1. State Sensors and State Feedback

a. Background

Consider a SMO system (MEMS or macro)

\[ m\ddot{x} + c\dot{x} + kx = f(t) \]

or \[ \dot{x} = \frac{f(t)}{m} - \frac{c}{m} \dot{x} - \frac{k}{m} x \]

where \( \omega_n = \sqrt{\frac{k}{m}} \) and \( \frac{c}{m} = \frac{\omega_n}{2} \)

\( x(t) = \) displacement = \( x_1 \)

\( \dot{x}(t) = \) velocity = \( x_2 \)

\( \ddot{x}(t) = \) acceleration = \( \dot{x}_2 \)

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t) \]

\( \dot{x}_1, \dot{x}_2 \) are the state variables

\( \odot \) Represent the system or "plant" in a block diagram

\[ f(t) \xrightarrow{\frac{1}{m}} x \xrightarrow{Sd+} x_1 \]

Notice

\( \odot \) Relationship between \( \frac{k}{m} \) and \( \omega_n \), and feeding back the displacement \( x \) or \( x_1 \)

\( \odot \) Relationship between \( \frac{c}{m} \) and \( \omega_n \), and feeding back the velocity \( \dot{x} \) or \( x_2 \)
b. Adding feedback to affect performance
   \( \rightarrow \) add a term proportional to displacement to change \( \omega_n \)
   \( \rightarrow \) add a term proportional to velocity to change \( Q \)

1. Why?
   i. SMPS (MEMS or macro) can be used for vibration isolation \( \rightarrow \) mechanical LPF.
      \( \rightarrow \) State feedback allows it to be tuned
   ii. MEMS resonator
      \( \rightarrow \) to adjust \( \omega_n \) and/or \( Q \)
   iii. MEMS gyroscope
      \( \rightarrow \) MEMS resonator application
      \( \rightarrow \) precise control of drive and sense SMPS systems
   iv. Vibrational Energy Harvester
      \( \rightarrow \) desire high \( Q \)
      \( \rightarrow \) desire to tune \( \omega_n \) to highest amplitude vibration frequency component in operating environment
   v. Applicable to electronic 2nd order biquad too
   vi. Applicable to other systems too: Fluidic, thermal, rotational, optical, etc.
C. Requirements

1. Displacement Sensor
2. Velocity Sensor
3. Actuators
4. Signal processing electronics
   → analog or digital

Note: 2 actuators are shown. One actuator could serve both purposes.

(2) More complex models could be used for sensors and actuators, such as \( \frac{a}{s+t} \)

Closed Loop System Becomes:

\[ \ddot{x} + \left( G_2 H_2 - \frac{c}{m} \right) \dot{x} + \left( G_1 H_1 - \frac{k}{m} \right) x + \frac{f(t)}{m} \]

Feedback can be used to increase or decrease \( \omega_n + Q \)

\[ \omega_n = \sqrt{\frac{k}{m} - G_1 H_1} \]

\[ \frac{\omega_n}{Q_{new}} = \frac{c}{m} - G_2 H_2 \]