1. State Sensors and State Feedback

a. Background

consider a SMO system (MEMS or macro)

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

or $\ddot{x} = \frac{f(t)}{m} - \frac{c}{m}\dot{x} - \frac{kx}{m}$

where $\omega_n = \sqrt{\frac{k}{m}}$ and $\frac{c}{m} = \frac{\omega_n}{Q}$

$x(t)$ = displacement $= x_1$

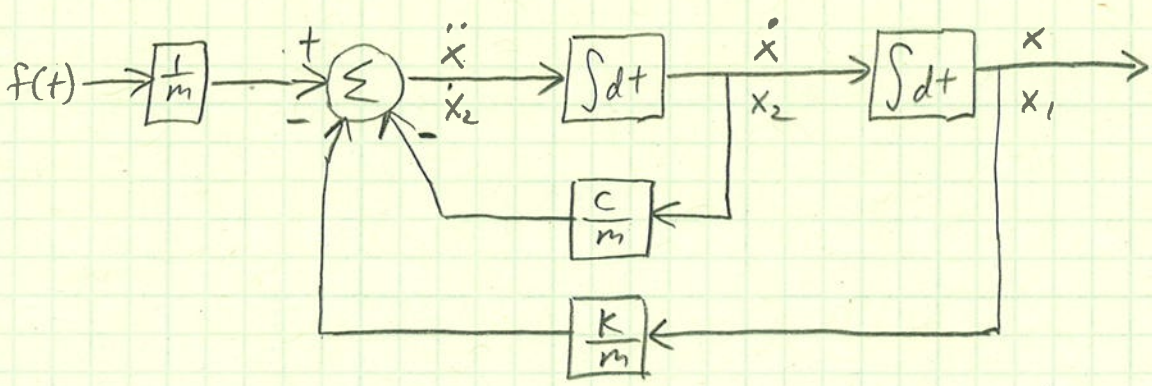
$\dot{x}(t)$ = velocity $= x_2$

$\ddot{x}(t)$ = acceleration $= \dot{x}_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$$ ← state space model

x_1, x_2 are the state variables

① Represent the system or "plant" in a block diagram



Notice

① relationship between $\frac{k}{m}$ and ω_n, and feeding back the displacement (x or x_1)

② relationship between $\frac{c}{m}$ and Q, and feeding back the velocity ($\dot{x}$ or x_2)

b. <u>Adding feedback to affect performance</u>

→ add a term proportional to displacement to change ω_n

→ add a term proportional to velocity to change Q

① <u>Why?</u>

i. SMDs (MEMS or macro) can be used for vibration isolation → mechanical LPF.

→ State feedback allows it to be tuned

ii. MEMS resonator

→ to adjust ω_n and/or Q

iii. MEMS gyroscope

→ mems resonator application

→ precise control of drive and sense SMD systems

iv. Vibrational Energy Harvester

→ desire high Q

→ desire to tune ω_n to highest amplitude vibration frequency component in operating environment

v. Applicable to electronic 2nd order biquad too

vi. Applicable to other systems too: fluidic, thermal, rotational, optical, etc.

C. Requirements

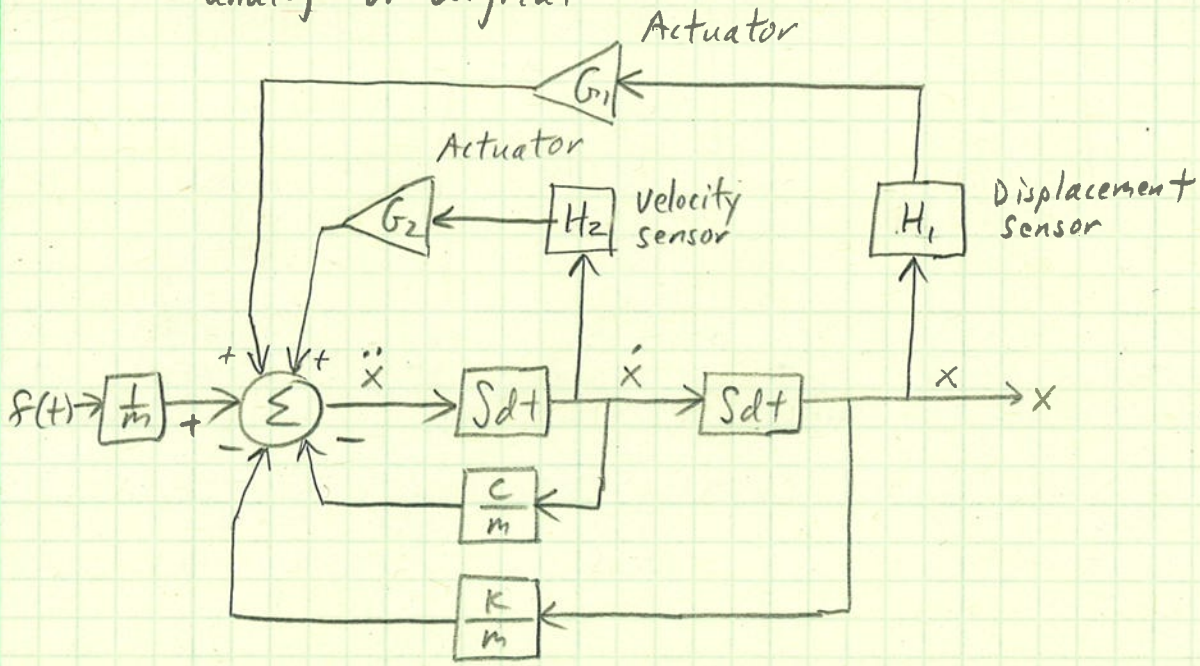
① Displacement Sensor

② Velocity Sensor

③ Actuators

④ Signal processing electronics

→ analog or digital



Note: (1) 2 Actuators are shown. One actuator could serve both purposes

(2) More complex models could be used for sensors and actuators, such as $\frac{a}{s+b}$

Closed Loop System Becomes:

$$\ddot{x} = \left(G_2H_2 - \frac{c}{m}\right)\dot{x} + \left(G_1H_1 - \frac{k}{m}\right)x + \frac{f(t)}{m}$$

feedback can be used to increase or decrease ω_n + Q

$$\omega_n\Big|_{new} = \sqrt{\frac{k}{m} - G_1H_1}, \quad \frac{\omega_n}{Q}\Big|_{new} = \frac{c}{m} - G_2H_2$$