

ex: 1 cm³ quartz cube under 2 kN force, correctly applied, can produce a voltage across it of 12,500 V

1. Mathematics of the Piezoelectric Effect

By Hooke's Law:

$$\vec{T}_{ij} = \vec{c}_{ijkl} \vec{S}_{kl}$$

where \vec{T}_{ij} is the mechanical stress tensor

\vec{c}_{ijkl} is the elastic stiffness constant tensor

↳ can be reduced to a 6x6 matrix. Some of the 36 elements may be zero, depending on the crystal symmetry

$$[\vec{c}_{ijkl}] = N/m^2$$

\vec{S}_{kl} is the strain field tensor

By Maxwell's equations: {non-piezoelectric materials}

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E} \quad \text{or electric flux density}$$

where \vec{D} ≡ electrical displacement vector

$\epsilon_0 \epsilon_r$ ≡ permittivity

\vec{E} ≡ electric field vector

For piezoelectric materials, both equations are coupled:

$$\vec{T}_{ij} = \vec{c}_{ijkl}^E \vec{S}_{kl} - \vec{e}_{kij} \vec{E}_k$$

where \vec{e}_{kij} is the piezoelectric constant

$$[\vec{e}_{kij}] = C/m^2$$

and \vec{c}_{ijkl}^E is measured under zero or constant electric field

Another related equation is:

$$\vec{D}_i = \vec{e}_{ikl} \vec{S}_{kl} + \varepsilon_{ik}^s \vec{E}_k$$

where ε_{ik}^s is measured at zero or constant strain

The two equations are sometimes shown in matrix form where they are called the piezoelectric constitutive equations:

$$[\vec{T}] = [\vec{c}] [\vec{S}] - [\vec{e}^T] \vec{E}$$

$$\vec{D} = [\vec{e}] [\vec{S}] + [\varepsilon] \vec{E}$$

\therefore Thoroughly understanding the piezoelectric effect requires an in depth knowledge Materials/Mechanical Engineering and Electromagnetics

2. Importance of Materials Engineering in ECE applications

- piezoresistivity
- piezoelectricity
- Seebeck effect \rightarrow thermocouples
- Peltier effect \rightarrow TECs
- photovoltaics
- CTE \rightarrow bimorphs
- SMA's
- etc.

Example of another form of the Piezoelectric Constitutive Equations, for a 4mm (C_{4v}) crystal class such as poled PZT):

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\ s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E = 2(s_{11}^E - s_{12}^E) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

<http://en.wikipedia.org/wiki/Piezoelectricity>

S → strain field tensor

T → mechanical stress tensor

D → elastic flux density

E → electric field vector