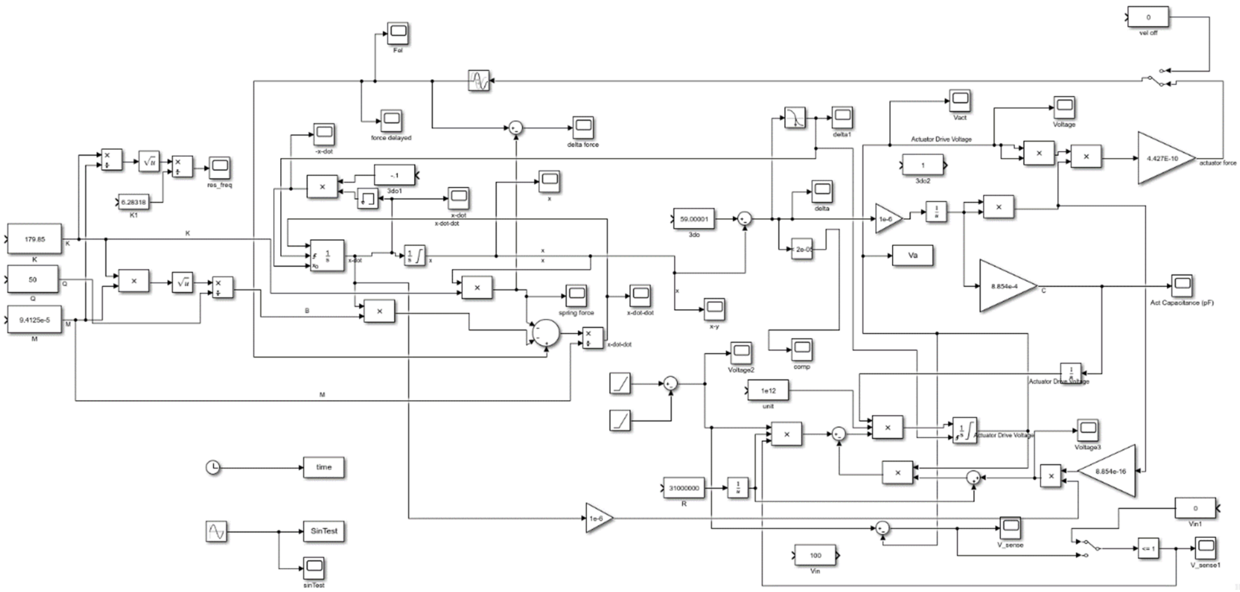
and since the movable electrode will bounce off the lower electrode and lose some energy:

$$\dot{x}_+ = -\gamma\dot{x}_-$$

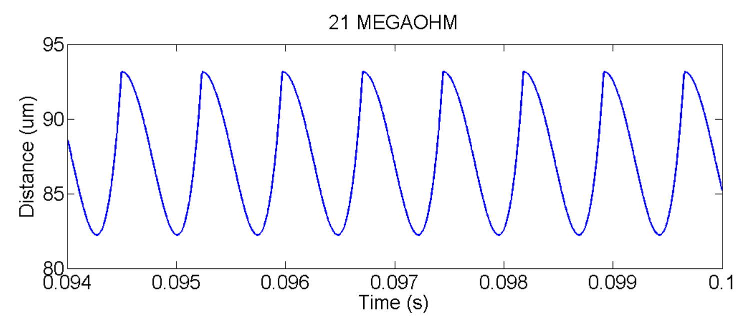
where γ is the percent momentum (really velocity) lost in the inelastic collision.

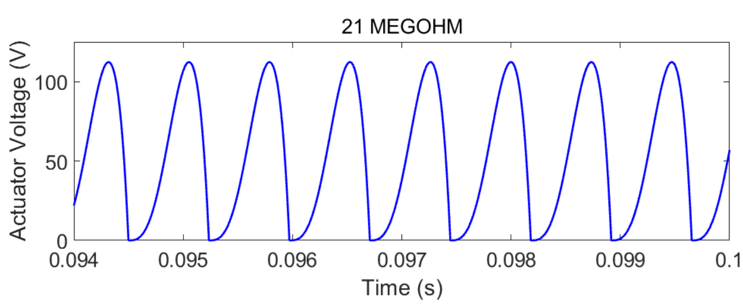
These three equations describe the motion and operation of this DC source, R and PPA system.

This is the Simulink model for this system:



Simulation response for a PPA with $V_{in} > V_{PI}$ and R = 21 MΩ:



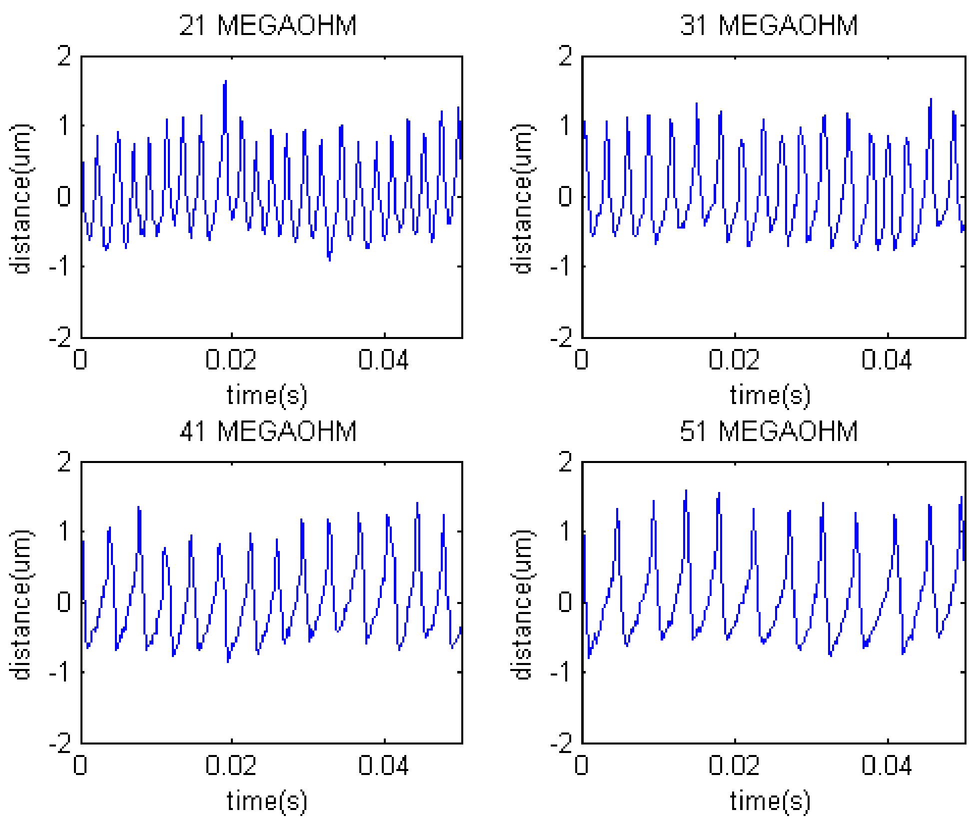


The movable electrode oscillates and the voltage across the PPA oscillates (nonlinearly).

So what is going on:

1) When the electrodes make contact due to snap-in, the PPA capacitor discharges and the electrostatic force goes to zero.
2) The spring force of the extended spring pulls the movable electrode back away from the fixed electrode.
3) The PPA cap starts recharging through R. But since R is so big, it takes a relatively long time for the PPA to charge up to where the pull-in condition is again reached. This allows the movable electrode to be move a sizable distance away from the fixed electrode.
4) Eventually the voltage across the PPA reached V_{PI}, and snap-in again occurs.

Actual recorded test data from a MEMS PPA in series with a large value resistor and a large voltage DC source:



The higher the resistance, the lower the oscillation frequency.

What about other similar systems?

A similar oscillating system could be realized using an electromagnet as the actuator (old fashioned DC doorbell or buzzer).

Can this system become chaotic?

Consider the describing equations:

$$\frac{dV_c}{dt} + V_c\left(\left(\frac{x_o - x}{R\varepsilon_o\varepsilon_r A}\right) + \frac{dx}{dt}\left(\frac{1}{x_o - x}\right)\right) - \frac{V_{in}(x_o - x)}{R\varepsilon_o\varepsilon_r A} = 0 \qquad (1)$$

$$m\ddot{x} + c\dot{x} + kx = F_e(t) = \frac{V^2\varepsilon_o\varepsilon_r A}{2(x_o - x)^2} \qquad (2)$$

$$\dot{x}_+ = -\gamma\dot{x}_- \qquad (3)$$

Consider the states: $\dot{x}$, $\ddot{x}$, $\dot{V}_c$

3 derivative terms and a nonlinearity (actually several nonlinearities), so it is possible.

In reality, this system can be made to go chaotic under the right conditions.

Consider this is regard to other oscillating systems discussed this semester:

> A DC voltage source, a resister, and a MEMS PPA are all that are required to generate mechanical oscillation and to produce an oscillating voltage waveform.