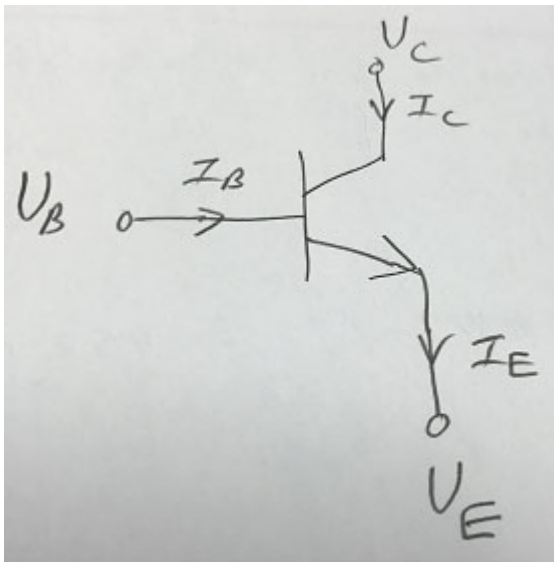


Tuesday 2/14/21

The next level of oscillator development is to replace the op amp with a transistor, which allows for higher frequency operation.

## Review of BJT transistors

NPN Transistor:



B: Base, C: Collector, E: Emitter

For amplifiers and oscillators, you want the BJT to operate in the Active Mode (Emitter Base Junction forward biased and Collector Base Junction reverse biased).

Therefore:  $V_C > V_B > V_E$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

$I_S$  is the saturation current.

$V_T$  is the thermal voltage:  $V_T = \frac{kT}{q}$

$k$  is the Boltzmann constant,  $1.38 \times 10^{-23}$  J/K

$T$  is absolute temperature in degrees Kelvin

$q$  is the magnitude of electronic charge:  $1.602 \times 10^{-19}$  C

$V_T$  is approximately 25 mV at room temperature.

$$I_C = \beta I_B = \alpha I_E$$

$\beta$  is the common emitter current gain, typically 100 to 200.

$$\alpha = \frac{\beta}{\beta + 1}$$

$$I_E = I_B + I_C$$

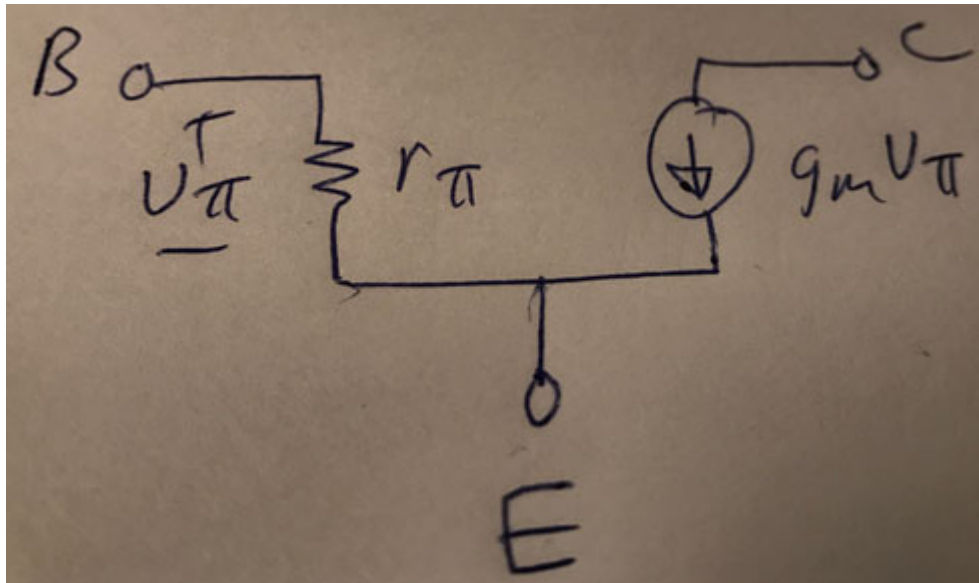
Assume  $V_{BE} = 0.7V$  for analysis purposes.

Small signal model:

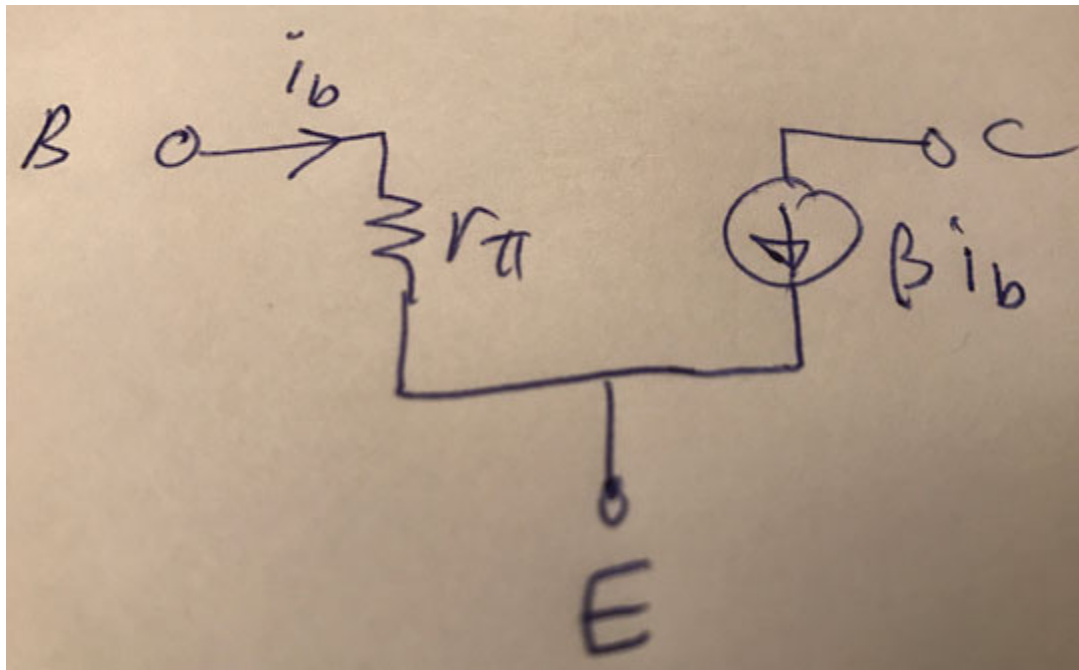
$$\text{Transconductance: } g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta}{g_m}$$

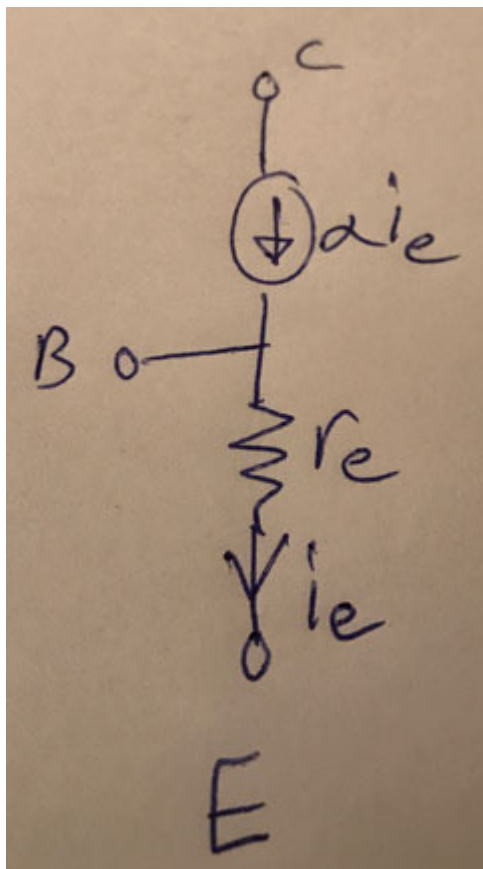
$$r_e = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1}$$



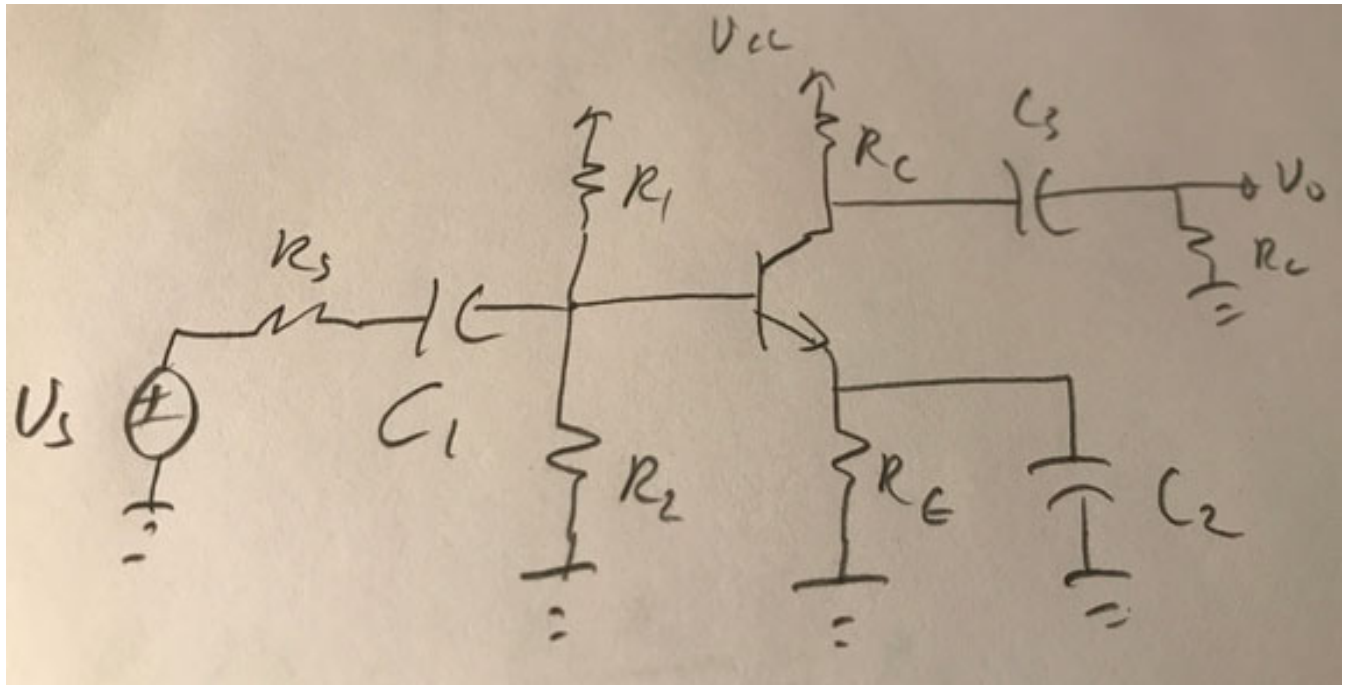
Or:



Or:

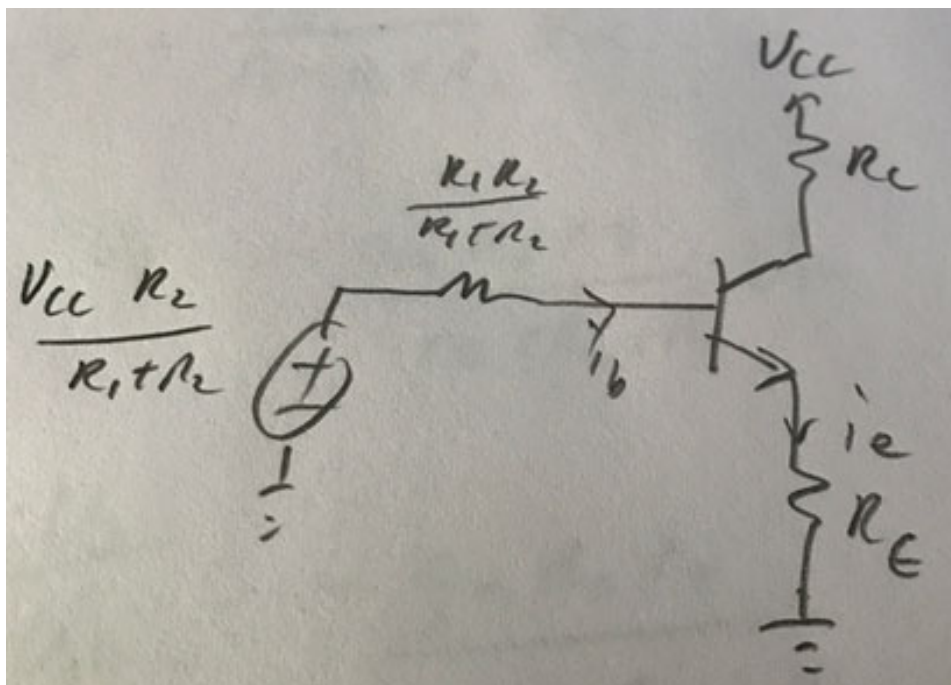


a. The Common Emitter Amplifier:



$C_1$ ,  $C_2$  and  $C_3$  are coupling capacitors: consider them to be open for DC analysis and shorts for higher frequency small signal analysis.

DC analysis model:



$$\frac{V_{CC}R_2}{R_1 + R_2} = I_E R_E + 0.7 + I_B \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{V_{CC}R_2}{R_1 + R_2} = \frac{I_C}{\alpha} R_E + 0.7 + \frac{I_C}{\beta} \frac{R_1 R_2}{R_1 + R_2}$$

Solve for  $I_C$

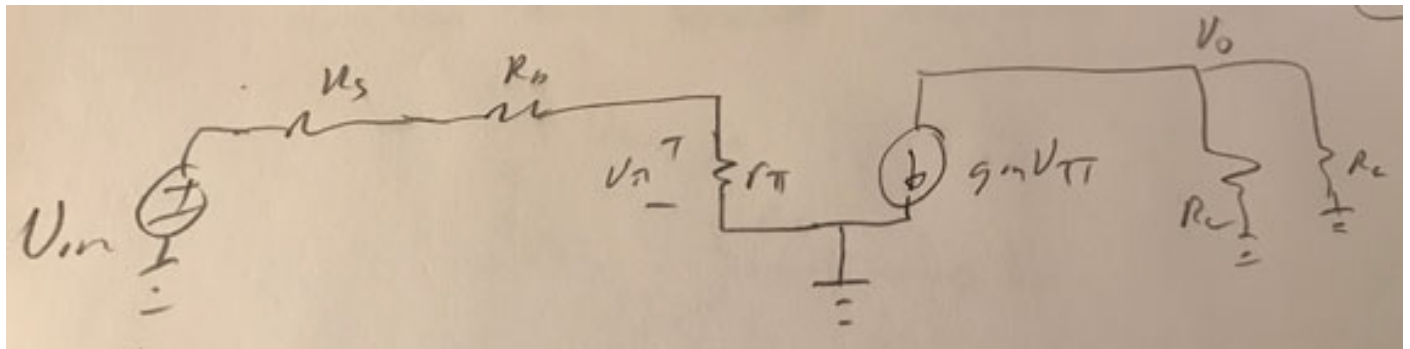
$$\text{Then: } V_C = V_{CC} - I_C R_C$$

$$\text{And: } V_B = I_E R_E + 0.7 = \frac{I_C}{\alpha} R_E + 0.7$$

Then verify Active Mode operation by  $V_C > V_B$

Then calculate  $g_m$  and  $r_\pi$

Small signal model:



$$R_b = \frac{R_1 R_2}{R_1 + R_2}$$

Let

$$R_o = \frac{R_C R_L}{R_C + R_L}$$

Then

$$V_o = -g_m V_\pi R_o$$

$$V_\pi = V_s \frac{r_\pi}{r_\pi + R_b + R_s}$$

$$\frac{V_o}{V_s} = -\frac{g_m R_o r_\pi}{r_\pi + R_b + R_s}$$

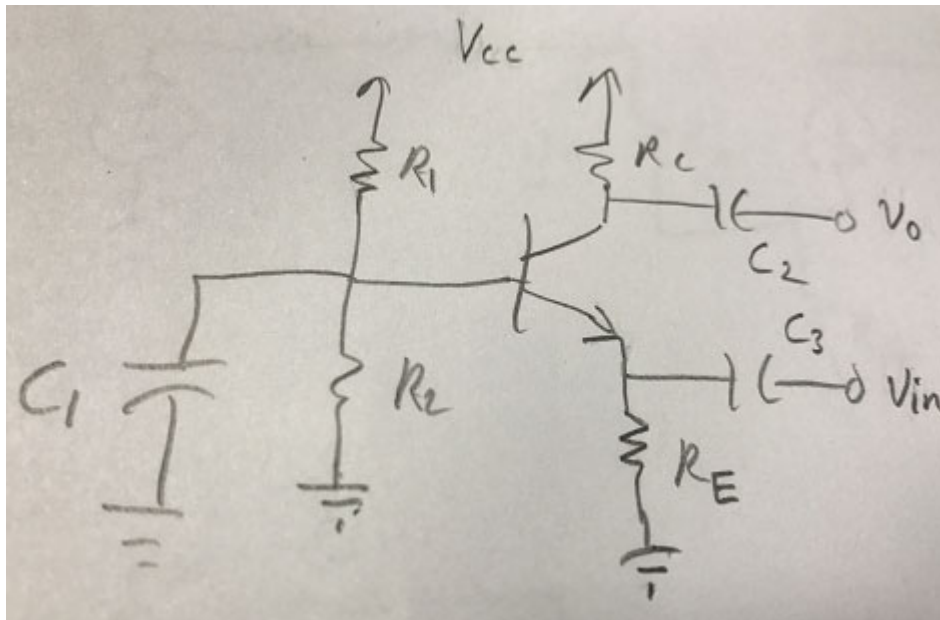
If  $R_E$  is not bypassed by a capacitor:

$$\frac{V_o}{V_s} = - \frac{g_m R_o r_\pi}{(\beta + 1)R_E + r_\pi + R_b + R_s}$$

This is called the common emitter amplifier with emitter degeneration.

b. The Common Base Amplifier:

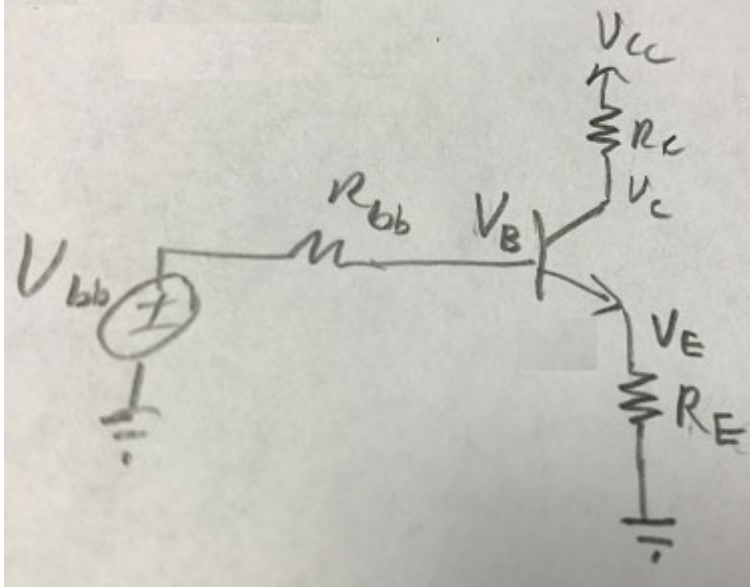
DC Analysis:



$C_1$ ,  $C_2$  and  $C_3$  are coupling capacitors

$$\text{Let } R_{bb} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\text{And } V_{bb} = V_{cc} \frac{R_2}{R_1 + R_2}$$



$$V_{bb} = I_E R_E + 0.7 + I_B R_{bb}$$

$$V_{bb} = \frac{I_C}{\alpha} R_E + 0.7 + \frac{I_C}{\beta} R_{bb}$$

Solve for  $I_C$

$$V_E = I_E R_E$$

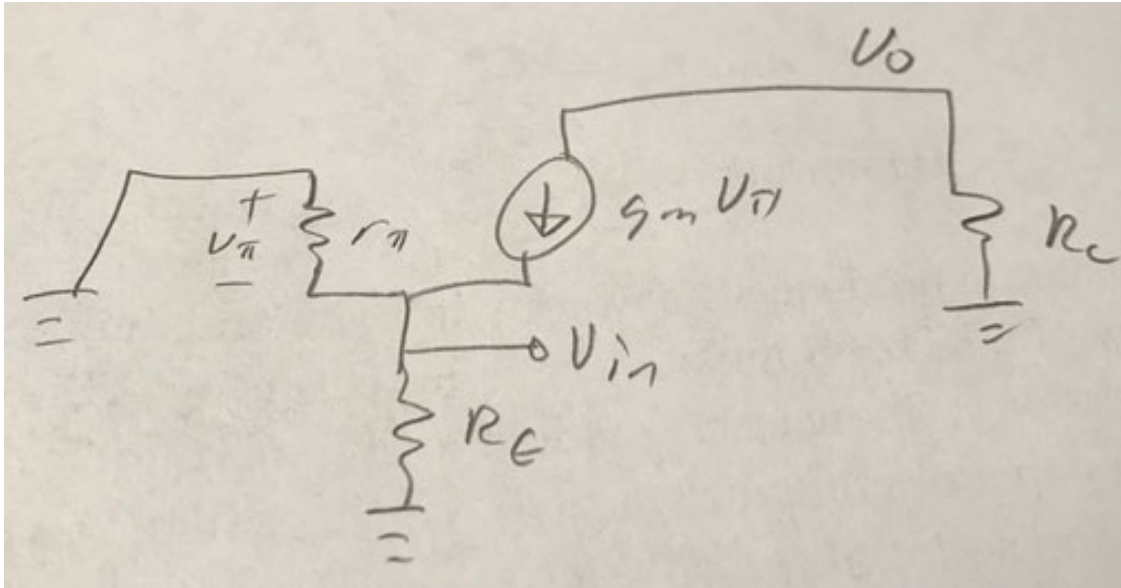
$$V_B = V_E + 0.7$$

$$V_C = V_{cc} - I_C R_c$$

Verify  $V_C > V_B$

Calculate  $g_m$  and  $r_\pi$

Small Signal Model:



$$V_{\pi} = -V_{in}$$

$$v_o = -g_m V_{\pi} R_C = v_{in} g_m R_C$$

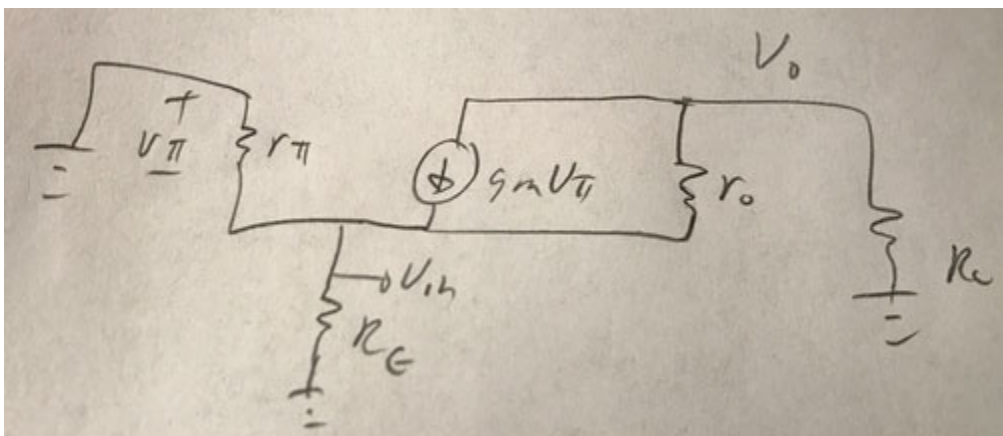
If  $r_o$  needs to be included:

$$r_o = \frac{V_A}{g_m V_T}$$

$V_A$  is the Early voltage, typically 50 V to 100 V.

For an  $I_C$  of 1 mA,  $r_o$  is typically 50 k $\Omega$  to 100 k $\Omega$ .

If  $r_o \gg R_C$ ,  $r_o$  can be neglected. Otherwise:





$$V_{\pi} = -V_{in}$$

$$\frac{v_o}{R_C} + \frac{v_o - v_{in}}{r_o} = -g_m v_{\pi} = g_m v_{in}$$

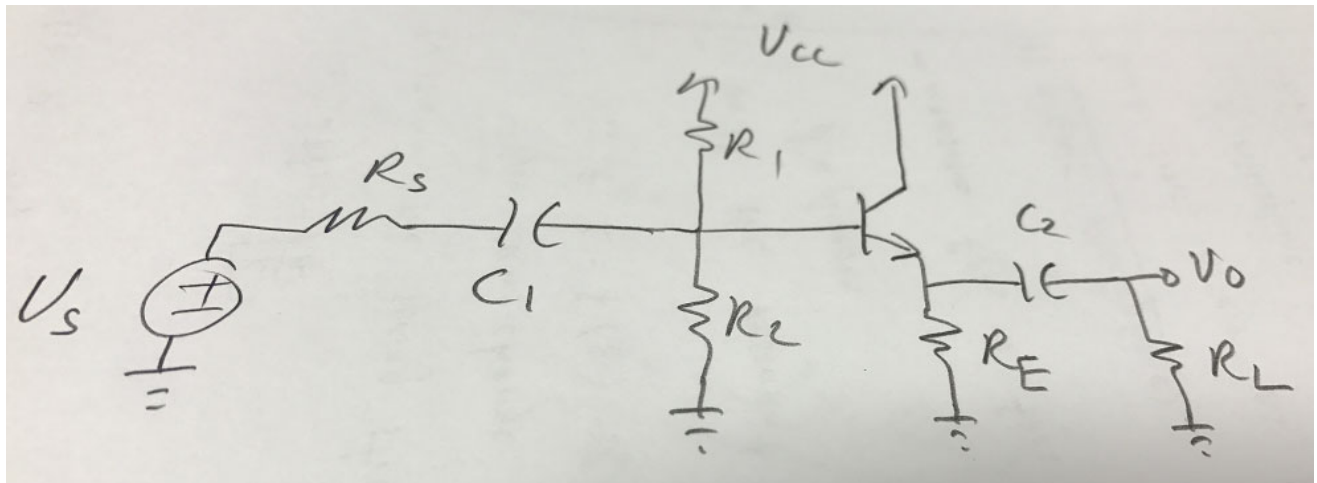
Therefore:

$$v_o \left( \frac{1}{R_C} + \frac{1}{r_o} \right) = v_{in} \left( g_m + \frac{1}{r_o} \right)$$

$$\frac{v_o}{v_{in}} = \frac{g_m + \frac{1}{r_o}}{\frac{1}{R_C} + \frac{1}{r_o}} = \frac{g_m + \frac{1}{r_o}}{\frac{r_o + R_C}{r_o R_C}} = \frac{(g_m r_o + 1) R_C}{r_o + R_C}$$

The common base amplifier has a good high frequency response, compared to the common emitter amplifier, due to the absence of the Miller effect, but it can suffer from relatively low input impedance, compared to the common emitter amplifier.

### c. The Common Collector (Emitter Follower) Amplifier



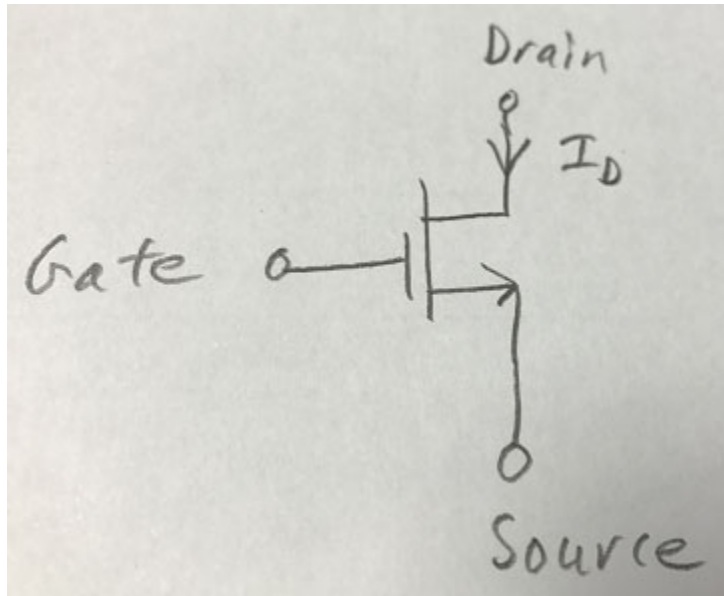
$$\frac{v_o}{v_s} < 1$$

Therefore, this amplifier is not really suitable for oscillators, other than as a buffer between multiple amplifier stages.

Keep in mind that the small signal models used here are simplified models. Computer simulation with tools such as LTspice should significantly improve the circuit modelling results.

## Review of MOSFET Transistors

NMOS Transistor:



DC signals:  $I_G = 0A$ ,  $I_D = I_S$

For amplifiers and oscillators, we are interested in the saturation region:

$$V_{DS} \geq V_{GS} - V_T$$

$V_T$  is the nMOSFET's threshold voltage, around 0.5V to 2V

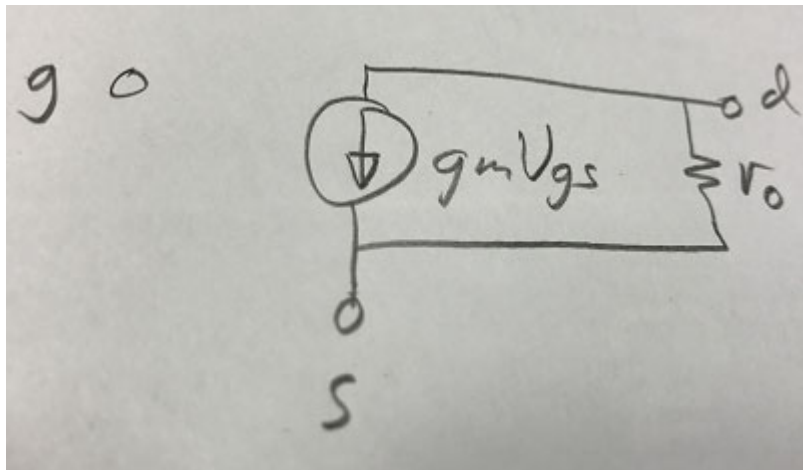
$$I_D = \frac{\mu_o C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 - \lambda V_{DS})$$

Often, this equation is simplified to:  $I_D = \frac{\beta}{2} (V_{GS} - V_T)^2 = \frac{K'}{2} \frac{W}{L} (V_{GS} - V_T)^2$ .

Typical value for  $\beta$ : 0.5 mA/V<sup>2</sup>.

W and L and the channel width and length, respectively.

Simplified small signal model for the nMOSFET:

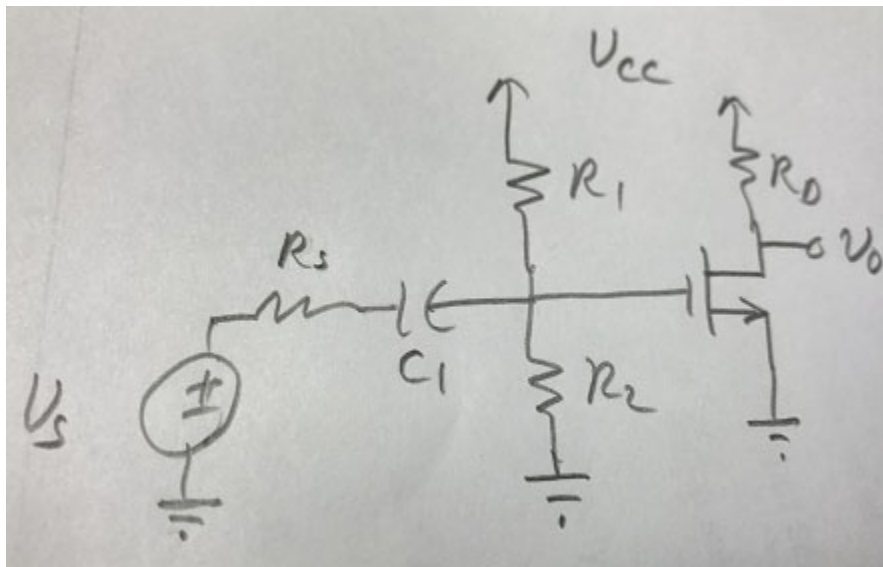


$$g_m \approx \beta(V_{GS} - V_T)$$

$$r_o = \frac{1 + \lambda V_{DS}}{I_D \lambda} \approx \frac{1}{I_D \lambda}$$

$\lambda$  is the channel length modulation parameter.

Consider this amplifier:



$C_1$  is a coupling capacitor: an open for the DC analysis.

$$V_G = V_{cc} \frac{R_2}{R_1 + R_2} = V_{GS}$$

$$I_D = \frac{\beta}{2} (V_{GS} - V_T)^2$$

$$V_D = V_{DS} = V_{CC} - I_D R_D$$

Then verify that  $V_{DS} \geq V_{GS} - V_T$ : Saturation mode.

Then calculate  $g_m$  and  $r_o$ .

Then perform a small signal analysis to calculate the gain.

Common Source amplifier: large negative gain.

Common Gate amplifier: large positive gain.

Common Drain (Source Follower) amplifier: gain a little less than one.

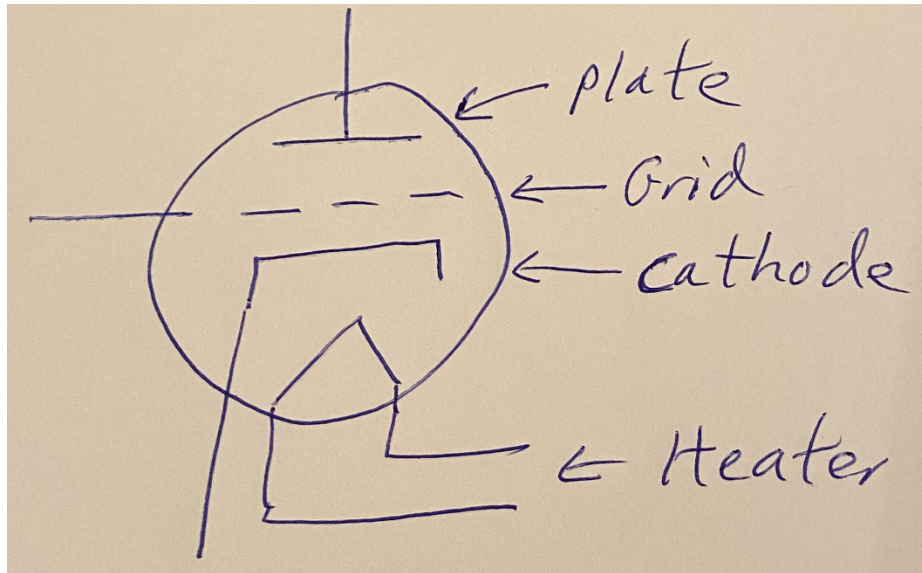
Comparing similar BJTs and MOSFETs:

- 1) BJT's have high transconductance values
- 2) MOSFETs have zero DC Gate current
- 3) MOSFETs have infinite input resistance looking into the gate

## Introduction to Vacuum Tubes

The three-electrode vacuum tube, the triode, dates back to 1906 and was invented by Lee De Forest (8/26/1873 to 6/30/1961). Various improvements have been made over the years. They are still used today for a few applications, such as high power RF amplifiers and for some audio applications.

Consider this schematic drawing for a triode:



The background pressure inside the vacuum tube might be on the order of  $1 \times 10^{-9}$  atm.

Joule heating is used to heat the heater filament and the nearby (possibly surrounding) cathode, which facilitates thermionic emission of electrons from the heated cathode. The cathode may be heated as high as  $1000^{\circ}\text{C}$ . The cathode is usually coated with a low work function material, such as thorium oxide, where the work function, a material property, is the minimum amount of energy required to remove an electron from the surface of the material into a vacuum.

A positive voltage (20 V to 1000s V) is applied to the plate (anode), which attracts the freed electrons from the cathode.

A negative voltage (relative to the cathode) is applied to the grid. As the grid voltage is reduced, the amount of electrons travelling through it to the plate decreases. At some large negative voltage, the cutoff voltage, all electrons will

cease flowing to the plate. Hence, the triode is somewhat similar in operation to the solid state jFET transistor.

Example triode equations:

$$I_a = G \left( 1 + \frac{V_g}{v_B} \right) \left( V_g + \frac{V_a + v_c}{\mu} \right)^{\frac{3}{2}}$$

and

$$I_g = \begin{cases} \alpha V_g^{1.5}, & \text{for } V_g > 0 V \\ \frac{\beta}{v_D - V_g}, & \text{for } V_g < 0 V \end{cases}$$

where  $I_a$  is the anode current and  $I_g$  is the grid current. Also,  $V_g$  is the grid voltage,  $V_a$  is the anode voltage, and  $G$ ,  $\mu$ ,  $\alpha$ ,  $\beta$ ,  $v_B$ ,  $v_C$ , and  $v_D$  are tube parameters.

Equations courtesy of <https://electromagneticworld.blogspot.com/2017/05/triode-vacuum-tube-modeling.html>.

By applying a negative bias voltage to the grid, a small AC signal can be added to the grid bias voltage. With an appropriately sized resistor connected to the plate, an amplified version of the input voltage can appear across that resistor.

Vacuum tubes are large, hot, high voltage, high power, and fragile compared to transistors.