Tuesday 3/21/23

Circuits for Implementing Nonlinear Functions

1) 4 Quadrant multiplier chips

Analog Devices makes several four quadrant multiplier (and related) chips <below>. Some are pretty fast now.

2) Diodes

Small signal: $i = I_s\left(e^{\frac{v}{nV_T}} - 1\right)$

Large signal: useful as a switch

3) MOSFET

Triode region: useful as a nonlinear voltage controlled resistance

Small signal: in saturation, the MOSFET is a square law device:
$I_D \approx \frac{\beta_n}{2}(V_{GS} - V_T)^2$

Larger signal: useful as an analog switch (analog comparator and digital logic used to trip it)

Analog Devices Multiplier/Divider chips:

Part Number	Transfer Function	Vin *min | V*	Vin *max | V*	Vin Range *max |*	BW -3 dB *typ | Hz*	Output Range	Total Error *max | %*	Price (1000+) *$ US*
Filter Parts 11 parts	11 Values Selected HIDE	-12.5 - -1 HIDE	1 - 12.5 HIDE	7 Values Selected HIDE	400k - 2G HIDE	8 Values Selected HIDE	0.25 - 2.7 HIDE	4.8 - 44.88 HIDE
ADL5391	a (XPLS-XMNS)(YPLS-YMNS)/1V + (ZPLS-ZMNS)	-1	1	2Vpp	2G	±2V	2.7	$4.95 (ADL5391ACPZ-R7)
AD835	[(X1–X2)(Y1–Y2)/U] + Z	-1	1	±1V	250M	±2.5V	0.3	$11.47 (AD835ARZ)
AD633	[(X1–X2)(Y1–Y2)/10] + Z	-10	10	±10V	1M	±11V	2	$4.80 (AD633ARZ)
AD734	[(X1–X2)(Y1–Y2)/(U1–U2)] + (Z1–Z2)	-12.5	12.5	±12.5V	10M	12V	0.4	$19.01 (AD734ANZ)
AD834	(4 mA)(XY)	-1	1	±1V	500M	±4.04mA	2	$22.85 (AD834JRZ)
AD538	[y(z/x)]^m	-10	10	±10V	400k	±11V	1	$44.88 (AD538ADZ)
AD539	-(vx*vy)/vu	-4.2	4.2	±4.2V	25M	±2.8mA	2.5	$29.94 (AD539JNZ)
AD632	[(X1–X2)(Y1–Y2)/10] + Z2	-12	12	±12V	1M	±11V	0.5	$17.34 (AD632AHZ)
AD534	[(X1-X2)(Y1-Y2)/10 V] + Z2	-10	10	±10V	1M	±11V	0.25	$26.38 (AD534JHZ)
AD534S	[(X1-X2)(Y1-Y2)/10 V] + Z2	-10	10	±10V	1M	±11V	0.25	-
AD532	(X1-X2)(Y1-Y2)/10 V	-10	10	±10V	1M	±10V	1	$36.54 (AD532JHZ)

ANALOG DEVICES

250 MHz, Voltage Output, 4-Quadrant Multiplier

Data Sheet **AD835**

FEATURES

Simple: basic function is W = XY + Z
Complete: minimal external components required
Very fast: Settles to 0.1% of full scale (FS) in 20 ns
DC-coupled voltage output simplifies use
High differential input impedance X, Y, and Z inputs
Low multiplier noise: 50 nV/√Hz

APPLICATIONS

Very fast multiplication, division, squaring
Wideband modulation and demodulation
Phase detection and measurement
Sinusoidal frequency doubling
Video gain control and keying
Voltage-controlled amplifiers and filters

FUNCTIONAL BLOCK DIAGRAM

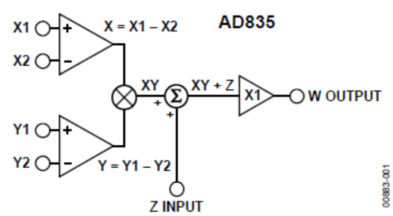


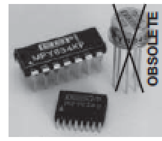
Figure 1.

GENERAL DESCRIPTION

The AD835 is a complete four-quadrant, voltage output analog multiplier, fabricated on an advanced dielectrically isolated complementary bipolar process. It generates the linear product of its X and Y voltage inputs with a −3 dB output bandwidth of 250 MHz (a small signal rise time of 1 ns). Full-scale (−1 V to +1 V) rise to fall times are 2.5 ns (with a standard R_L of 150 Ω), and the settling time to 0.1% under the same conditions is typically 20 ns.

PRODUCT HIGHLIGHTS

1. The AD835 is the first monolithic 250 MHz, four-quadrant voltage output multiplier.
2. Minimal external components are required to apply the AD835 to a variety of signal processing applications.
3. High input impedances (100 kΩ||2 pF) make signal source loading negligible.
4. High output current capability allows low impedance loads to be driven.



MPY634

SBFS017A – DECEMBER 1995 – REVISED DECEMBER 2004

Wide Bandwidth
PRECISION ANALOG MULTIPLIER

FEATURES

- WIDE BANDWIDTH: 10MHz typ
- ±0.5% MAX FOUR-QUADRANT ACCURACY
- INTERNAL WIDE-BANDWIDTH OP AMP
- EASY TO USE
- LOW COST

APPLICATIONS

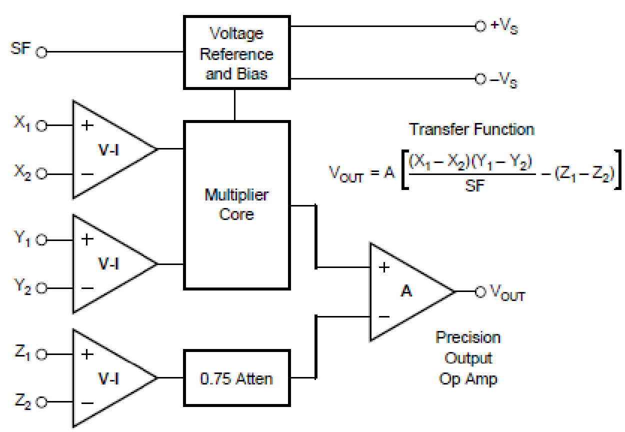
- PRECISION ANALOG SIGNAL PROCESSING
- MODULATION AND DEMODULATION
- VOLTAGE-CONTROLLED AMPLIFIERS
- VIDEO SIGNAL PROCESSING
- VOLTAGE-CONTROLLED FILTERS AND OSCILLATORS

DESCRIPTION

The MPY634 is a wide bandwidth, high accuracy, four-quadrant analog multiplier. Its accurately laser-trimmed multiplier characteristics make it easy to use in a wide variety of applications with a minimum of external parts, often eliminating all external trimming. Its differential X, Y, and Z inputs allow configuration as a multiplier, squarer, divider, square-rooter, and other functions while maintaining high accuracy.

The wide bandwidth of this new design allows signal processing at IF, RF, and video frequencies. The internal output amplifier of the MPY634 reduces design complexity compared to other high frequency multipliers and balanced modulator circuits. It is capable of performing frequency mixing, balanced modulation, and demodulation with excellent carrier rejection.

An accurate internal voltage reference provides precise setting of the scale factor. The differential Z input allows user-selected scale factors from 0.1 to 10 using external feedback resistors.



ANALOG DEVICES

10 MHz, Four-Quadrant Multiplier/Divider

AD734

FEATURES

High accuracy
- **0.1% typical error**

High speed
- **10 MHz full power bandwidth**
- **450 V/μs slew rate**
- **200 ns settling to 0.1% at full power**

Low distortion
- **−80 dBc from any input**
- **Third-order IMD typically −75 dBc at 10 MHz**

Low noise
- **94 dB SNR, 10 Hz to 20 kHz**
- **70 dB SNR, 10 Hz to 10 MHz**

Direct division mode
- **2 MHz BW at gain of 100**

APPLICATIONS

High performance replacement for AD534
Multiply, divide, square, square root
Modulators, demodulators
Wideband gain control, rms-to-dc conversion
Voltage-controlled amplifiers, oscillators, and filters
Demodulator with 40 MHz input bandwidth

FUNCTIONAL BLOCK DIAGRAM

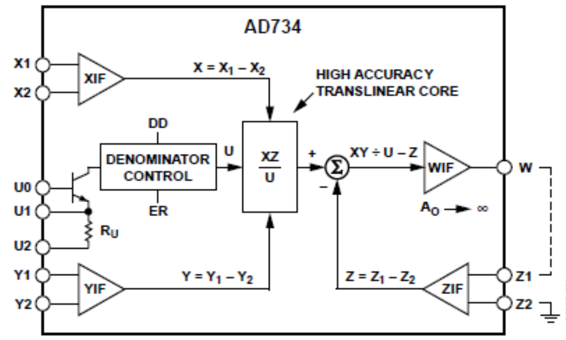


Figure 1.

GENERAL DESCRIPTION

The AD734 is an accurate high speed, four-quadrant analog multiplier that is pin compatible with the industry-standard AD534 and provides the transfer function W = XY/U. The AD734 provides a low impedance voltage output with a full power (20 V p-p) bandwidth of 10 MHz. Total static error (scaling, offsets, and nonlinearities combined) is 0.1% of full scale. Distortion is typically less than −80 dBc and guaranteed. The low capacitance X, Y, and Z inputs are fully differential. In most applications, no external components are required to define the function.

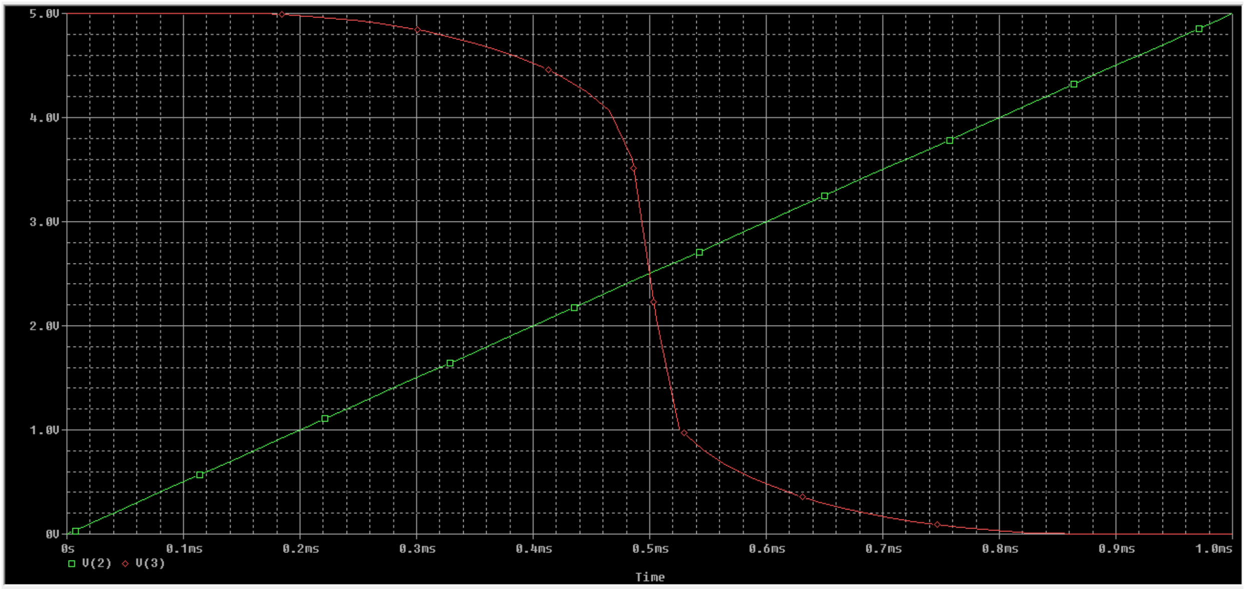
The internal scaling (denominator) voltage, U, is 10 V, derived from a buried-Zener voltage reference. A new feature provides the option of substituting an external denominator voltage, allowing the use of the AD734 as a two-quadrant divider with a 1000:1 denominator range and a signal bandwidth that remains 10 MHz to a gain of 20 dB, 2 MHz at a gain of 40 dB, and 200 kHz at a gain of 60 dB, for a gain-bandwidth product of 200 MHz.

The advanced performance of the AD734 is achieved by a combination of new circuit techniques, the use of a high speed complementary bipolar process, and a novel approach to laser trimming based on ac signals rather than the customary dc methods. The wide bandwidth (>40 MHz) of the AD734's input stages and the 200 MHz gain-bandwidth product of the multiplier core allow the AD734 to be used as a low distortion demodulator with input frequencies as high as 40 MHz as long as the desired output frequency is less than 10 MHz.

The AD734AQ and AD734BQ are specified for the industrial temperature range of −40°C to +85°C and come in a 14-lead CERDIP and a 14-lead PDIP package. The AD734SQ/883B, available processed to MIL-STD-883B for the military range of −55°C to +125°C, is available in a 14-lead CERDIP.

4) CMOS Inverter

Amplifier with nonlinear saturation



V_o vs. V_{in} for approximately balanced CMOS inverter. Green: V_{in}, red: V_o.

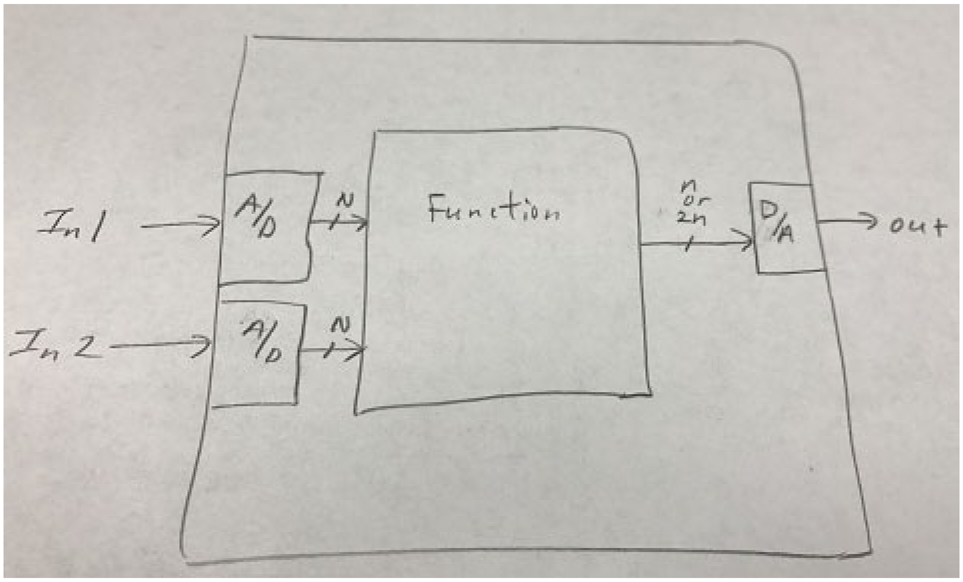
5) OP amp and related circuits

Useful as a saturating amplifier

Useful as a comparator (signum function): $sgn(x) = \begin{cases} -1 \ \ if\ x < 0 \\ 0\ if \quad x = 0 \\ 1\ if\ x > 0 \end{cases}$

Useful in hysteresis circuits (positive feedback amplifier)

6) Digital Nonlinearity



The input could be one, two, or even more A/D's. Could be made from discrete components or a single microcontroller.

The function could be a memory lookup table, a multiport SRAM for a time delay, or a microcontroller arithmetic processing unit.

It could be a simple multiplication, or almost any mathematical function or nonlinearity, even time varying.

It would need to be clocked at a considerably higher frequency than the BW of the rest of the system. A LPF on the D/A output might be required.

Issues to consider: operating speed, converter resolution, propagation delay, filtering requirements, added noise to the system.

7) Other components possibly useful for circuit nonlinearity generation:
    a. BJTs
    b. jFETs
    c. SCR's and triacs
    d. Electrolytic capacitors
    e. LED with a CdS photocell
    f. MEMS voltage controlled capacitor
    g. High current resistor that heats up due to I^2R losses (light bulb)
    h. Saturating inductor
    i. Current mirror circuits
    j. Switches and relays
    k. Plasma breakdown devices (neon bulb, etc.)

The Logistic Map and Chaos

Chaos can be observed in a simple discrete model for the population dynamics within a species. For this:

r is the reproductive rate, which decreases as the population increases,

x is a measure of the population as a fraction of the carrying capacity,

x_0 is the starting population,

x_k is the population at the k^{th} interval,

x_{k+1} is the population at the $(k+1)^{th}$ interval,

In this simple system, the population completely dies out each interval, and the next interval's population is a function of this interval's reproductive rate and current population.

$$r(x) = r(1 - x)$$

Therefore, this equation models the population at successive generations (intervals), k:

$$x_{k+1} = rx_k(1 - x_k)$$

The plot of x(k) vs. k is called the logistic map.

Consider the logistic maps for some cases for different reproductive rates, r, with an initial population, x_0, of 0.1.

If r<1, the population eventually dies out.

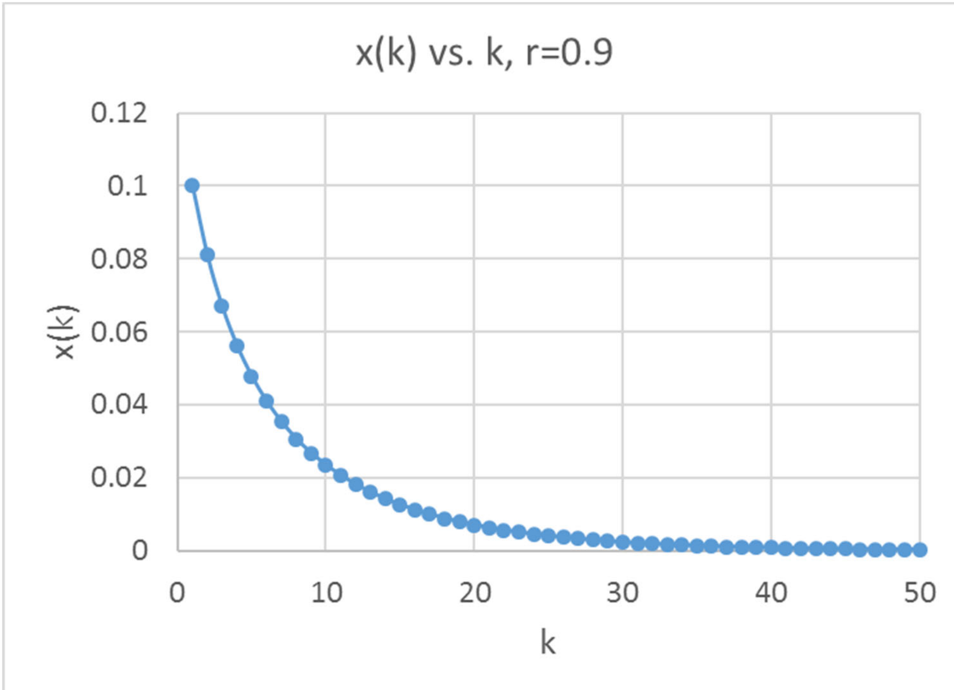
If r=1.9, the population settles out at 0.47 (47% of the carrying capacity).

If r=2.9, the population stabilizes after several small boom and bust cycles with a population at 65.6% of the carrying capacity.

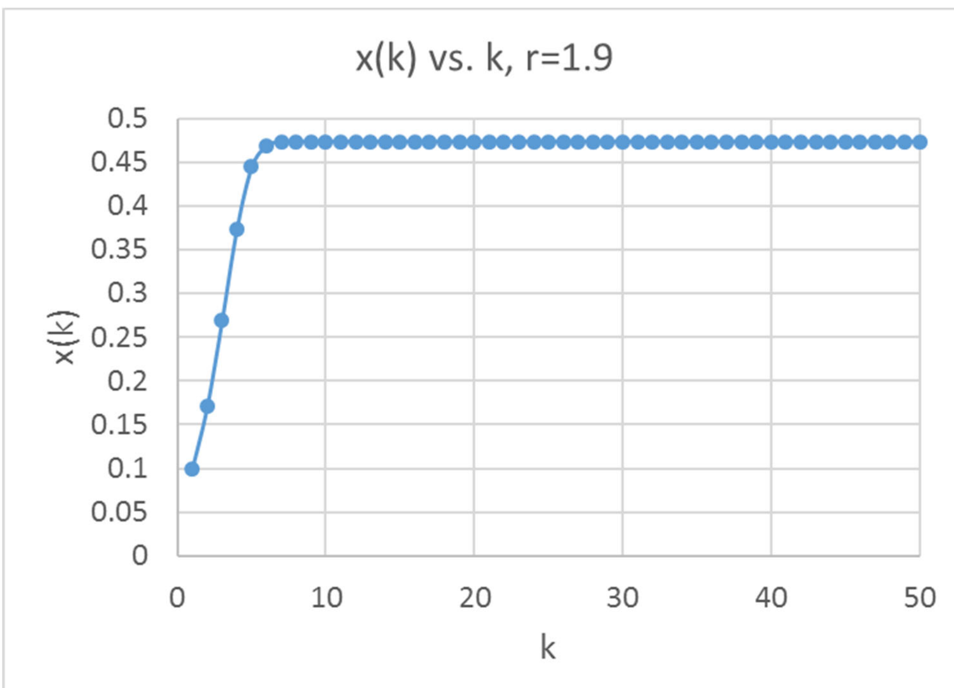
If r=3.3, the population settles to continually having cyclical boom and bust cycles.

If r=3.6, the population is now chaotic.

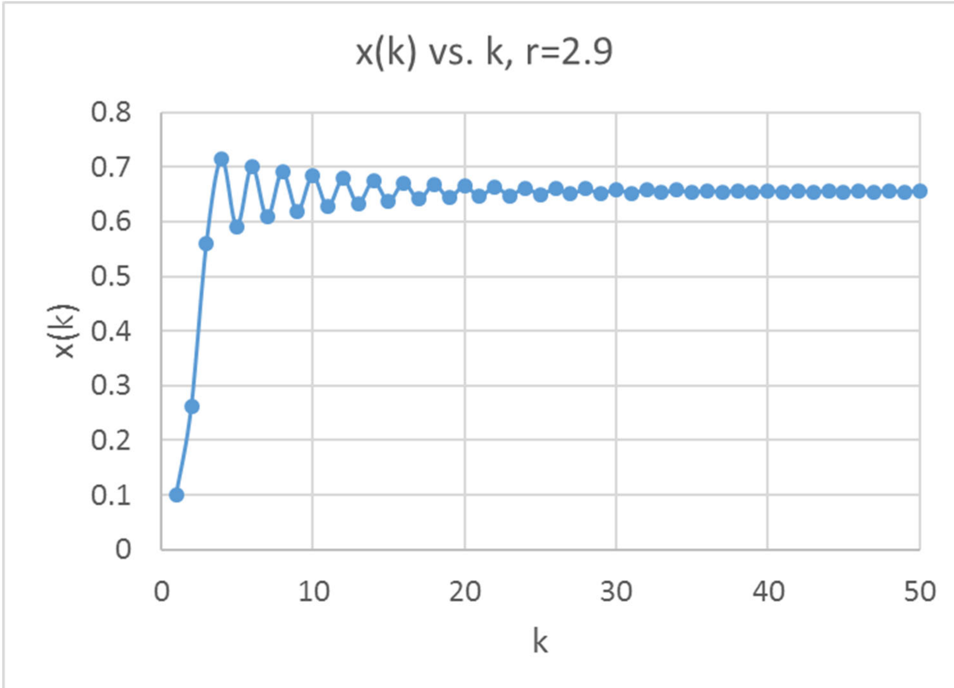
A bifurcation diagram is useful in understanding the dynamics of this and other systems that can have chaotic regions. A bifurcation diagram shows the values visited (or asymptotically approached) of a system as a function of a system bifurcation parameter.

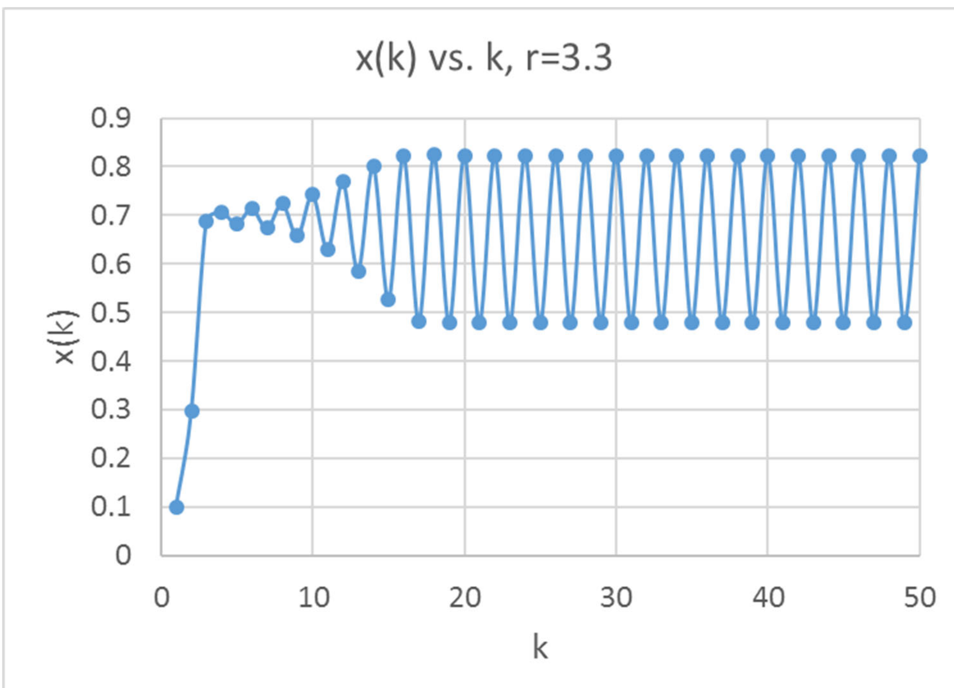


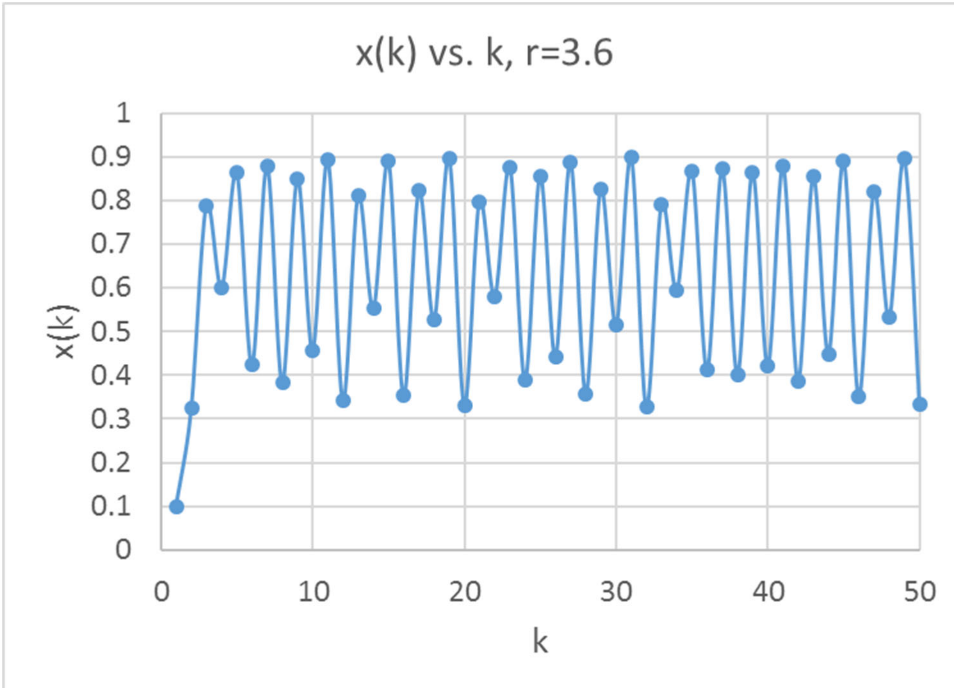
x(k) vs. k, r=0.9
0.12
0.1
0.08
0.06
0.04
0.02
0
x(k)
0
10
20
30
40
50
k

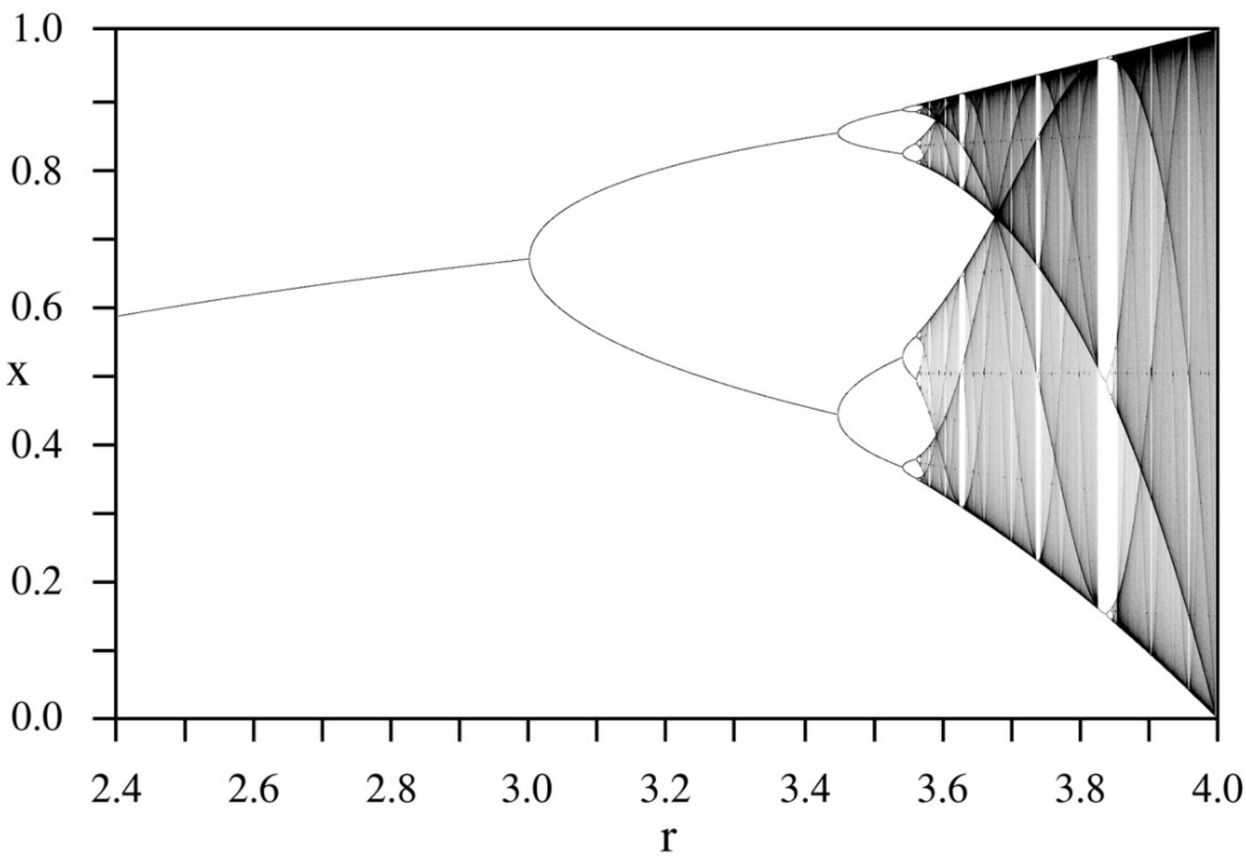


x(k) vs. k, r=1.9
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
x(k)
0
10
20
30
40
50
k

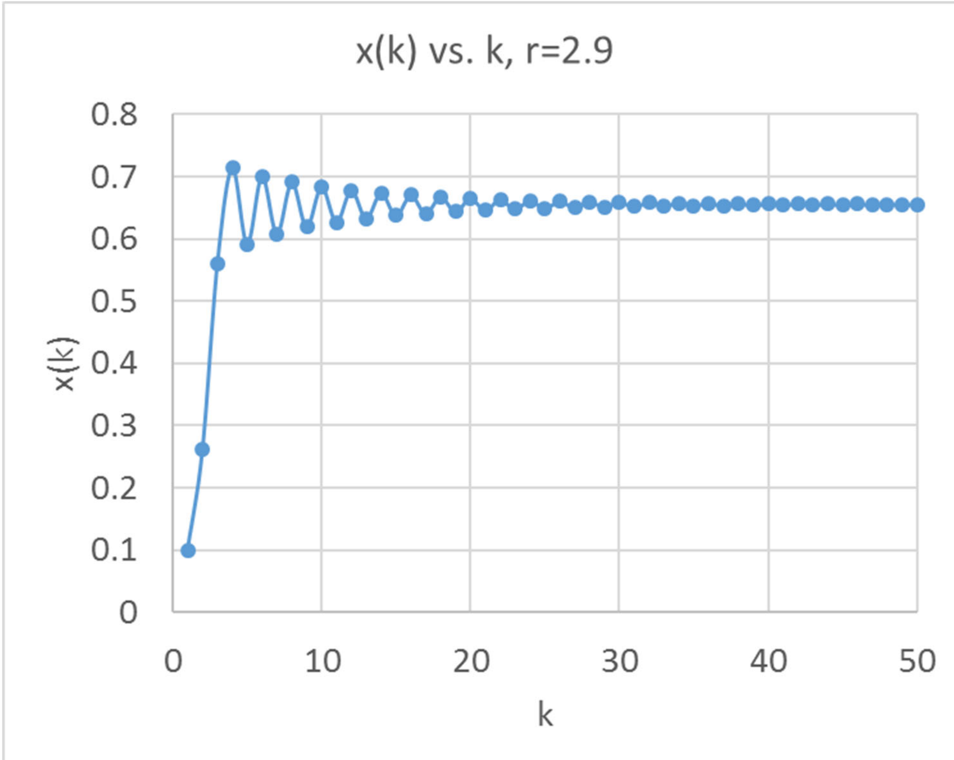

x(k) vs. k, r=2.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x(k)
0
10
20
30
40
50
k


x(k) vs. k, r=3.3
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x(k)
0
10
20
30
40
50
k

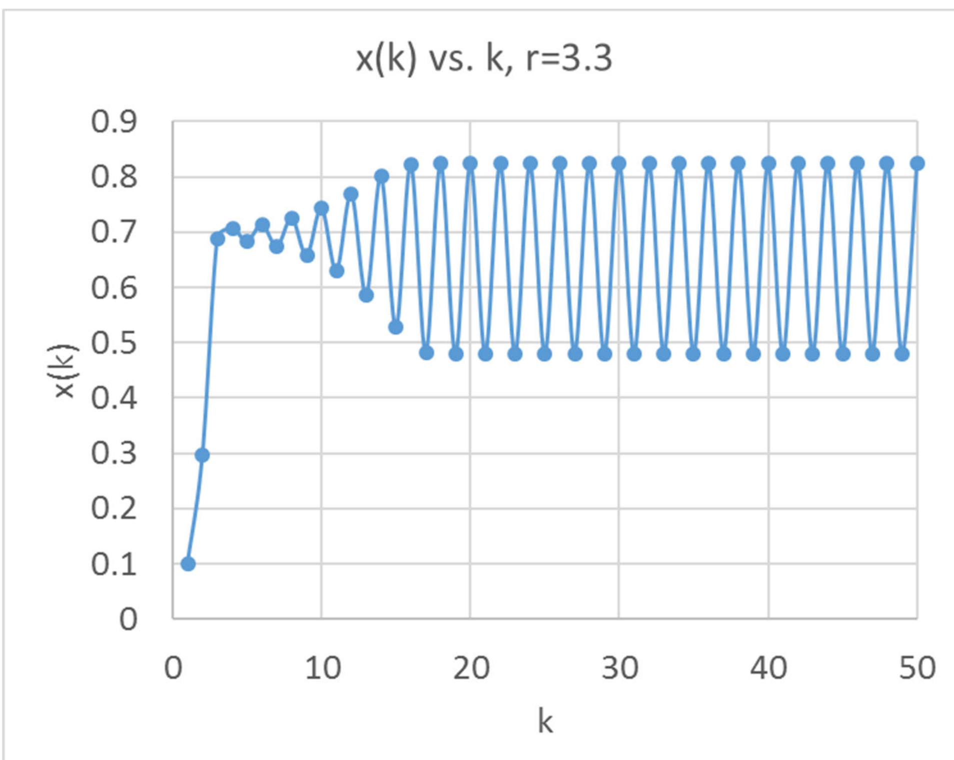




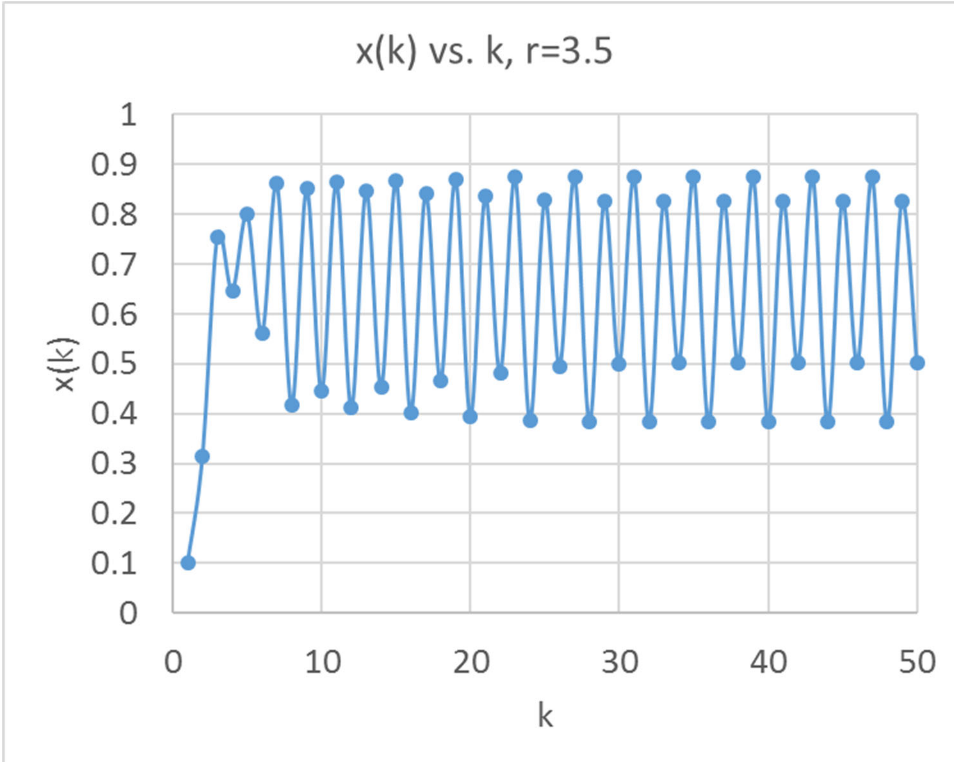
Bifurcation diagram for this system.



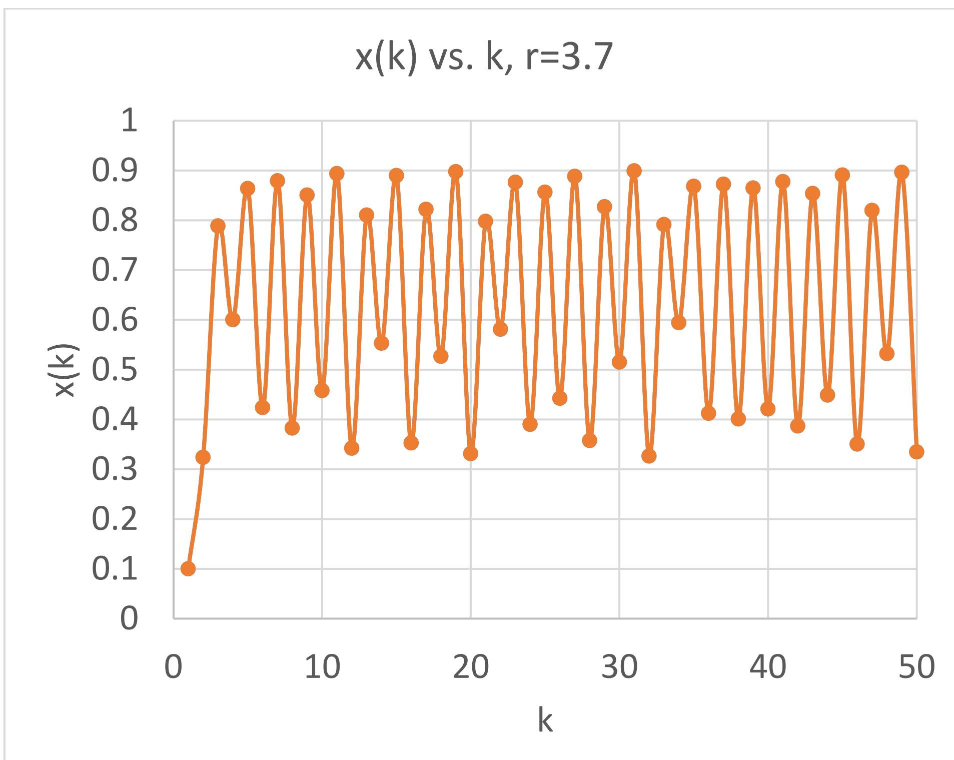
Relate to Bifurcation diagram (steady state x(k) ~ .66)



Relate to Bifurcation diagram (steady state x(k) ~ .40 & .82)



Relate to Bifurcation diagram (steady state x(k): period doubling)



Relate to Bifurcation diagram (steady state x(k): chaotic response)