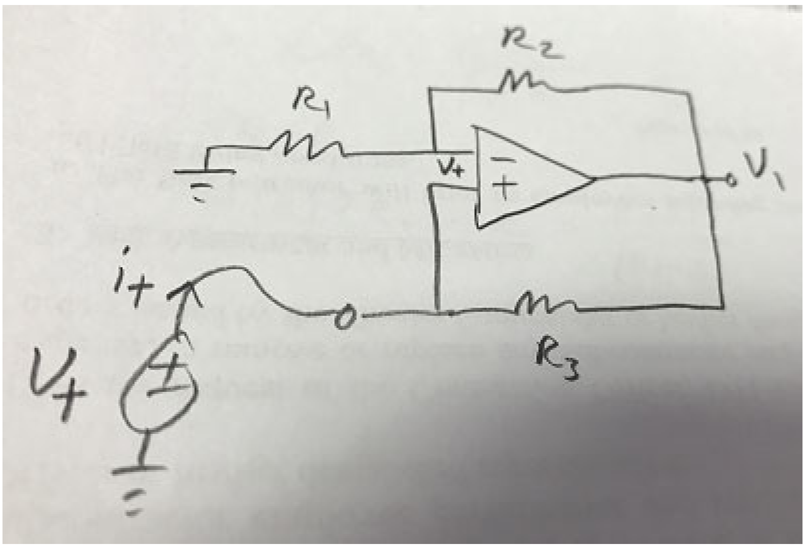
Thursday 2/9/23

5) Negative Resistance LC Oscillators

Consider this op amp circuit:



V_t is a "test source" generating a current I_t, and used to calculate the impedance, Z_{in}, looking into the terminals that the V_t source is connected to:

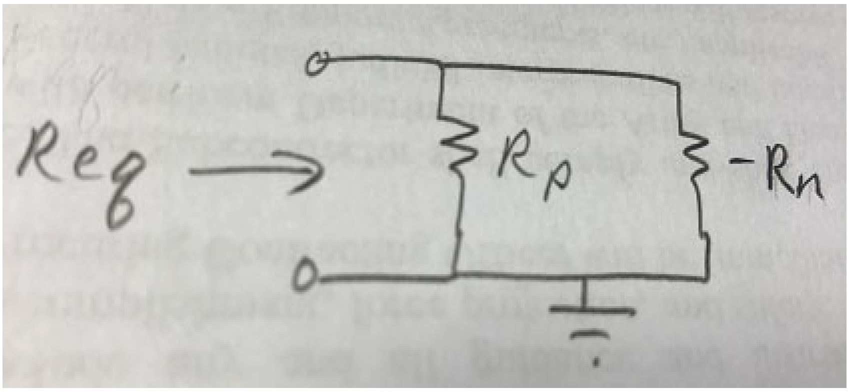
$$Z_{in} = \frac{V_t}{I_t}$$

V_t appears at both op amp inputs.

Therefore, the current through R_1, i_1, is $i_1 = \frac{V_t}{R_1}$.
The current, i_1, flows through R_2.
Therefore $V_1 = V_t + i_1 R_2 = V_t\left(1 + \frac{R_2}{R_1}\right)$
Then: $I_t = \frac{V_t - V_1}{R_3} = \frac{V_t - V_t\left(1+\frac{R_2}{R_1}\right)}{R_3} = -V_t \frac{R_2}{R_1 R_3}$

Leading to: $Z_{in} = -\frac{R_1 R_3}{R_2}$, a negative resistance!

Consider the effect of placing this negative resistance, $-R_n$, in parallel with a positive resistance, R_p, as shown below:



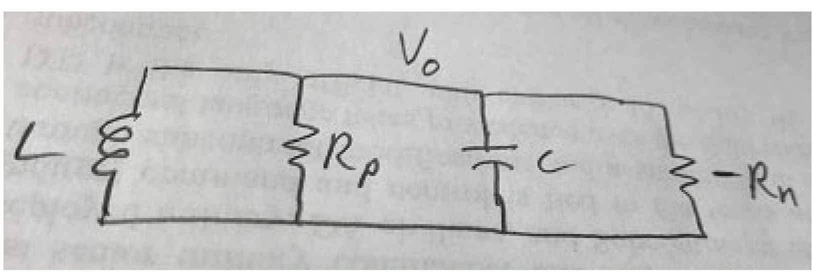
$$R_{eq} = \frac{R_p(-R_n)}{R_p - R_n}$$

If $|-R_n| = R_p$, $R_{eq} \to \infty$: an open circuit

If $|-R_n| > R_p$, $R_{eq} > 0$

If $|-R_n| < R_p$, $R_{eq} < 0$

Consider the LCR tank circuit below with an attached negative resistance:



$$V_o\left(\frac{1}{sL} + \frac{1}{R_p} + sC - \frac{1}{R_n}\right) = 0$$

$$V_o\left(s^2 + \frac{s}{CR_p} - \frac{s}{CR_n} + \frac{1}{LC}\right) = 0$$

$$V_o\left(s^2 + \frac{s}{C}\left(\frac{R_n - R_p}{R_p R_n}\right) + \frac{1}{LC}\right) = 0$$

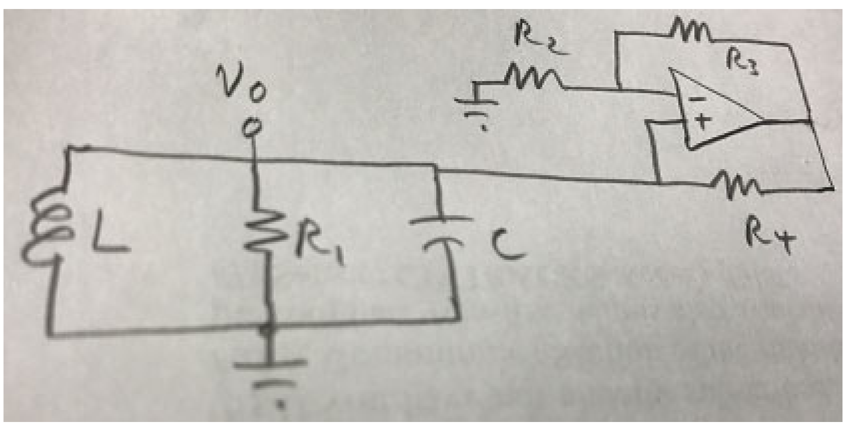
If $|-R_n|=R_p$, the damping term goes to 0.

If $|-R_n|>R_p$, the damping is positive, the oscillation will die out.

If $|-R_n|<R_p$, the damping is negative, the system is unstable and the oscillation amplitude with grow.

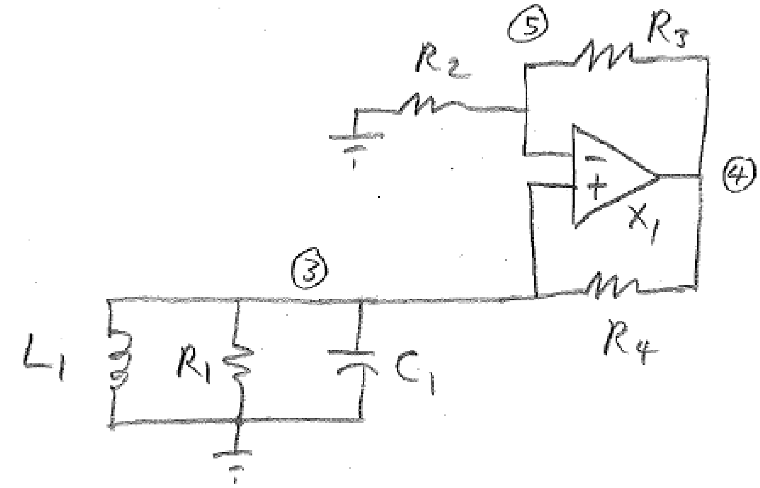
The use of negative resistance is a circuit technique for cancelling out the damping term in this dynamical system, similar to what we did using feedback to satisfy the BSC in previously discussed oscillators. In fact, you could model this circuit as a feedback control system too.

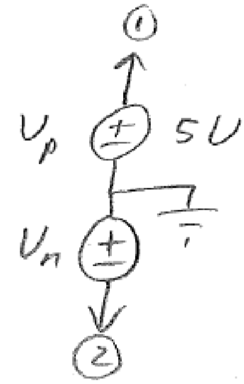
Consider the circuit below, where R_1 below is R_p in the previous circuit:



Here, $R_n = -\frac{R_2 R_4}{R_3}$

The circuit should begin to oscillate for: $R_1 > |-\frac{R_2 R_4}{R_3}|$





Let $L_1 = 1mH$ and $C_1 = 1\mu F$

$$f = \frac{1}{2\pi\sqrt{LC}} = 5033\ Hz$$

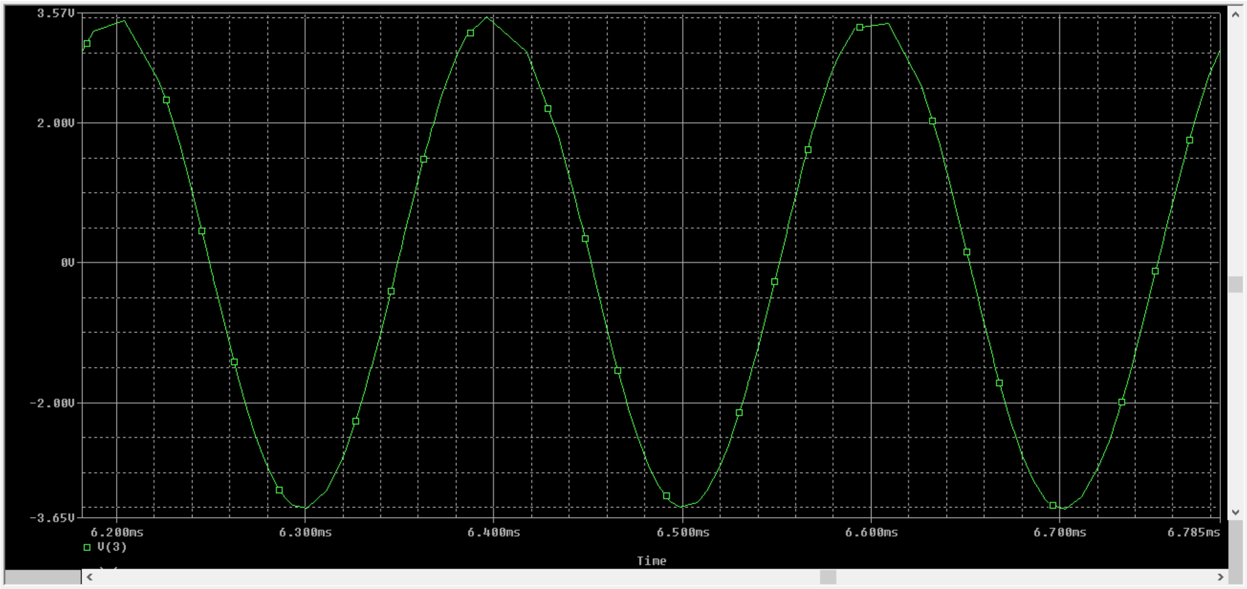
Let $R_1 = 10k\Omega$

Let $R_2 = R_3 = R_4 = 1k\Omega$

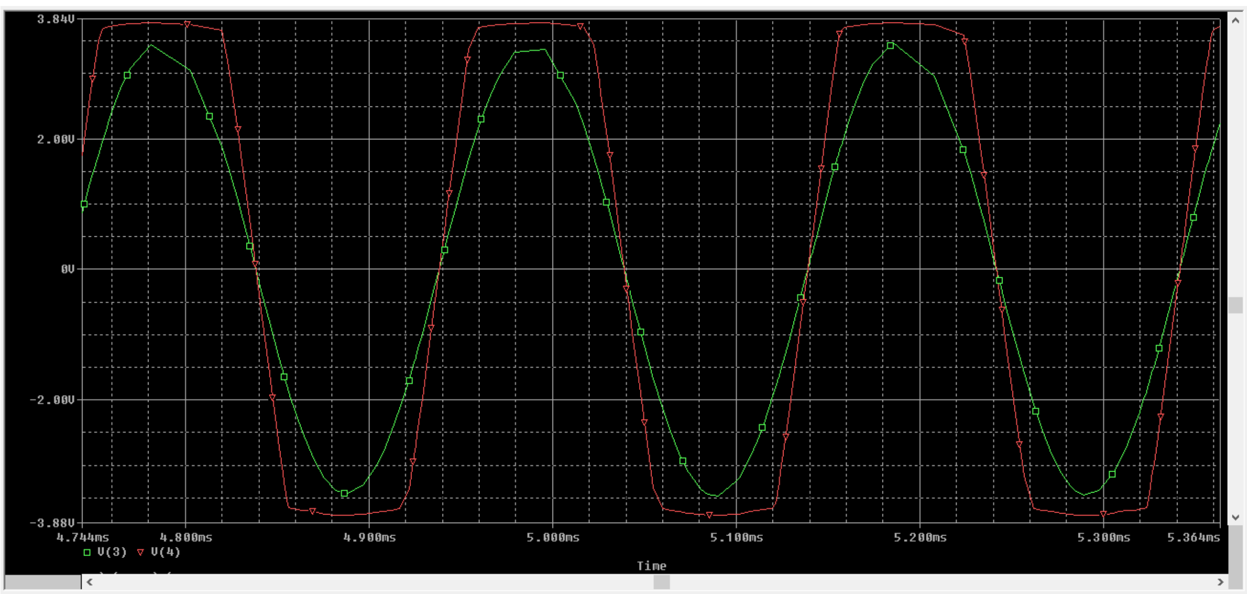
$$\therefore\ R_n = -\frac{R_2 R_4}{R_3} = -\frac{1k(1k)}{1k} = -1k\Omega$$

$R_1 > |R_n| \rightarrow$ it should oscillate

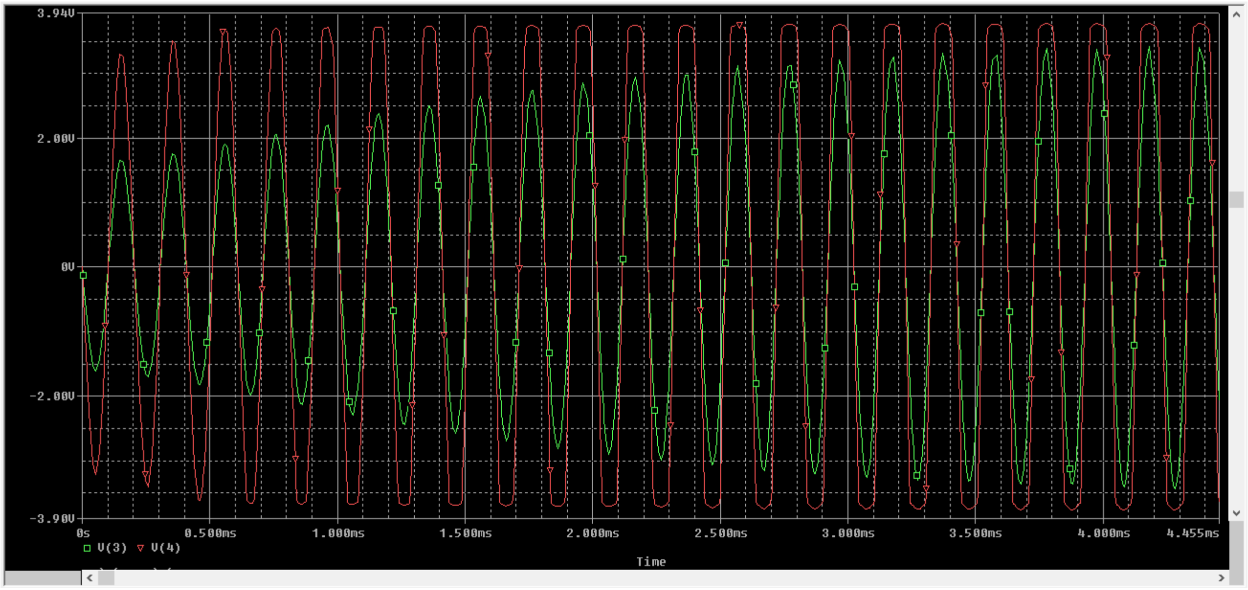
Simulated in PSpice:



V(3) in steady state. Measured frequency: 4992.5 Hz.

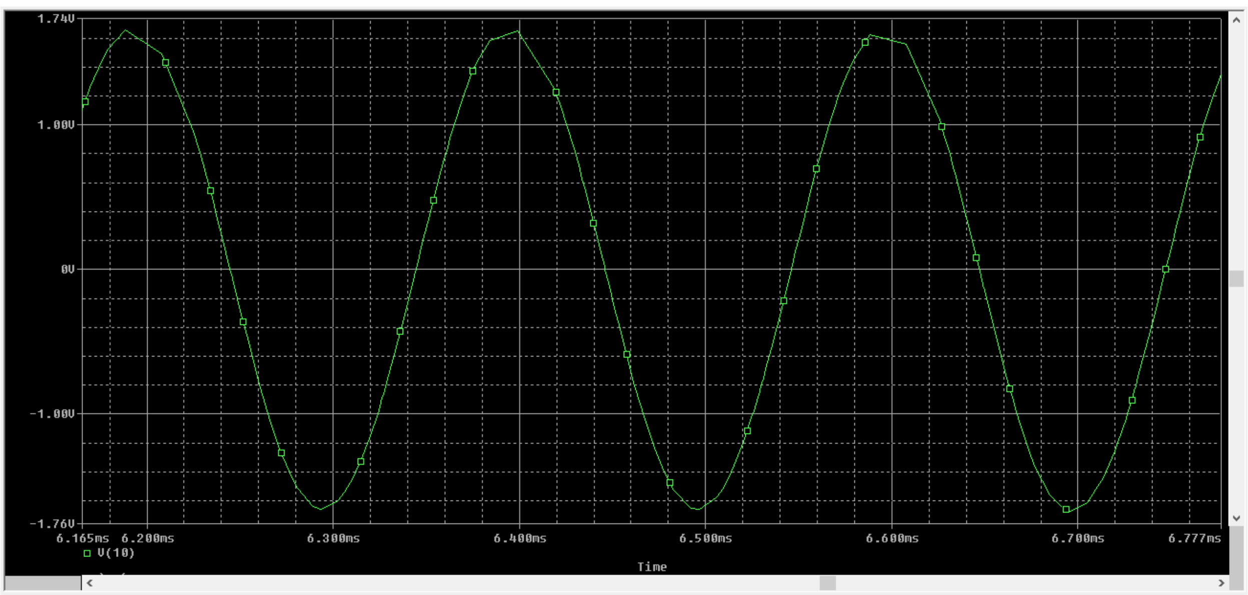


V(3) and V(4). Saturation of V(4) is due to gain roll off in the op amp when this signal nears the power supply rails. This nonlinearity reduces the negative damping back to zero damping, limiting the amplitude of the voltage in the tank circuit, V(3).



V(3) and V(4) from startup to op amp saturation.

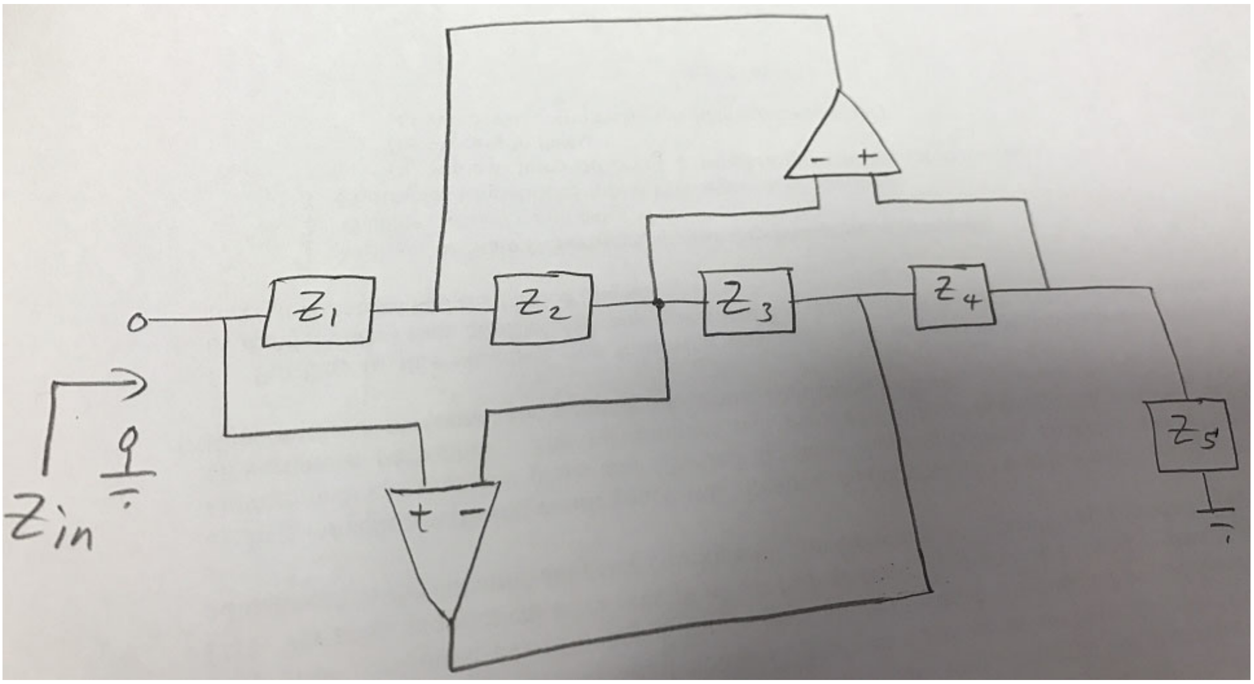
Is an AGC necessary in this circuit? Why not just add a 10 kΩ pot in parallel with the LCR tank, and pull off the required amplitude through the wiper:



Voltage output at the 10 kΩ pot wiper, with 5 kΩ on each side. It will slightly affect V(3) compared to not having the pot in the circuit. Note: add a buffer if it will be driving a load.

What about the inductor? Can we eliminate it in low frequency oscillators? Inductors for low frequency applications tend to be large, heavy, lossy, and have significant self-capacitance between windings.

Consider the circuit below:



It is a GIC (Generalized Impedance Converter), where:

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

Consider Z_2 being a capacitor (C_2), and the other Z's being resistors:

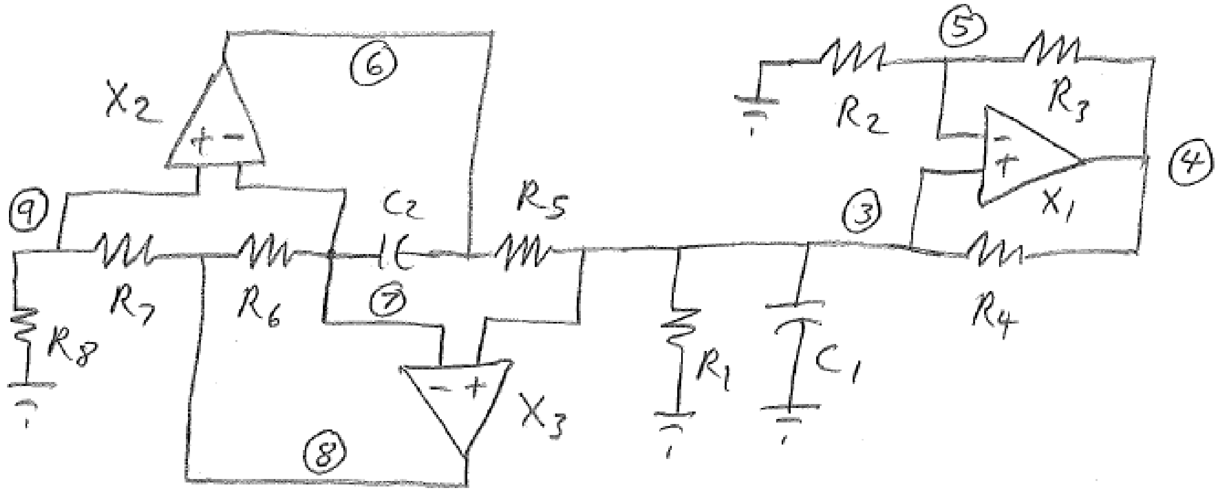
$$Z_{in} = \frac{sC_2 R_1 R_3 R_5}{R_4}$$

Z_{in} is now effectively an inductor!

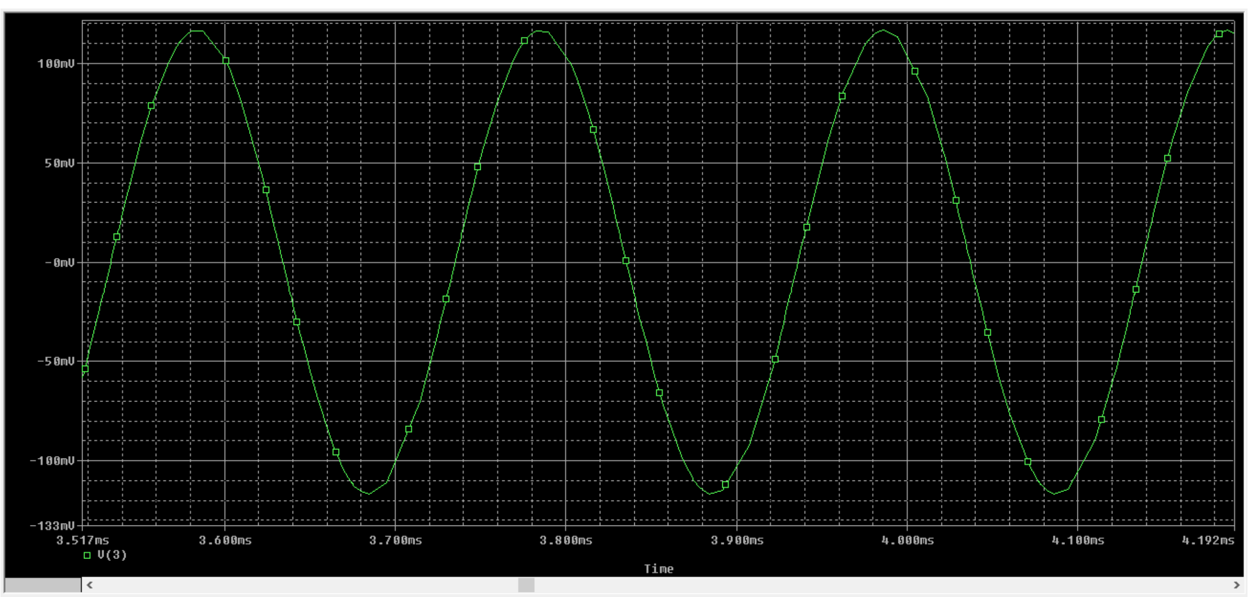
Let C_1 = 10 nF, R_1=1 kΩ, R_3=2 kΩ, R_4=100 kΩ, and R_5=5 kΩ

Then Z_{in}=1 mH.

The GIC replaced the inductor, using the AD8610 op amp for all 3 op amps:



R_1, C_1, R_2, R_3 and R_4 the same is before.



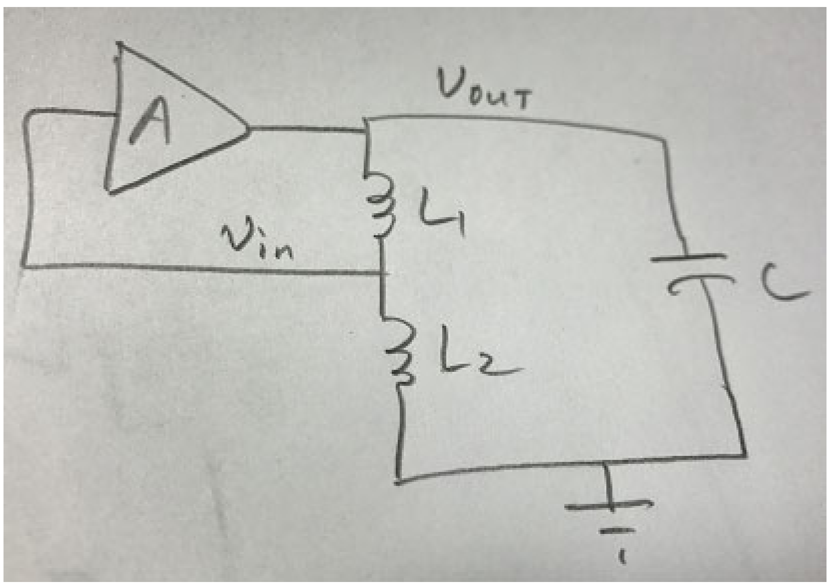
V(3). 4980 Hz measured frequency. 116.59 mV amplitude. Small amplitude may be due to the op amps used to make the GIC inductor. An amplifier can be used to amplify the output sinusoid.

6) Classic LC Oscillators

a. Hartley Oscillator

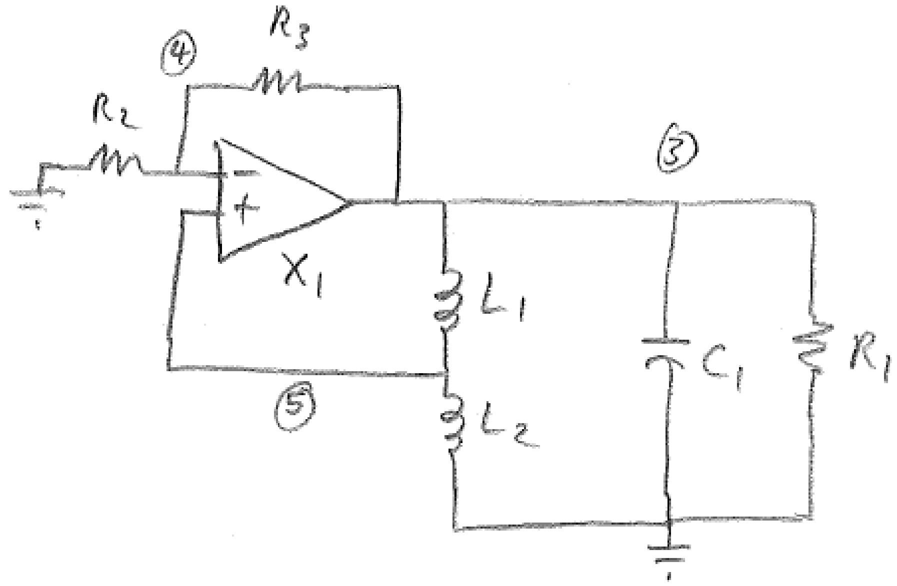
Invented by Ralph Hartley (11/30/1888 - 5/1/1970) in 1915. He was a U.S. engineer working for the Research Laboratory of Western Electric Company.

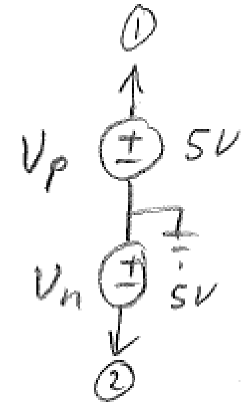
The circuit is a tank circuit consisting of 2 series inductors in parallel with a capacitor. The inductors serve as a voltage divider. An amplifier inputs the voltage from the inductor voltage divider, amplifies it, and feeds the amplified signal back to the top of the tank circuit, in phase with the tank circuit voltage at that point.



$$f = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

Consider the circuit below:

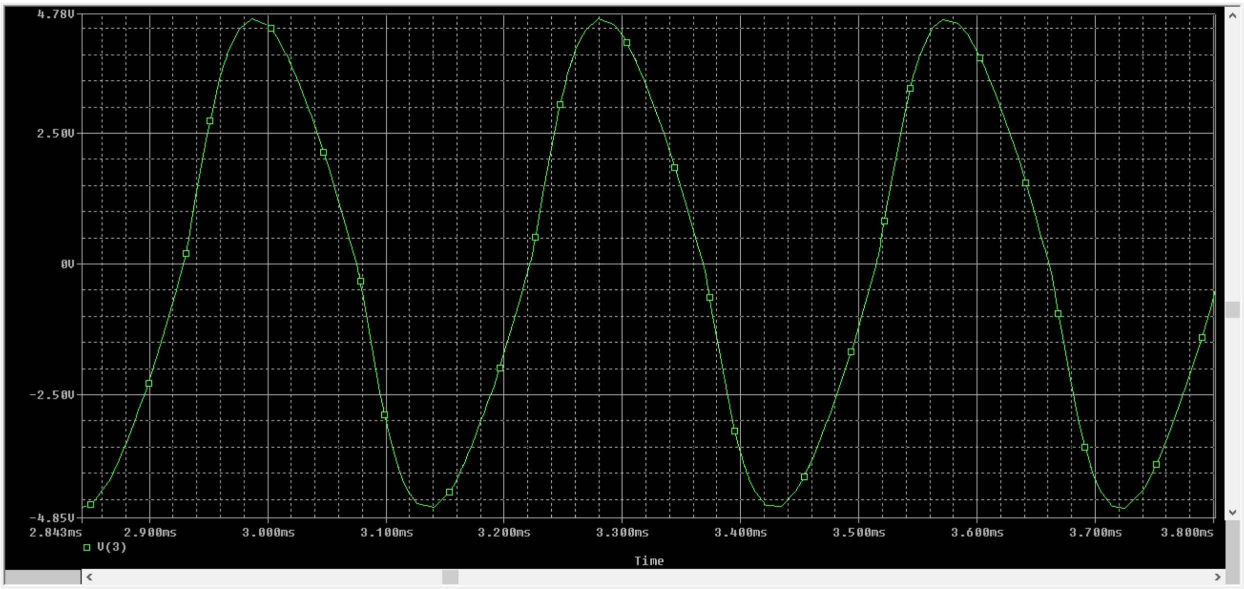




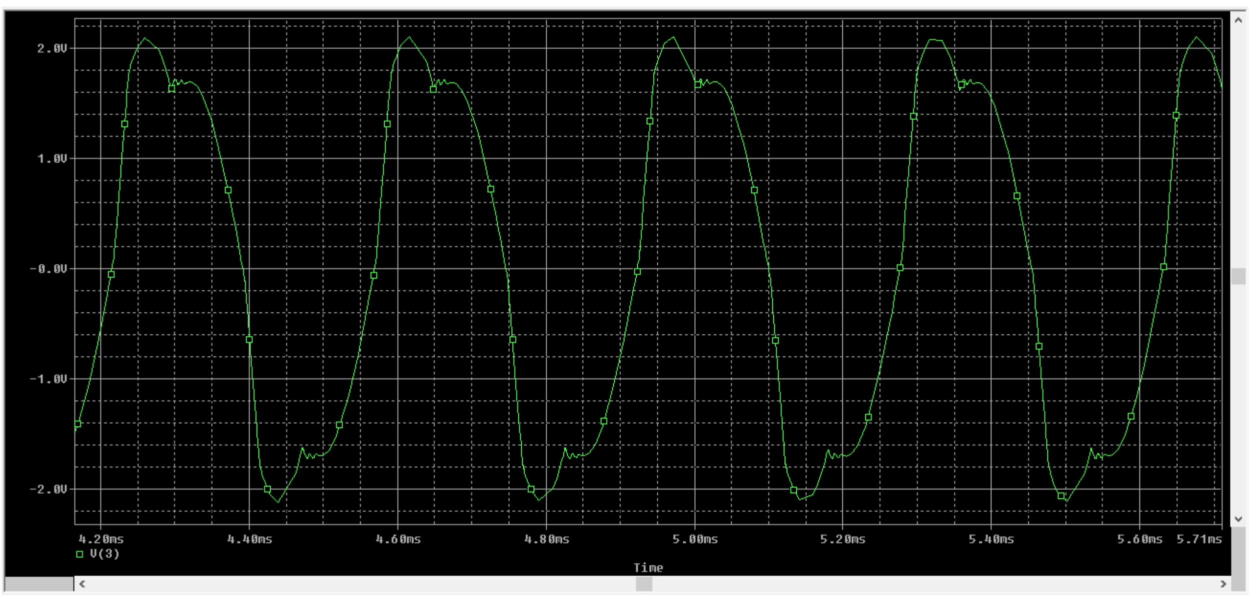
X1: AD8610 op amp
C1: 1μF
L1, L2: 1mH
R1: 10 kΩ
R2: 1 kΩ
R3: 1.5 kΩ

Calculated oscillation frequency: 3558.8 Hz
Simulated in PSpice

R3 replaced with a 2 kΩ pot and 2 Schottky diodes between the wiper and node 3, to realize a simple AGC.



V(3): slightly distorted sinewave (without AGC). Oscillation frequency: 3423 Hz

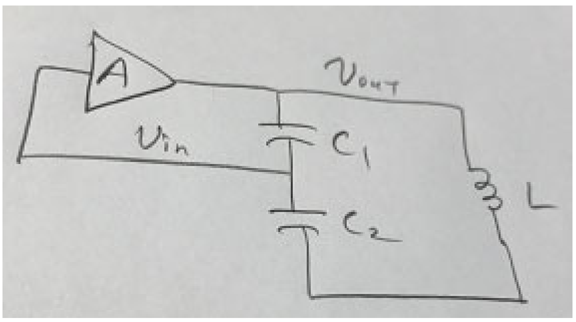


V(3) with the 2 kΩ pot and Schottky diodes (1.75 kΩ between diodes). Amplitude is reduced, but signal is highly distorted. Oscillation frequency approximately 2847 Hz.

b. <u>Colpitts Oscillator</u>

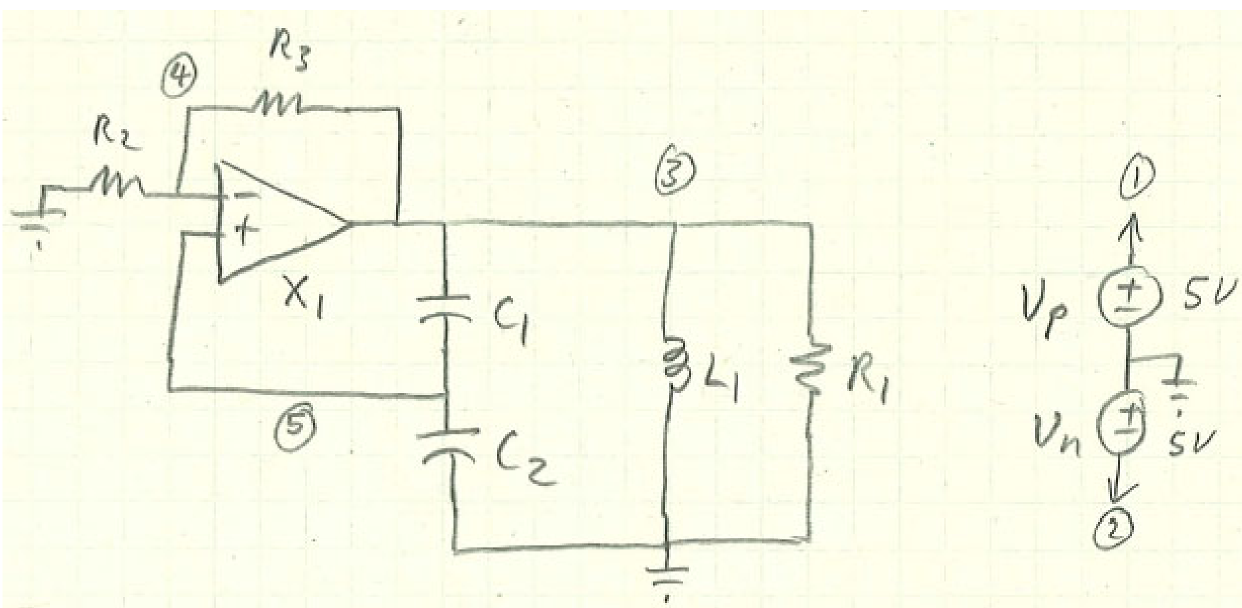
Invented by U.S. engineer, Edwin Colpitts (1/19/1872 – 3/6/1949) in 1918.

It is the electrical dual of the Hartley oscillator, with 2 capacitors in series, in parallel with an inductor:

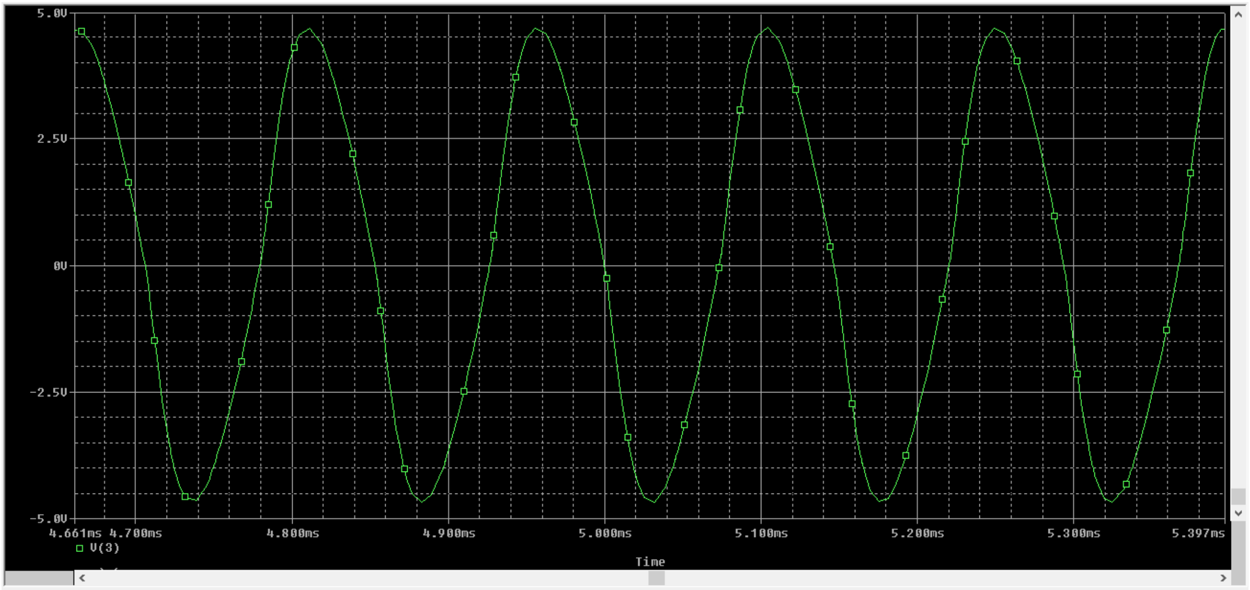


$$f = \frac{1}{2\pi\sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

Op amp version:



X1: AD8610 op amp
C1, C2: 1μF; L1: 1mH; R1: 10 kΩ; R2: 1 kΩ; R3: 1.5 kΩ
Calculated oscillation frequency: 7117.6 Hz



V(3) from PSpice simulation. Oscillation frequency: 6826 Hz. Slightly distorted sinewave.