## Tuesday 2/21/23

d. Oscillators Using Crystal and Ceramic Resonators

Some limiting factors in LC oscillators include:
(1) The size and weight of inductors (particularly at low frequencies)
(2) Resistive losses in inductors (low inductor Q)
(3) Inductive accuracy of inductors
(4) Long term stability

Piezoelectric crystal and ceramic resonators are therefore used to improve these issues.

Quartz is a piezoelectric material. Piezoelectric materials experience a small dimensional change is response to an applied voltage. Also, they generate an electric charge in response to an applied mechanical stress. Some ceramics also exhibit piezoelectric properties.

Schematic symbol for piezoelectric crystal and ceramic resonators:


Overall equivalent circuit:

$\mathrm{C}_{1}$ : equivalent capacitance, motional capacitance
$\mathrm{L}_{1}$ : equivalent inductance, motional inductance
$\mathrm{R}_{1}$ : equivalent resistance, series resistance
$\mathrm{C}_{0}$ : inner electrode capacitance, holder or shunt capacitance

The resonator has an impedance $\mathrm{Z}=\mathrm{R}+\mathrm{j} \mathrm{X}$
$\mathrm{f}_{\mathrm{r}}$ is the resonant frequency: $f_{r}=\frac{1}{2 \pi \sqrt{L_{1} C_{1}}}$
$\mathrm{f}_{\mathrm{a}}$ is the anti-resonant frequency: $f_{a}=f_{r} \sqrt{\left(1+C_{1}+C_{0}\right)}$
$\mathrm{Q}_{\mathrm{m}}$ is the mechanical quality factor: $Q_{m}=\frac{1}{2 \pi F_{r} C_{1} R_{1}}$
In between $f_{r}$ and $f_{a}, Z$ is normally inductive. See example on next page.
In between $f_{r}$ and $f_{a}$, the resonator can be modelled as:

$R_{E}$ is the effective resistance over this frequency range.
$L_{E}$ is the effective inductance over this frequency range.

$$
\mathrm{Z}=\mathrm{R}_{\mathrm{E}}+\mathrm{j} \omega \mathrm{~L}_{\mathrm{E}}
$$

Example $\mathrm{L}_{1}, \mathrm{C}_{1}, \mathrm{R}_{1}, \mathrm{C}_{\mathrm{o}}$ and $\mathrm{Q}_{\mathrm{m}}$ values for some ceramic and crystal resonators <below>.

Note: higher oscillation modes (overtones) do exist due to the mechanical resonance. They only occur at odd multiples of the fundamental frequency ( $3^{\text {rd }}, 5^{\text {th }}$, etc.). Crystals specified for frequencies over 30 MHz often use overtones. Your oscillator circuit must therefore be designed appropriately if using an overtone mode.


Figure 1.5) Impedance and Phase Characteristics for Ceramic Resonators

| FREQUENCY | CERAMIC RESONATOR |  |  |  | CRYSTAL |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 455 KHz | 2.50 MHz | 4.00 MHz | 8.00 MHz | 453.5 KHz | 2.457 MHz | 4.00 MHz | 8.00 MHz |
| L1 ( $\mu \mathrm{H}$ ) | $8.8 \times 10^{3}$ | $1.0 \times 10^{3}$ | 385 | 72 | $8.6 \times 10^{3}$ | $7.2 \times 10^{3}$ | $2.1 \times 10^{3}$ | $1.4 \times 10^{4}$ |
| $\mathrm{C}_{1}$ (pF) | 14.5 | 4.2 | 4.4 | 5.9 | 0.015 | 0.005 | 0.007 | 0.027 |
| $\mathrm{C}_{0}(\mathrm{pF})$ | 256.3 | 33.3 | 36.3 | 39.8 | 5.15 | 2.39 | 2.39 | 5.57 |
| R1 ( $\Omega$ ) | 9.0 | 17.6 | 8.7 | 4.8 | 1060 | 37.0 | 22.1 | 8.0 |
| Qm | 2734 | 912 | 1134 | 731 | 23000 | 298869 | 240986 | 88677 |
| $\Delta \mathrm{F}(\mathrm{KHz})$ | 12 | 147 | 228 | 555 | 0.6 | 3 | 6 | 19 |

Figure 1.1 Comparisons of equivalent Circuit Constants for Ceramic and Crystal Resonators
Courtesy ECE International, Inc.

Crystal and ceramic resonators can be used in linear or square wave oscillators. In a linear oscillator, they are used to replace the discrete inductor. Example linear oscillator circuits:


Pierce oscillator circuit.
The Pierce oscillator is a derivative of the Colpitts oscillator.


Clapp oscillator circuit with collector grounded.
The Clapp oscillator is also a derivative of the Colpitts oscillator, except that a $3^{\text {rd }}$ capacitor, $\mathrm{C}_{3}$, is used in series with the inductor (inherent in the resonator).
$\omega_{o}=\sqrt{\frac{1}{L}\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}\right)}$


Clapp oscillator circuit with base grounded.
With each of these circuits, the crystal should be modelled with $\mathrm{C}_{1}, \mathrm{~L}_{1}, \mathrm{R}_{1}$ and $\mathrm{C}_{0}$ included.

These oscillator circuits are referred to as aperiodic oscillators, because they will occasionally operate in an overtone (higher frequency) mode. $\mathrm{C}_{2}$ can be replaced with a resonant circuit to prevent this condition from occurring.

Note, these oscillators do not have AGC circuits. If the gain is too high, you may get a square wave output instead of a sinusoidal output.


Example crystal oscillator circuit using a common emitter amplifier in a Colpitts configuration.

## Nonlinear Oscillators

Our definition: an oscillator with a periodic non-sinusoidal output, such as a square wave.

1) Sinusoidal oscillator circuit with a really high amplifier gain, much higher than necessary to satisfy the BSC

Let's revisit the Wein bridge oscillator (shown on next page)
$R_{3}$ of $15 \mathrm{k} \Omega$ achieved oscillation. Let's make $R_{3}$ by $25 \mathrm{k} \Omega$ and evaluate the difference. As $\mathrm{R}_{3}$ gets large, the tops of the output signal flatten out, more closely approximating a square wave. This forces more of the time varying signal to reside in the nonlinear range of the op amp, effectively clipping it.

However, the bottom part of the output square wave is close to -4 V , not 0 V , as would be expected of a digital signal. A clipping diode can be added off the op amp output signal and applied to a $100 \mathrm{k} \Omega$ load resistor. This removes most of the negative going part of the periodic signal.

Two CMOS inverters were added after the $100 \mathrm{k} \Omega$ load resistor, and fed to another $100 \mathrm{k} \Omega$ load resistor to square up the signal $<$ see below $>$.

Pspice Simulated Wain Bridge Oscillator


$$
\begin{aligned}
& f=\frac{1}{2 \pi R C}=\frac{1}{2 \pi(159.155)\left(1 \times 10^{-6}\right)}=1000 \mathrm{kHz} \\
& C_{1}=C_{2}=1 \mu \mathrm{~F} \\
& R_{1}=R_{2}=159.155 \Omega \\
& 1+\frac{R_{3}}{R_{4}}=3 \rightarrow R_{4}=5 \mathrm{~K}, R_{3}=10 \mathrm{k} \Omega \\
& X_{1}: A D 8610 \text { op amp } \rightarrow 25 \mathrm{mHz} \text { GBW product }
\end{aligned}
$$

Result $\rightarrow$ no oscillation
$\therefore R_{3}$ increased to $15 \mathrm{k} \Omega$ to achieve oscillation Output: Distorted sinusoid: $f=857,93 \mathrm{~Hz}$
Possible causes for lower freq:
(1) phase shift in op amp
(2) nonlinear distortion near power supply rails

$\mathrm{V}(2)$ with $\mathrm{R} 3=15 \mathrm{k} \Omega$

$\mathrm{V}(2)$ with $\mathrm{R} 3=25 \mathrm{k} \Omega$


Voltage waveform after the diode. Observe that the peak voltage has dropped due to the forward voltage drop across the diode.


Voltage waveform after two CMOS inverters following the clipping diode circuit.

The duty cycle is less than $50 \%$, but this is usually not a problem for many applications.
2) A rail-to-rail comparator added to the sinusoidal oscillator's output

This circuit can convert the sinusoidal output waveform into a square wave.


## More on Nonlinear Oscillators

## a. Terminology

The term Multivibrator is often used in texts describing nonlinear oscillators. A multivibrator is an electronic circuit that switches between 2 states and consists of two amplifying devices cross-coupled with resistors and capacitors:

1. Astable Multivibrator: a multivibrator circuit where neither state is stable. It continually switches back and forth between the two states.
2. Monostable Multivibrator: a multivibrator circuit with one stable state and one unstable state. A trigger pulse causes the circuit to temporarily switch to the unstable state, where the circuit eventually switches back to the stable state and then stays there until another trigger pulse is applied. A "One Shot" is a multivibrator.
3. Bistable Multivibrator: a multivibrator with two stable states. A trigger pulse is required to change either state. A flipflop is a bistable multivibrator.

A Relaxation Oscillator is an astable multivibrator that uses an active circuit element to charge an inductor or a capacitor through a resistor until a threshold current or voltage is reached, where the circuit then changes states and discharges or recharges the inductor or capacitor. The time constant of the inductor or capacitor circuit determines the oscillation frequency.

A Ring Oscillator consists of an odd number ( $\mathrm{n}>1$ ) of inverters (NOT gates) connected in a continuous chain. The odd number results in the impossibility of a stable condition, so the circuit oscillates, and the oscillation frequency is due to the propagation delay through each inverter $\left(\mathrm{T}=2 \mathrm{n} \tau_{\mathrm{D}}\right)$. Adding more inverters to the chain reduces the oscillation frequency.
b. Time constant trigger for a state change

Consider the RC circuit below:


Initially, C has no stored charge $\left(\mathrm{V}_{\mathrm{o}}=0 \mathrm{~V}\right)$. At $\mathrm{t}=0$, a positive DC voltage, $\mathrm{V}_{\mathrm{s}}$, is applied to the resistor. $\mathrm{V}_{\mathrm{o}}$ will increase until it reaches a trip voltage level, $\mathrm{t}_{\mathrm{tr}}$ sec later.
$V_{o}(s)=V_{s}(s) \frac{\frac{1}{s C}}{R+\frac{1}{s C}}$
$V_{o}(s)=V_{s}(s) \frac{\frac{1}{R C}}{s+\frac{1}{R C}}$
$V_{s}(s)=\frac{v_{s}}{s}$
$V_{o}(s)=\frac{v_{s}}{s} \frac{\frac{1}{R C}}{s+\frac{1}{R C}}=\frac{A}{s}+\frac{B}{s+\frac{1}{R C}}$

Using partial fraction expansion:
$A=\frac{v_{s} \frac{1}{R C}}{0+\frac{1}{R C}}=v_{s}$
$B=\frac{v_{s} \frac{1}{R C}}{-\frac{1}{R C}}=-v_{s}$
Therefore:
$V_{o}(s)=\frac{v_{s}}{s}-\frac{v_{s}}{s+\frac{1}{R C}}$
$v_{o}(t)=v_{s}\left(1-e^{-\frac{t}{R C}}\right)$
Rearranging terms:
$\frac{v_{o}}{v_{s}}=1-e^{-\frac{t}{R C}}$
$e^{-\frac{t}{R C}}=1-\frac{v_{o}}{v_{s}}$
$-\frac{t}{R C}=\ln \left(1-\frac{v_{o}}{v_{s}}\right)$
$t=-R C\left[\ln \left(1-\frac{v_{o}}{v_{s}}\right)\right]$
$\tau$ is defined here as the time, $t$, it takes for $v_{0}(t)$ to reach the trip voltage level to cause the multivibrator circuit to change states. For an astable multivibrator with 2 time constant trip circuits, $\tau_{1}$ and $\tau_{2}: T \approx \tau_{1}+\tau_{2}$.
c. 2 transistor classical astable multivibrator circuit

$\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are identical NPN transistors. Let's let $\mathrm{V}_{\mathrm{cc}}=5 \mathrm{~V}$.
$\mathrm{C}_{1}=\mathrm{C}_{2}$
$\mathrm{R}_{1}=\mathrm{R}_{4}$, and are relatively small
$\mathrm{R}_{2}=\mathrm{R}_{3}$
For $\mathrm{v}_{\mathrm{BE}}<0.7 \mathrm{~V}, \mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are in cutoff $\left(\mathrm{I}_{\mathrm{c}}=0 \mathrm{~A}\right)$

Assume that $\mathrm{Q}_{1}$ is on $\left(\mathrm{v}_{\mathrm{BE}}=0.7 \mathrm{~V}\right)$ and $\mathrm{Q}_{2}$ is in cutoff.
The left plate of $\mathrm{C}_{1}$ is approximately at 0 V (gnd) while the right plate is tied to $\mathrm{V}_{\mathrm{cc}}$ through $\mathrm{R}_{3}$, charging it up.

The left plate of $\mathrm{C}_{2}$ is held at approx. $0.7 \mathrm{~V}\left(\mathrm{v}_{\mathrm{BE}}\right.$ of $\left.\mathrm{Q}_{1}\right)$ while the right plate is held at approximately $\mathrm{V}_{\mathrm{cc}}(\sim 5 \mathrm{~V})$ through $\mathrm{R}_{4}$ with $\left.\mathrm{Q}_{2}{ }^{\prime} \mathrm{s}_{\mathrm{c}}=0 \mathrm{~V}\right)$

Once the right plate of $\mathrm{C}_{1}$ reaches $0.7 \mathrm{~V}, \mathrm{Q}_{2}$ turns on (the right plate of $\mathrm{C}_{2}$ drops to approx. 0 V (gnd). Since the voltage across a capacitor cannot change instantaneously, the left plate voltage of $\mathrm{C}_{2}$ drops from 0.7 V to -4.3 V , turning off Q1.
$\mathrm{C}_{2}$ then starts charging through $\mathrm{R}_{2}$ until the $\mathrm{Q}_{1}$ 's $\mathrm{v}_{\mathrm{BE}}$ reaches $0.7 \mathrm{~V} \ldots$

Therefore, $\tau_{1}$ is set by $\mathrm{C}_{1} \mathrm{R}_{3}$ and $\tau_{2}$ is set by $\mathrm{C}_{2} \mathrm{R}_{2}$.
$\mathrm{f} \approx \frac{1}{\tau_{1}+\tau_{2}}$


Approximate waveforms for $\mathrm{v}_{\mathrm{ol}}(\mathrm{t})$ and $\mathrm{v}_{\mathrm{o} 2}(\mathrm{t})$.
The output voltages, $\mathrm{v}_{\mathrm{ol}}(\mathrm{t})$ and $\mathrm{v}_{\mathrm{o} 2}(\mathrm{t})$, are inverted. The logic level " 1 " is $\mathrm{V}_{\mathrm{cc}}$. The exponential rise when either output voltage goes high is due to the capacitors charging through $R_{1}$ and $R_{4}$, respectively.

## Application

Let $\mathrm{V}_{\mathrm{cc}}$ be 12 V (instead of 5 V ) and "gnd" be -12 V (instead of 0 V ). Connect $\mathrm{V}_{\mathrm{ol}}(\mathrm{t})$ to $\mathrm{V}_{\mathrm{o} 2}(\mathrm{t})$ through the secondary of a 10:1 transformer. The voltage across the primary of the transformer switches between 120 V and -120 V and is approximately a square wave. The circuit is now a simple DC to AC inverter for power applications, useful for non-sinusoidal power applications.

